

RESEARCH ARTICLE



# Parametric Analysis of Heat and Mass Transfer in Nanofluid Flow Through a Porous Channel with Brownian Motion and Thermophoresis Effects

Ghulam Rasool<sup>1</sup>, Saeed Islam<sup>1</sup>, Sajjad Hussain<sup>2</sup> and Tao Sun<sup>3</sup>

## **Abstract**

This study investigates the combined effects medium properties, of porous nanoparticle dynamics, and fluid characteristics on heat and mass transfer in nanofluid flow through a channel bounded by permeable walls. A comprehensive mathematical model is developed incorporating the Brinkman-Darcy momentum equation, energy and nanoparticle concentration equations, and key nanofluid transport mechanisms such as Brownian motion and thermophoresis. resulting nonlinear boundary value problem is solved numerically using a robust BVP4C approach in MATLAB. Parametric analyses are conducted to assess the influence of the Schmidt number (Sc), porosity parameter  $(\lambda)$ , Darcy number (Dc), Prandtl number (Pr), and Brownian motion and thermophoresis parameters (Nb and Nt), on velocity, temperature, and concentration distributions. Results reveal that higher Sc and  $\lambda$  suppress both velocity and temperature fields, while increasing Dc, Pr, and nanoparticle activity (Nb, Nt) enhance thermal and solutal transport. The findings provide valuable insights for optimizing nanofluid-based thermal systems and designing advanced porous channel configurations for improved heat and mass transfer performance.

**Keywords**: nanofluid, porous channel, brownian motion, thermophoresis, heat and mass transfer.

# 1 Introduction

In recent years, the study of nanofluids in porous media has attracted significant attention due to their enhanced heat and mass transfer capabilities and their potential in energy and thermal management systems. Numerous investigations have addressed nanofluid preparation to collectively emphasize the sensitivity of nanofluid transport to fluid properties, nanoparticle dynamics, and medium structure, laying a strong foundation for advanced modeling and optimization



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in engineering applications. Turkyilmazoglu [1] derived multiple exact solutions for free convection flows in saturated porous media under variable heat flux conditions. His work provides analytical insights into how variable surface heating significantly influences thermal boundary layer development in porous structures. Mahbubul [2] provides a comprehensive overview of nanofluid preparation techniques, including both single-step and two-step The chapter elaborates on key factors influencing nanofluid stability, such as particle size, surfactant use, and sonication parameters. These insights serve as a fundamental basis for developing stable and homogeneous nanofluids for enhanced heat transfer applications. Feng et al. [3] developed a comprehensive thermal property model for nanofluids and applied it to analyze flow and heat transfer between parallel disks. Their study demonstrates that nanofluid thermophysical properties significantly affect velocity and temperature distributions, offering valuable insights into the design of advanced cooling systems using nanofluids. Turkyilmazoglu [4] investigated unsteady magnetohydrodynamic flow and coupled heat and mass transfer over a porous rotating disk subjected to a uniform outer radial flow. The study highlighted the significant influence of magnetic field strength and disk rotation on thermal and solutal boundary layer characteristics. Sohail et al. [31] analyzed entropy generation in a hyperbolic tangent nanofluid containing motile gyrotactic microorganisms under modified Darcy-Forchheimer characteristics with ion-slip effects. Their study revealed that ion-slip influx and porous resistance significantly influence irreversibility and transport behavior in peristaltic flows. Naseem et al. [6] investigated axisymmetric flow and heat transfer in a TiO<sub>2</sub>/H<sub>2</sub>O nanofluid over a porous stretching sheet with slip boundary conditions using a reliable computational strategy. Their findings indicated that slip effects and nanoparticle concentration strongly enhance thermal performance in porous Goyal et al. [7] examined the stretching flows. pulsatile flow of Casson hybrid nanofluids in an inclined channel, considering the effects of temperature-dependent viscosity. Their findings highlight the enhanced heat and momentum transfer characteristics of ternary-hybrid nanofluids compared to conventional nanofluids, providing new perspectives for thermal management in engineering applications. Turkyilmazoglu [8] proposed a two-parameter family of basic states in porous media that leads to Darcy-Bénard convection. The analysis

provides deeper understanding of stability thresholds and flow patterns in thermally driven porous systems. Rasool et al. [9] investigated the entropy generation and binary chemical reaction effects in magnetohydrodynamic (MHD) Darcy-Forchheimer Williamson nanofluid flow over a non-linearly stretching surface. Their analysis reveals the significant influence of magnetic field, porous medium parameters, and chemical reaction rates on entropy generation, offering deeper insights into the thermodynamic optimization of nanofluid-based systems. Hayat et al. [10] explored the hydromagnetic boundary layer flow of Williamson fluids, accounting for the combined effects of thermal radiation and Ohmic dissipation. Their study demonstrates that magnetic and radiative parameters substantially alter velocity and temperature profiles, providing important guidelines for controlling heat transfer in electrically conducting non-Newtonian fluids. Turkyilmazoglu et al. [36] examined viscous flow through a porous-walled pipe under asymptotic magnetohydrodynamic (MHD) effects. Their results showed that strong magnetic fields and wall permeability significantly modify the velocity field and can be used to control transport phenomena in porous channels. Some furthers studies that authors may be interested to read in [11–13, 15, 16, 37].

