

RESEARCH ARTICLE



# Three-Dimensional Nanofluid Flow with Convective and Slip Condition with Thermal Radiation Effect via Stretching/Shrinking Surface

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## **Abstract**

A numerical technique for the nonlinear thermal radiation effect on 3D ("Three Dimensional") nanofluid (NFs) motion through shrinking or stretching surface with convective boundary condition is examined. In this investigation we use the convective and velocity slip conditions. The governing equations were converted into a set of couple non-linear ODE's by suitable similarity transformations. The converted nonlinear equations are obtained by applying R-K-F ("Range-Kutta-Fehlberg") procedure along with shooting technique. The physical parameters are explained graphically on velocity, temperature and concentration. Moreover, we found the coefficient of skin friction, rate of heat transfer with various nanofluid parameters. It is very good agreement when compared with previous study.



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\*Corresponding author: ☑ Nainaru Tarakaramu nainaru143@gmail.com **Keywords**: nanofluid, Magnetohydrodynamics, Shrinking/stretching surface, velocity slip, nonlinear thermal radiation.

#### 1 Introduction

In most of the investigation, developed the no-slip condition is established Kn ("Knudson Number"), but in some circumstances such as suspensions, emulsions, foams and polymer solutions [1–4], the no-slip condition is not acceptable for the slip flow range 0.01Kn0.1, still the basic equations of energy and Navier-Stokes can be used by taking into account velocity slip and temperature jump. It has a lot of importance in physics. The stagnation point flow of NFs via surface with different slip conditions examined by [5–8]. Wang et al. [9] introduce boundary layer slip with microscale gas motion. Magnetohydrodynamic (MHD) slip motion over a sheet/wall was investigated by Mahabaleshwar et al. [10] for hybrid nanofluid flow with Navier's slip, Aziz and Jamshed [11] for non-Newtonian nanofluid

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with temperature-dependent conductivity, and Abbas and Sheikh [12] for ferrofluid with nonlinear slip Nojoomizadeh et al. [13] studied permeability effects on slip velocity in microchannels, while Farooq et al. [14] explored second-order slip in Sisko nanomaterial flow. Abd El-Aziz and Afify [15] analyzed MHD stagnation-point flow with induced magnetic fields, and Majeed et al. [16] investigated nanoparticle magnetic properties under momentum slip in aqueous media. Nayak et al. [17] and Sharma et al. [18] examined MHD slip flow with porous media and heat transfer. Ahmad et al. [19] noticed that the solid volume fraction diminutions the velocity and enhances the temperature distribution. Three dimensional couple stress casson liquid motion via bidirectional surface was created Tarakaramu et al. [20]. Noor et al. [21] finding that wall shear stress and fluid velocity increase as the plates approaching each other.

The word NFs is created Choi [22, 23] which define the size of base fluids ("like water, oil kerosene and ethylene glycol"). These fluids have addition of NPs, the properties of the fluids can be rapidly enhanced. It has a lot of demand placed upon them in terms of growing or declining energy release to systems and their effects depend on thermal conductivity, heat capacity and other physical properties in manufacturing presses [24–26]. The heat and mass transfer on 3D ("Three-Dimensional") MHD ("Magnetohydrodynamic") motion of Maxwell NFs via stretching sheet was discover Sreedevi and Sudarsana Reddy [27]. Eid et al. [28] found that the increase of Fe<sub>3</sub>O<sub>4</sub> ("nanoparticle") NPs concentration enhances heat transfer rate of hybrid NFs in a shrinkable case and opposite happens in a stretchable state. Satya Narayana et al. [29] finding reviled that the thermal and species boundary layer thickness is enhanced due to rising values of thermal radiation.

Moreover, the boundary layer flow through a shrinking/stretching sheet has a motivate work in fluid dynamic. Some of the practical situations of shrinking sheets ("like shrinking spring, shrinking balloon, and shrinking plastic"). Which is very useful in packaging of bulk products. The shrinking/stretching sheet has many tremendous applications ("crystal growing, extraction of polymer and rubber sheet, like glass-fibre production, paper production, annealing and tinning of copper wires, wire drawing, metal and polymer processing and many others") in manufacturing industries and technological process. Jat et al. [30] developed the 3D boundary layer