Nadeem et al. [17] analyzed the influence of magnetohydrodynamics (MHD) on nanofluid flow with variable viscosity in porous media. Their work highlights the critical role of magnetic fields and viscosity variations in modulating flow behavior and heat transfer, thereby advancing the understanding of nanofluid transport phenomena in complex porous Nadeem et al. [17] examined the effects of magnetohydrodynamics (MHD) on a modified nanofluid model with variable viscosity in a porous medium. Their study reveals that both the magnetic field and viscosity variation have a pronounced impact on flow structure and thermal performance, providing valuable insights for optimizing nanofluid-based porous media systems. Sheremet et al. [18] investigated free convection in a square cavity filled with a porous medium saturated by nanofluid, employing the Tiwari and Das model. Their results highlight the influence of nanoparticle volume fraction and porous medium properties on heat transfer enhancement, offering new perspectives for optimizing thermal systems involving nanofluids in porous structures. Kumar Pundir et al. [19] studied double diffusive convection in a saturated rotating

couple-stress nanofluid layer embedded in a porous medium. Their findings demonstrate the significant effects of rotation, couple-stress, and nanoparticle concentration on convective stability and transport phenomena, providing valuable insights into complex nanofluid behavior in porous environments. Kumar Pundir et al. [19] explored double diffusive convection in a rotating couple-stress nanofluid layer saturated with a porous medium. Their analysis emphasizes how rotation, couple-stress effects, and nanoparticle concentration collectively impact convective stability and enhance mass and heat transfer characteristics in porous systems [20, 21].

The referenced volume [22] presents foundational concepts of mass and heat transfer, offering a rigorous treatment of fundamental principles and transport phenomena. The text serves as a key resource for understanding analytical and practical approaches to mass and heat transfer in engineering systems. Panigrahi et al. [23] investigated magnetohydrodynamic (MHD) Casson nanofluid flow and heat transfer over a linear stretching sheet embedded in a porous medium. Their results reveal that magnetic field strength, porosity, and Casson fluid parameters substantially influence velocity and temperature profiles, thus offering key insights for the thermal management of non-Newtonian nanofluid systems. Maiti and Maiti [24] analyzed mixed convective nanofluid flow and heat transfer generated by a stretchable rotating disk in a porous medium. Their study highlights the combined effects of disk rotation, stretching, and porous structure on enhancing heat and mass transfer, providing valuable guidance for the design of advanced thermal systems using nanofluids. Imtiaz et al. [25] investigated the convective heat transfer of hybrid nanofluids adjacent to a tilted hemisphere in the atmosphere, with relevance to climate change mitigation. Their results demonstrate that hybrid nanofluids can significantly enhance thermal transport, providing new avenues for environmental temperature regulation in atmospheric systems. Rauf et al. [26] analyzed MHD mixed convection flow of Maxwell hybrid nanofluids, incorporating Soret, Dufour, and morphology effects. The study highlights the pronounced influence of magnetic fields and coupled mass-heat diffusion phenomena on flow stability and heat transfer enhancement in advanced fluid systems. Waseem et al. [27] performed an entropy generation analysis of magnetohydrodynamic hybrid nanofluid flow using OHAM, considering viscous dissipation and thermal

radiation effects. The study showed that increasing magnetic field intensity and radiative heat enhance irreversibility, while hybrid nanoparticles improve thermal transport efficiency. Alhazmi et al. [28] investigated the influence of modified heat and mass fluxes on thermal and solutal transport in Williamson Their results highlighted that modified flux boundary conditions significantly enhance heat and mass transfer rates in non-Newtonian fluid flows. Wang et al. [29] employed the Crank-Nicolson numerical technique to study thermal enhancement in 3D convective Walter-B fluid flow around a circular cylinder with dispersed nanoparticles. Their results demonstrated that nanoparticle insertion significantly improves heat transfer characteristics in viscoelastic Algehyne et al. [33] conducted a fluid systems. computational study on the effects of an aligned magnetic field in a plume generated by an energy line source. Their findings elucidate the interplay between magnetic field orientation and plume dynamics, offering valuable insights for controlled heat transfer and fluid flow management [34, 35].

Sohail et al. [31] investigated thermally radiated boundary layer flow over a stretched sheet by incorporating modified heat and mass fluxes using OHAM analysis. Their findings revealed that thermal radiation and altered flux conditions strongly influence temperature and concentration gradients in boundary layer flows. Sohail et al. [31] analyzed the boundary layer flow of Prandtl liquid considering heat generation and thermal radiation effects under the Cattaneo-Christov double diffusion framework. The study demonstrated that relaxation time parameters and radiative heat significantly modify thermal and solutal boundary layer thicknesses. Enamul et al. [32] explored magnetohydrodynamic Darcy-Forchheimer flow of a non-Newtonian second-grade hybrid nanofluid between double-revolving disks with variable thermal conductivity. Their entropy generation analysis revealed that magnetic field strength, Forchheimer resistance, and thermal conductivity variations strongly affect heat transfer and irreversibility rates. Khan et al. [37] emphasized the role of thermo-fluid sciences in achieving sustainability, providing a conceptual roadmap for integrating thermal systems with resilient energy solutions.

Imran et al. [38] studied peristaltic motion under modified Darcy's principle with a Hartmann boundary layer, revealing the significant influence of magnetic fields and porous resistance on heat



transfer enhancement. Ijaz et al. [39] used homotopic computations to explore non-Darcian modified Eyring-Powell incompressible liquid flow over a thickened surface, demonstrating strong effects of surface thickening and fluid elasticity on flow resistance. Sarafraz et al. [40] developed a novel catalytic model for hydrogen production in a micro-reactor via methane and water vapor conversion, optimizing reaction efficiency for sustainable fuel generation. Dave et al. [41] experimentally and economically analyzed a humidifier-dehumidifier desalination system enhanced by an absorption refrigeration cycle, showing improved water production in humid climates. Gabayan et al. [42] investigated functionalized nano-graphene oxide using density functional theory, showing its potential to improve biodiesel production efficiency from jatropha oil. Mujtaba et al. [43] integrated enzyme-mimicking nanomaterials with analytical techniques to fabricate microfluidic systems, advancing biomedical diagnostic applications. Rumon et al. [44] synthesized antibacterial and biocompatible polymer nanocomposites based on polysaccharide gum hydrogels, highlighting their suitability for biomedical and tissue engineering applications.

While substantial progress has been made in understanding nanofluid transport in porous and non-porous channels, the combined influences of porous medium resistance, Brownian motion, and thermophoresis on heat and mass transfer remain less explored. Prior research highlights the importance of key dimensionless parameters, such as Darcy, Prandtl, Schmidt, Brownian motion, and thermophoresis numbers, in shaping the velocity, temperature, and concentration fields. However, further numerical and parametric analyses are essential to fully elucidate the interplay between porous media characteristics and nanoparticle-driven transport phenomena for the development of efficient thermal systems.

# 2 Novelty and Significance

The present model extends earlier nanofluid flow analyses in porous and non-porous channels by combining the Darcy–Brinkman momentum formulation with Brownian and thermophoretic energy interactions. In contrast to classical nanofluid studies, which either neglected porous resistance or treated it under simplified Darcy assumptions, the current formulation explicitly incorporates the porous drag term  $-\mu_{nf}u/K$  in the momentum equation. This term represents the resistive force exerted by the solid

matrix of the porous medium, which dampens the fluid motion as the permeability K decreases.

In the non-dimensional form, the porous effects are captured by the Darcy number

$$D_c = \frac{K}{H^2}$$

and the porosity parameter  $\lambda$ . Physically, larger values of  $\lambda$  enhance the solid matrix resistance, thickening the hydrodynamic boundary layer and reducing fluid velocity, whereas higher  $D_c$  increases the effective permeability, lowering the drag and promoting momentum and heat transfer.

Additionally, the present model uniquely investigates the coupled influence of  $\lambda$ ,  $D_c$ , and nanoparticle transport parameters (Nb, Nt) on velocity, temperature, and concentration fields, which has not been comprehensively addressed in earlier porous-channel nanofluid studies. The integration of Brownian motion and thermophoretic diffusion terms,  $Nb\,\theta'\phi'$  and  $Nt\,(\theta')^2$ , within the energy equation further differentiates this work by accounting for cross-diffusion heat transfer effects inside porous structures. This comprehensive approach provides deeper physical insight and offers practical guidelines for optimizing nanofluid-based thermal systems employing porous channels.

# 3 Problem Formulation

The present work addresses the parametric analysis of nanofluid flow and transport in a channel bounded by permeable walls and saturated with a porous medium, accounting for both Brownian motion and thermophoretic diffusion of nanoparticles. A coupled system of nonlinear partial differential equations, based on the Brinkman–Darcy momentum model and augmented by energy and nanoparticle concentration equations, is formulated to describe the combined influence of porous medium properties, fluid characteristics, and nanoparticle dynamics. Appropriate similarity transformations are applied to reduce the governing equations to an ordinary differential equation system, which is then solved numerically to quantify the effects of key controlling parameters on flow, heat, and mass transfer profiles. Turkyilmazoglu [8] analyzed Darcy-Bénard convection in porous media by introducing a two-parameter family of basic states. His findings emphasize the sensitivity of thermal instability thresholds to porous resistance, which is directly relevant to our Brinkman-Darcy formulation.

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Turkyilmazoglu et al. [36] investigated viscous flow in a porous-walled configuration under asymptotic MHD effects. Their study highlights how porous drag and wall permeability influence flow resistance, providing a physical basis for porous-channel nanofluid models.