flow due to shrining sheet. Recently, some of the scientists [31–34] working on stagnation point flow towards a shrinking/stretching sheet. 2D ("Two Dimensional") stagnation point flow of NFs via stretching sheet was presented Najib et al. [35]. Khan et al. [36] discussed analytical and numerical analysis for the viscous and heat transfer flow over a nonlinear stretching sheet. Murtaza et al. [37] explored the biomagnetic fluid flow and heat transfer in 3D unsteady shrinking/stretching sheet. Jusoh et al. [38] analysed the 3D rotating and heat transfer flow of ferrofluid over exponential permeable shrinking/stretching sheet with suction effect. 3D flow of nanofluid passing through an exponential stretching sheet. Further, the boundary layer flow via stretching or shrinking sheet was examined [39–46].

# 2 Mathematical Analysis

The nonlinear effect on 3D NFs motion over a shrinking or stretching sheet with velocity slip in the presence of magnetohydrodynamic is considered. The physical model of the coordinate system is explored in Figure 1. It is considered that the surface is stretched or shrink in the  $x^*y^*$  surface with velocity slip. The  $z^*$  direction is orthogonally to stretching  $b_1 > 0$  or shrinking surface  $b_1 < 0$ . The nanofluid flow occupies the region  $z^* > 0$ . The velocity components of the surface trough  $x^*$  and  $y^*$  directions are  $u_w^*(x) = a_q x^*$  and  $v_w^*(y^*) = b_1 y^*$ , respectively. The liquid is electrically conducting under the effect of uniform magnetic field  $B_0$ . Moreover, the velocity mass flux is  $w_1 = w_0$ , where  $w_0 < 0$  then it is called suction and  $w_0 > 0$  then it is known as injection. Under these considerations, the basic governing equations are:

 $\frac{\partial u_1^*}{\partial x^*} + \frac{\partial v_1^*}{\partial u^*} + \frac{\partial w_1^*}{\partial z^*} = 0$ 

$$u_{1}\frac{\partial u_{1}}{\partial x^{*}} + v_{1}\frac{\partial u_{1}}{\partial y^{*}} + w_{1}\frac{\partial u_{1}}{\partial z^{*}} = v_{1}\frac{\partial^{2} u_{1}}{\partial (z^{*})^{2}} - \frac{\sigma_{1}B_{0}^{2}}{\rho_{f}}u_{1} \quad (2)$$

$$u_{1}\frac{\partial v_{1}}{\partial x^{*}} + v_{1}\frac{\partial v_{1}}{\partial y^{*}} + w_{1}\frac{\partial v_{1}}{\partial z^{*}} = v_{1}\frac{\partial^{2} v_{1}}{\partial (z^{*})^{2}} - \frac{\sigma_{1}B_{0}^{2}}{\rho_{f}}v_{1} \quad (3)$$

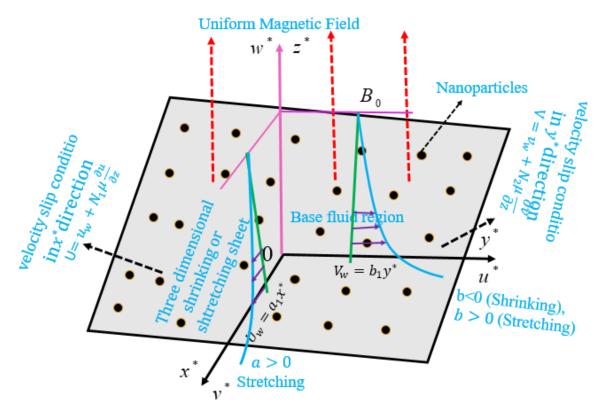
$$u_{1}\frac{\partial T^{*}}{\partial x^{*}} + v_{1}\frac{\partial T^{*}}{\partial y^{*}} + w_{1}\frac{\partial T^{*}}{\partial z^{*}} = \alpha_{m}\left(\frac{\partial^{2} T^{*}}{\partial (z^{*})^{2}}\right) - \frac{1}{(\rho c_{p})_{f}}\left(\frac{\partial q_{r}}{\partial z^{*}}\right)$$

$$+ \tau \left\{ D_{B}\left(\frac{\partial C^{*}}{\partial z^{*}}\frac{\partial T^{*}}{\partial z^{*}}\right) + \frac{D_{T^{*}}}{T_{\infty}^{*}}\left(\frac{\partial T^{*}}{\partial z^{*}}\right)^{2} \right\}$$