# 3.1 Novelty of the Present Model

The present study introduces several key features that differentiate it from earlier works on nanofluid transport in porous media. The following aspects highlight the novelty of the proposed model:

- 1. **Darcy–Brinkman Porous Formulation:** The inclusion of both the Brinkman viscous shear term and the Darcy drag resistance term  $-\mu_{nf}u/K$  in the momentum equation allows simultaneous consideration of viscous and porous medium effects, which are often treated separately in earlier studies.
- 2. **Explicit Treatment of Porous Parameters:** The parametric investigation of the **Darcy number**

$$D_c = \frac{K}{H^2}$$

and the **porosity parameter**  $\lambda$  enables detailed analysis of how permeability and solid matrix resistance influence velocity, temperature, and concentration fields. Most earlier works assumed a constant or weakly varying porous resistance.

 Coupled Brownian and Thermophoretic Energy Interactions: The energy equation incorporates cross-diffusion heat transfer effects through the terms

$$Nb \theta' \phi'$$
 and  $Nt (\theta')^2$ ,

accounting for nanoparticle migration-driven heat transport inside the porous channel, which has rarely been investigated in a Brinkman–Darcy framework.

- 4. Simultaneous Parametric Analysis of Nanoparticle Transport and Porous Effects: Unlike many earlier works that examined either nanofluid effects or porous media resistance separately, the present model systematically explores the combined influence of (Nb, Nt),  $D_c$ , and  $\lambda$  on heat and mass transfer.
- 5. Engineering-Relevant Channel Configuration: The channel bounded by permeable walls with suction/injection boundary conditions more realistically represents practical applications such as thermal energy storage devices, filtration

systems, and heat exchangers using porous nanofluid media.

These features collectively provide a more comprehensive understanding of nanofluid transport in porous channels and offer practical guidelines for optimizing thermal system design.

# 3.2 Governing Equations

Consider steady, incompressible, two-dimensional flow of a nanofluid between two parallel, permeable plates at y=0 and y=H, in a porous medium. Let u and v be the velocity components in x and y directions.

**Continuity:** 

$$\nabla \cdot \mathbf{V} = 0 \tag{1}$$

Momentum (Darcy-Brinkman):

$$\rho_{nf}\left(\mathbf{V}\cdot\nabla\mathbf{V}\right) = -\nabla p + \mu_{nf}\nabla^{2}\mathbf{V} - \frac{\mu_{nf}}{K}\mathbf{V}$$
 (2)

Energy (with Brownian motion and thermophoresis):

$$\rho_{nf}c_{p,nf}\left(\mathbf{V}\cdot\nabla T\right) = k_{nf}\nabla^{2}T + \tau \left[D_{B}\left(\nabla C\cdot\nabla T\right) + \frac{D_{T}}{T_{\infty}}\left(\nabla T\cdot\nabla T\right)\right]$$
(3)

Nanoparticle concentration:

$$\mathbf{V} \cdot \nabla C = D_B \nabla^2 C + \frac{D_T}{T_\infty} \nabla^2 T \tag{4}$$

where:

V = (u, v) is the velocity vector.

such that,

Continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{5}$$

Momentum (Darcy-Brinkman):

$$\rho_{nf}\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu_{nf}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{\mu_{nf}}{K}u$$
(6)

$$\rho_{nf} \left( u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu_{nf} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \frac{\mu_{nf}}{K} v$$
(7)



# **Energy (with Brownian and Thermophoresis):**

$$\rho_{nf}c_{p,nf}\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k_{nf}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right]$$
(8)

# Nanoparticle concentration:

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) + \frac{D_T}{T_\infty} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \quad f(0) = S_0, \quad f'(0) = U_0, \quad \theta(0) = 1, \quad \phi(0) = 1 \quad (15)$$

$$(9) \quad f(1) = S_H, \quad f'(1) = U_H, \quad \theta(1) = 0, \quad \phi(1) = 0 \quad (16)$$

where:

- u, v =velocity components,
- p = pressure,
- T = temperature,
- C = nanoparticle concentration,
- $\rho_{nf}, \mu_{nf}, c_{p,nf}, k_{nf} = \text{density, viscosity, specific}$ heat, and thermal conductivity of nanofluid,
- K = permeability of the porous medium,
- $D_B$  = Brownian diffusion coefficient,
- $D_T$  = thermophoretic diffusion coefficient,
- $T_{\infty}$  = reference temperature,
- $\tau$  = ratio of nanoparticle heat capacity to base fluid,

## 3.3 Boundary Conditions

At y = 0 (lower wall):

$$u = u_0, \quad v = v_0, \quad T = T_0, \quad C = C_0$$
 (10)

At y = H (upper wall):

$$u = u_H, \quad v = v_H, \quad T = T_H, \quad C = C_H$$
 (11)

where  $u_0, v_0, T_0, C_0, u_H, v_H, T_H, C_H$  are wall velocities, suction/injection, temperatures, and concentrations.