 $u_1 \frac{\partial C^*}{\partial x^*} + v_1 \frac{\partial C^*}{\partial y^*} + w_1 \frac{\partial C^*}{\partial z^*} = D_B \left( \frac{\partial^2 C^*}{\partial (z^*)^2} \right) + \frac{D_T^*}{T_\infty^*} \left( \frac{\partial^2 T^*}{\partial (z^*)^2} \right)$ 

(1)





**Figure 1.** Physical model of the problem.

Corresponding B.Cs are

$$\begin{aligned} u_1 &= u_w^*(x) = U_w(x) + N_1 \gamma_0 \frac{\partial u_1}{\partial z^*}, \\ v_1 &= v_w^*(x) = V_w(x) + N_2 \gamma_0 \frac{\partial v^*}{\partial z^*}, \\ w_1 &= w_0, \quad T^* = T_w^*, \\ -k \frac{\partial C^*}{\partial z^*} &= h_1 (T_f - T^*) \end{aligned} \text{ at } \quad z^* = 0$$
 
$$u_1 \to 0, v_1 \to 0, T^* \to T_\infty^*, C^* \to C_\infty^* \quad \text{as } Z^* \to \infty$$

According to the Roseland's approximation [47] the non-linear radiative heat flux  $q_r$  as given by

$$q_r = \frac{-4\sigma_1}{3k^*} \frac{\partial T^{*4}}{\partial z^*} = \frac{-16\sigma_1 T^{*3}}{3k^*} \frac{\partial T^*}{\partial z^*}$$
 (7)

Differentiate above the heat flux equation, we get

$$\frac{\partial q_r}{\partial z} = \frac{-16\sigma_1}{3k^*} \frac{\partial}{\partial z} \left( T^{*3} \frac{\partial T^*}{\partial z} \right) \tag{8}$$

Eq. (4) is transfer by utilizing above Eq. (6), we get

$$u_{1}\frac{\partial T^{*}}{\partial x^{*}} + v_{1}\frac{\partial T^{*}}{\partial y^{*}} + w_{1}\frac{\partial T^{*}}{\partial z} = \alpha_{m}^{*} \left(\frac{\partial^{2} T^{*}}{\partial z^{*2}}\right) + \frac{1}{(\rho C)_{f}} \left[\frac{16\sigma_{1}}{3K^{*}}\frac{\partial}{\partial z^{*}}\left(T^{*3}\frac{\partial T^{*}}{\partial z^{*}}\right)\right] + \tau \left\{D_{B}\left(\frac{\partial C^{*}}{\partial z^{*}}\frac{\partial T^{*}}{\partial z^{*}}\right) + \frac{D_{T}}{T_{\infty}^{*}}\left(\frac{\partial T^{*}}{\partial z^{*}}\right)^{2}\right\}$$

$$(9)$$

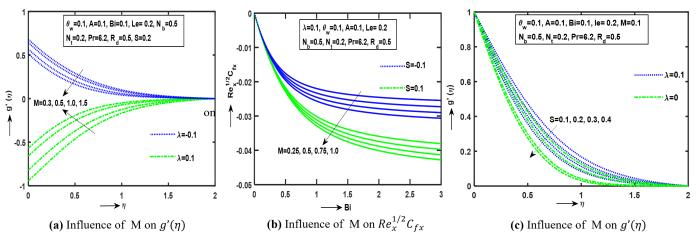
The following dimensionless functions and the similarity variables are:

$$\eta = z \sqrt{\frac{a_1}{\nu_f}}, \quad u_1 = a_1 x^* f'(\eta), 
v_1 = a_1 y^* g'(\eta) 
w_1 = -\sqrt{a_1 \nu_f} \left( f'(\eta) + g(\eta) \right), 
\theta(\eta) = \frac{T^* - T^*_{\infty}}{T^*_w - T^*_{\infty}}, 
\varphi(\eta) = \frac{C^* - C^*_{\infty}}{C^*_w - C^*_{\infty}}$$
(10)

Utilizing the above dimensions, Eq. (1) is identically satisfied and translate Eqs. (2)-(4) and Eq. (9) becomes:

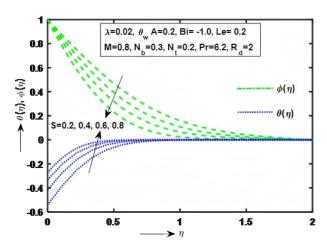
$$f''' + ff''(f+g) - (f')^2 - Mf' + 1 = 0$$
 (11)



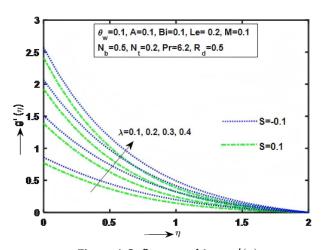


**Figure 2.** Effect of M on fluid motion and skin friction coefficient.

(14)



**Figure 3.** Influence *S* of on  $\theta(\eta)$ ,  $\phi(\eta)$ .



**Figure 4.** Influence of  $\lambda$  on  $g'(\eta)$ .

$$g''' + gg''(f+g) - (g')^{2} - Mg' + 1 = 0$$

$$\left( \left( 1 + R_{d} \left( 1 + (\theta_{w} - 1)\theta \right)^{3} \theta \right) \right)'$$

$$+ \Pr\left( (f+g)\theta' + N_{b}\theta'\varphi' + N_{t}\theta''^{2} \right)$$
(13)

$$\varphi'' + Le \Pr(f+g)\varphi' + \left(\frac{N_t}{N_b}\right)\theta'' = 0$$

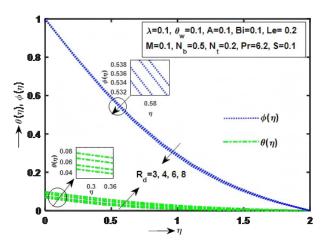
With subject to the boundary conditions are:

$$f(0) = S, g(0) = 0, f'(0) = 1 + Af''(0), g'(0) = \lambda + Bg''(0)$$

$$\varphi(0) = 1, \theta'(0) + Bi(1 - \theta(0)) = 0$$

$$f'(\eta) \to 0, g'(\eta) \to 0, \theta(\eta) \to 0, \varphi(\eta) \to 0 \quad \text{as } \eta \to \infty$$
(15)

The physical quantities of practical interest are the



**Figure 5.** Influence  $R_b$  of on  $\theta(\eta)$ , $\phi(\eta)$ .

local skin friction coefficient  $C_{fx}$  and  $C_{fy}$ , and Nusselt number  $Nu_x$ ; it is defined as

$$Nu_x = \frac{x^* q_x}{k^* (T_w^* - T_\infty^*)} \tag{16}$$

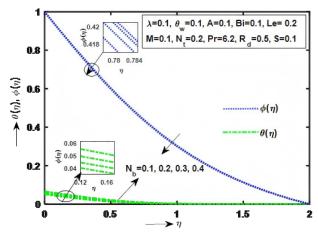
Defines the skin friction or shear stresses along  $x^*$ ,  $y^*$  direction, heat flux  $q_w$  and mass flux  $q_m$  from surface of the sheet are

$$q_w = -k^* \left(\frac{\partial T^*}{\partial z^*}\right)_{z^*=0} \tag{17}$$

Substituting the  $u_1$ ,  $v_1$ ,  $T^*$  from the Eq. (7) onto Eq. (15) and using Eq. (14), we getting

$$Nu_x Re_x^{-1/2} = -\theta'(0)$$
 (18)





**Figure 6.** Influence  $N_t$  of on  $\theta(\eta)$ , $\phi(\eta)$ .

where  $Re_x=\frac{U_wx^*}{\nu}$  and  $Re_y=\frac{U_wy^*}{\nu}$  are the local Reynolds number.

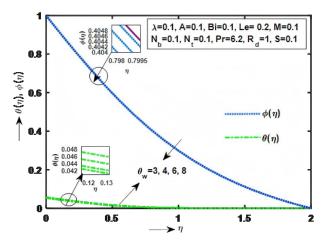
## 3 Results and Discussion

The physical effect of M ("Magnetic Parameter") on fluid motion component  $g'(\eta)$  for the cases of sheet is stretching ( $\lambda > 0$ ) and shrink ( $\lambda < 0$ ),  $Re_x^{1/2}C_f$ (coefficient of skin friction) for the cases of suction (S > 0) and injection (S < 0) is predicted on Figures 2 (a)-2(b). It is clear that the nanofluid flow velocity is slowing down along *y*-direction when the sheet is stretching  $(\lambda > 0)$  while opposite motion displays when sheet is shrink ( $\lambda < 0$ ) with various growth numerical values of M as illustrated in Figure 2(a). Moreover, the coefficient of skin friction along  $x^*$ -direction for both cases of suction (S > 0) and injection (S < 0) as explored in Figure 2(b). Physically, a drag force like resistive type force (Lorentz force) is created disturbance by the fluid particles of the vertical magnetic field to the electrically conducting fluid. This force has to reduce the motion of the fluid over a stretching surface.