## 3.4 Similarity Transformation

Let

$$\eta = \frac{y}{H}, \quad \psi = xf(\eta), \quad u = xf'(\eta), \quad v = -f(\eta)$$
$$\theta(\eta) = \frac{T - T_H}{T_0 - T_H}, \quad \phi(\eta) = \frac{C - C_H}{C_0 - C_H}$$

#### **Reduced ODEs:**

$$(1 + \lambda) f^{iv} - \frac{1}{D_c} f'' + \text{Re} \left[ f f''' - f' f'' \right] = 0$$
 (12)

$$\theta'' + \Pr \cdot \operatorname{Re} f \theta' + Nb\theta' \phi' + Nt(\theta')^2 = 0 \tag{13}$$

$$\phi'' + \operatorname{Sc} \cdot \operatorname{Re} f \phi' + \frac{Nt}{Nb} \theta'' = 0 \tag{14}$$

## With boundary conditions:

$$f(0) = S_0, \quad f'(0) = U_0, \quad \theta(0) = 1, \quad \phi(0) = 1 \quad (15)$$
  
 $f(1) = S_H, \quad f'(1) = U_H, \quad \theta(1) = 0, \quad \phi(1) = 0 \quad (16)$ 

## Parameter definitions:

- $Re = \frac{\rho_{nf}u_0H}{\mu_{nf}}$  (Reynolds number)
- $Pr = \frac{\mu_{nf}c_{p,nf}}{k_{nf}}$  (Prandtl number)
- $Sc = \frac{\mu_{nf}}{\rho_{nf}D_B}$  (Schmidt number)
- $Nb = \frac{\tau D_B(C_0 C_H)}{\nu}$  (Brownian motion parameter)
- $Nt = \frac{ au D_T (T_0 T_H)}{
  u T_\infty}$  (Thermophoresis parameter)
- $D_c = \frac{K}{H^2}$  (Darcy number)

## 4 Results and Discussion

Figure 1 illustrates the effect of the Schmidt number (Sc) on the velocity profile  $f'(\eta)$  within the channel. As the Schmidt number increases from 0.25 to 1.25, a noticeable reduction in the magnitude of the velocity profile is observed, accompanied by a more flattened profile across the channel. This trend is attributed to the reduced mass diffusivity at higher Sc values, which suppresses solutal-driven momentum transport and thus restricts fluid motion. The resulting thicker boundary layer demonstrates the significant role of mass diffusivity in regulating flow characteristics in nanofluid systems. Overall, the findings highlight that controlling Sc can be an effective means of modulating flow and transport properties in engineering applications involving nanofluids.

Figure 2 presents the influence of the porosity parameter  $(\lambda)$  on the velocity profile  $f'(\eta)$  within the channel. It is evident that as  $\lambda$  increases from 0.25 to 1.25, the peak velocity in the center of the channel diminishes and the overall profile becomes less pronounced. This outcome is due to the enhanced resistance provided by the porous medium at higher  $\lambda$  values, which restricts fluid flow and suppresses

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momentum transfer. The increased porosity parameter effectively thickens the hydrodynamic boundary layer and leads to a more uniform but reduced velocity distribution. These observations highlight the critical role of porous media properties in modulating nanofluid transport and optimizing channel flow performance.

Figure 3 displays the effect of the Darcy number (Dc) on the velocity profile  $f'(\eta)$  in the channel. As Dc increases from 0.25 to 1.25, there is a marked rise in the velocity magnitude and the profile becomes more prominent. This behavior is associated with the enhanced permeability of the porous medium at higher Dc values, which reduces the drag force experienced by the fluid and facilitates greater momentum transfer. Consequently, the flow accelerates and the boundary layer near the walls becomes thinner. The results underscore the importance of the Darcy number in controlling flow dynamics in porous channels, particularly for optimizing nanofluid transport.

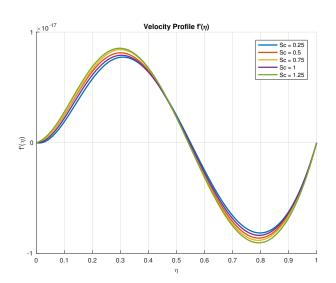
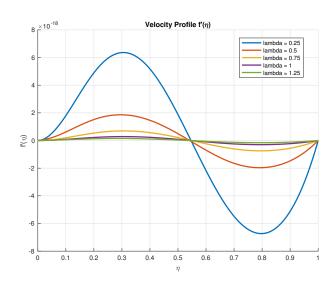
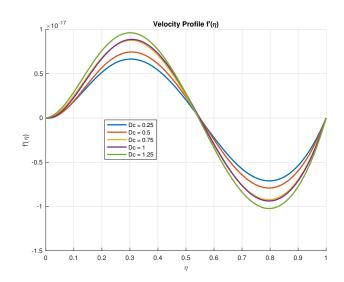


Figure 1. Effect of Schmidt number (Sc) on the velocity profile  $f'(\eta)$ . An increase in Sc reduces the velocity magnitude and leads to a flatter velocity distribution within the channel.

Figure 4 illustrates the impact of the Brownian motion parameter (Nb) on the velocity profile  $f'(\eta)$  within the channel. As Nb increases from 0.25 to 1.25, the velocity profile exhibits a noticeable reduction in magnitude, and the flow becomes more suppressed across the channel. This effect is attributed to the enhanced random motion of nanoparticles at higher Nb, which promotes additional momentum exchange and increases effective viscosity, thereby impeding the overall fluid motion. The trend reveals that stronger Brownian motion acts to diminish peak velocities and



**Figure 2.** Effect of porosity parameter  $(\lambda)$  on the velocity profile  $f'(\eta)$ . Increasing  $\lambda$  leads to greater flow resistance, resulting in a reduced and more uniform velocity distribution.



**Figure 3.** Effect of Darcy number (Dc) on the velocity profile  $f'(\eta)$ . Higher values of Dc enhance the permeability of the porous medium, resulting in increased velocity and a more pronounced profile.

contributes to a thicker hydrodynamic boundary layer. These findings highlight the key role of nanoparticle dynamics in shaping flow behavior in nanofluid systems.

Figure 5 demonstrates the influence of the thermophoresis parameter (Nt) on the velocity profile  $f'(\eta)$  within the channel. With increasing Nt from 0.25 to 1.25, there is a clear reduction in the velocity magnitude and a flattening of the profile. This trend arises because higher thermophoresis causes nanoparticles to migrate more strongly from



hot to cold regions, which disturbs the local flow and increases the overall resistance to fluid motion. Consequently, the momentum transport is hindered, resulting in lower velocities and a thicker velocity boundary layer. These results highlight the significant role of thermophoretic effects in modulating nanofluid flow in porous channels.

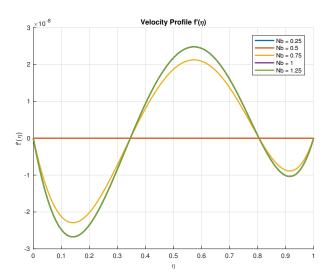


Figure 4. Effect of Brownian motion parameter (Nb) on the velocity profile  $f'(\eta)$ . Increasing Nb reduces velocity magnitude and thickens the boundary layer due to enhanced nanoparticle motion.

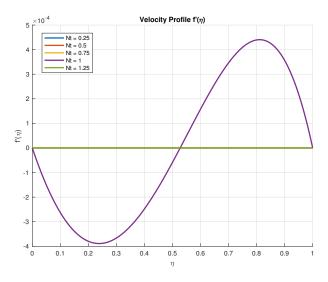
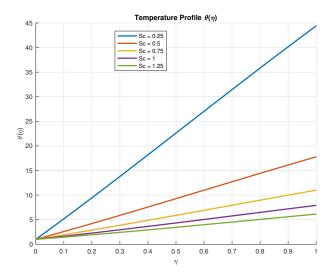


Figure 5. Effect of thermophoresis parameter (Nt) on the velocity profile  $f'(\eta)$ . Higher Nt values lead to reduced velocity and a more uniform velocity profile due to intensified nanoparticle migration.

Figure 6 illustrates the effect of the Schmidt number (Sc) on the temperature profile within the channel. As Sc increases from 0.25 to 1.25, the temperature throughout the domain decreases significantly. This behavior is due to reduced mass diffusivity at higher

Sc, which suppresses solutal effects and thus limits energy transfer associated with nanoparticle migration. Consequently, the temperature profile becomes more uniform, and the maximum temperature diminishes with increasing Sc. These results demonstrate the strong coupling between mass diffusivity and thermal transport in nanofluid systems.

Figure 7 presents the influence of the porosity parameter  $(\lambda)$  on the temperature profile in the channel. As  $\lambda$  increases from 0.25 to 1.25, the temperature profile decreases sharply, especially near the channel walls. This is attributed to the enhanced resistance offered by the porous medium at higher  $\lambda$  values, which restricts fluid flow and reduces convective heat transfer. As a result, the thermal boundary layer becomes thinner and the temperature gradients become steeper. These findings highlight the critical role of porous medium properties in regulating thermal transport in nanofluid flows.



**Figure 6.** Effect of Schmidt number (Sc) on the temperature profile. Higher Sc values reduce overall temperature and produce a more uniform temperature distribution across the channel.

Figure 8 shows the effect of the Darcy number (Dc) on the temperature profile. With an increase in Dc from 0.25 to 1.25, the temperature profile rises significantly, indicating that higher permeability in the porous medium enhances convective transport of heat. This results in higher temperature values across the channel and a broader thermal boundary layer. The results confirm that increasing the Darcy number facilitates heat transfer in nanofluid flows through porous structures.

Figure 9 demonstrates the variation in temperature



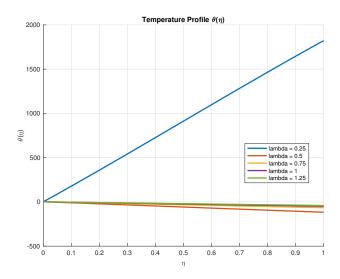
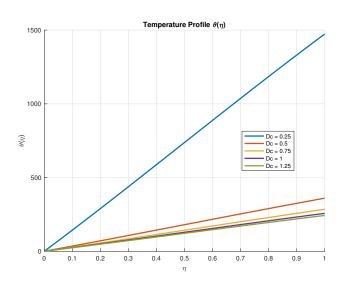


Figure 7. Influence of porosity parameter  $(\lambda)$  on the temperature profile. Increasing  $\lambda$  leads to lower temperatures and sharper thermal gradients near the walls.

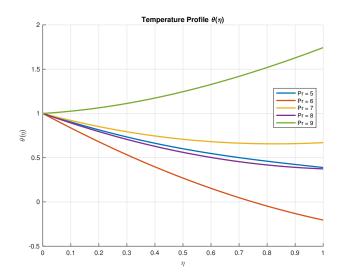


**Figure 8.** Effect of Darcy number (Dc) on the temperature profile. Higher Dc increases permeability and promotes higher temperatures and wider thermal boundary layers.

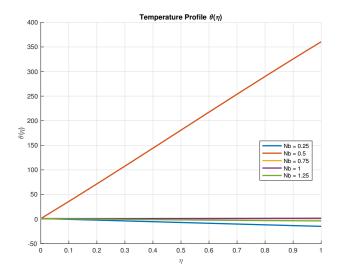
profile with respect to the Prandtl number (Pr). As Pr increases from 5 to 9, the temperature decreases more rapidly from the hot wall towards the channel center, resulting in a thinner thermal boundary layer. This is due to the lower thermal diffusivity at higher Pr, which limits the spread of heat and sharpens the temperature gradient. Such behavior is characteristic of fluids with high Prandtl numbers, where conduction dominates over convection in thermal transport.

Figure 10 depicts the effect of the Brownian motion parameter (Nb) on the temperature profile in the channel. As Nb increases from 0.25 to 1.25, the temperature within the channel rises, and the thermal

boundary layer becomes broader. This behavior results from enhanced nanoparticle diffusion due to Brownian motion, which promotes greater energy transport and raises the overall temperature. These results indicate that stronger Brownian motion can effectively increase heat transfer in nanofluid flows.



**Figure 9.** Variation of temperature profile with Prandtl number (Pr). Increasing Pr produces a thinner thermal boundary layer and sharper temperature gradients.



**Figure 10.** Effect of Brownian motion parameter (Nb) on the temperature profile. Higher Nb leads to greater energy transport, higher temperatures, and broader thermal boundary layers.

Figure 11 depicts the effect of the Schmidt number (Sc) on the concentration profile within the channel. As Sc increases from 0.25 to 1.25, the concentration profile becomes more negative and flatter, indicating a reduction in the mass diffusivity and enhanced suppression of solute transport. This trend suggests that higher Schmidt numbers effectively inhibit



nanoparticle dispersion, resulting in more uniform but diminished concentration gradients across the channel. The outcome underlines the importance of mass diffusivity in regulating solute transport in nanofluid flows.

Figure 12 illustrates the influence of the porosity parameter ( $\lambda$ ) on the concentration profile within the channel. With increasing  $\lambda$  from 0.25 to 1.25, the concentration profile shows a strong negative shift and a more uniform distribution. This pattern is caused by greater flow resistance and reduced convective mixing at higher porosity parameter values, which limits nanoparticle transport and results in lower concentration gradients. The findings emphasize the significant role of porous medium characteristics in shaping solute transport in nanofluids.

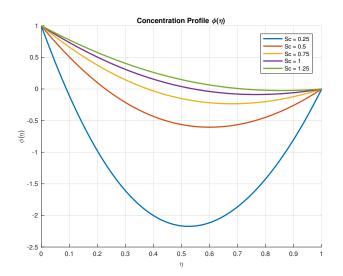
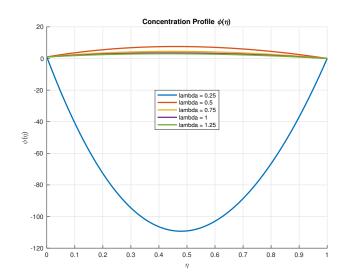


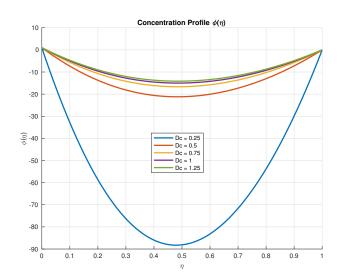
Figure 11. Effect of Schmidt number (Sc) on the concentration profile. Higher Sc values lead to flatter and more negative concentration profiles, highlighting reduced mass diffusivity.

Figure 13 presents the effect of the Darcy number (Dc) on the concentration profile in the channel. As Dc increases from 0.25 to 1.25, the concentration profile rises, reflecting enhanced permeability and convective transport within the porous medium. This effect facilitates greater nanoparticle movement and leads to increased concentration levels throughout the domain. The results confirm that higher Darcy numbers promote solute transfer in nanofluid channels.

Figure 14 displays the variation of the concentration profile with the Prandtl number (Pr). As Pr increases from 5 to 9, the concentration profiles become progressively higher and steeper, indicating



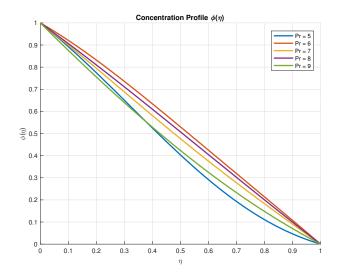
**Figure 12.** Influence of porosity parameter  $(\lambda)$  on the concentration profile. Increasing  $\lambda$  produces a more negative and uniform concentration field due to enhanced flow resistance.



**Figure 13.** Effect of Darcy number (Dc) on the concentration profile. Higher Dc enhances permeability and elevates concentration across the channel.

that greater Prandtl numbers intensify the coupling between thermal and solute transport. This trend signifies more efficient nanoparticle transfer in fluids with higher Pr, as reduced thermal diffusivity enhances concentration gradients. These results reveal the interplay between heat and mass transfer in nanofluids.

Figure 15 demonstrates the effect of the Brownian motion parameter (Nb) on the concentration profile within the channel. With an increase in Nb from 0.25 to 1.25, the concentration profile rises markedly, indicating that stronger Brownian diffusion enhances



**Figure 14.** Variation of concentration profile with Prandtl number (Pr). Higher Pr produces increased concentration and sharper gradients across the channel.

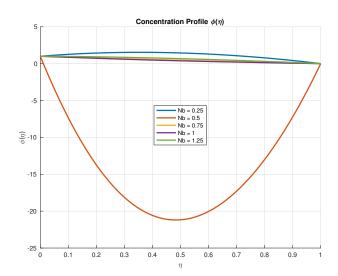


Figure 15. Effect of Brownian motion parameter (Nb) on the concentration profile. Higher Nb enhances nanoparticle dispersion, resulting in increased concentration throughout the channel.

nanoparticle dispersion and raises solute concentration throughout the domain. This behavior highlights the critical role of Brownian motion in augmenting mass transfer and shaping the concentration field in nanofluid flows.

#### 5 Conclusion

A detailed numerical investigation of nanofluid flow through a porous channel, incorporating Brownian motion and thermophoresis effects, has been presented. Based on the parametric analysis, the following key conclusions can be drawn:

- Increasing the Schmidt number (Sc) and porosity parameter  $(\lambda)$  significantly suppresses velocity and temperature fields due to reduced mass diffusivity and enhanced flow resistance.
- Higher Darcy numbers  $(D_c)$  enhance permeability, reduce drag resistance, and consequently increase fluid velocity, heat transfer, and solute transport across the channel.
- The Prandtl number (Pr) strongly influences the thermal boundary layer; higher Pr leads to steeper temperature gradients and thinner thermal layers.
- Brownian motion (Nb) and thermophoresis (Nt)
  parameters enhance thermal and solutal transport
  by promoting nanoparticle migration, with Nb
  showing a stronger impact on temperature rise
  and concentration enrichment.
- The combined effects of porous medium properties and nanoparticle dynamics provide effective means to control momentum, heat, and mass transfer in porous-channel configurations, which can be useful for designing nanofluid-based heat exchangers, filtration systems, and thermal energy storage devices.

# 6 Limitations of the Present Study

While the present analysis provides useful insights into nanofluid transport through porous channels, it is subject to the following limitations:

- The nanofluid is assumed to behave as a single-phase continuum with local thermal equilibrium; interfacial slip effects between nanoparticles and base fluid are neglected.
- The flow is considered steady, laminar, and incompressible, thus excluding transient, turbulent, or compressibility effects that may occur in practical applications.
- Non-Newtonian rheology of the base fluid is not considered; the analysis is restricted to Newtonian nanofluids.
- Thermal radiation, viscous dissipation, and chemical reactions are neglected, which may influence heat and mass transfer in high-temperature or reactive systems.
- The channel walls are assumed to be smooth and permeable with prescribed temperature and concentration, whereas surface roughness or



variable wall properties can alter flow behavior in real systems.

These assumptions provide a simplified framework for fundamental understanding but may limit the direct applicability to complex industrial or biological systems. Future work may relax these assumptions by incorporating unsteady, turbulent, or non-Newtonian effects and validating the model experimentally.

# **Data Availability Statement**

Data will be made available on request.

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#### **Conflicts of Interest**

The authors declare no conflicts of interest.

## **Ethical Approval and Consent to Participate**

Not applicable.

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