Figures 3 and 4 presented the S ("Mass Flux Parameter") on base fluid flow component  $g'(\eta)$ ,  $y^*$ -direction for the cases of sheet is stretching ( $\lambda > 0$ ), shrink ( $\lambda < 0$ ) and  $\theta(\eta)$ ,  $\varphi(\eta)$ . It is clear the fluid velocity motion,  $\varphi(\eta)$  is slow reduction through a stretching and shrinking sheet while reverse behaviour shows temperature various enlarge values of S. Physically, the larger values of mass flux effect in fluid particles and the fluid resistance slow down then its fluid flow boundary layer thickness is reducing.

The impact of  $\lambda$  ("Stretching/shrinking Parameter") on velocity component  $g'(\eta)$  along  $y^*$ -direction is explored through in Figure 5 for the cases of suction (S>0)

and injection (S < 0). It is clear the fluid motion is monotonically decrease over a stretching surface with various enlarge values of  $\lambda$ . Because, the fluid flow convergent to surface area very fast then the surface is injection case.



**Figure 7.** Influence  $\theta_w$  of on  $\theta(\eta)$ ,  $\phi(\eta)$ .

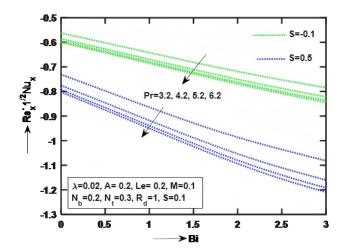


Figure 8. Influence Pr of on  $Re_x^{-1/2}N_x$ .

The significant effect of  $R_d$  ("Nonlinear thermal radiation") on  $\theta(\eta)$ ,  $\varphi(\eta)$  as discussed in Figure 6. It is noticed that fluid  $\theta(\eta)$  flow enhances, while opposite direction of  $\varphi(\eta)$  with higher enlarge values of  $R_d$ . Physically, nonlinear thermal radiation is inversely proportional to the thermal conductivity. In this fact the weaker thermal conductivity of the nanofluid flow and large values of  $R_d$  than that it has produce more heat on surface.

Figure 7 depicts the physical parameter  $N_t$  ("Thermophoresis Parameter") on  $\theta(\eta)$ ,  $\varphi(\eta)$ . It is noticed that  $\theta(\eta)$  of the nanofluid flow enhances over the surface while opposite motion of fluid  $\varphi(\eta)$  with higher values of  $N_t$ . Physically, when huge amount of temperature variance exists, then the



**Table 1.** Evaluation of Skin friction coefficient -f''(0) for  $A = B = Bi = R_d = 0.$ 

M	Present study	Sarah et al. [48]	Nadeem et al. [49]	Gupta et al. [50]	Ahmad et al. [51]
0.0	1.000000	1.00000	1.0004	1.0003181	1.0042
10	3.316624	3.31662	3.3165	3.3165824	3.3165
100	10.04987	10.04987	10.049	10.049864	10.049

**Table 2.** Comparison of Skin friction coefficient  $-f''(\infty)$  for  $A = B = Bi = R_d = 0.$ 

M	Present study	Nadeem et al. [49]
0.0	1.173719	_
10	3.367222224	3.3667
100	10.06646642	10.066

suspended particles tends to move more closer to the surface to reduce the concentration and subsequently enhances the heat somewhere, far from the cold area surface. This force produces the effect called "thermophoresis force".

Figure 8 displays the heat transfer of Pr ("Prandtl Number") for both stretching and shrinking sheet. It is observed that the  $Re_x^{-1/2}N_x$  ("Heat Transfer rate") boundary layer decreases for both present and absents of suction and injection cases. Here, the boundary layer of heat transfer rate is weaker when absents of suction case while comparing to injection case, because of the higher thermal conductivity of nanofluid flow with enlarge values of Pr and associate boundary layer thickness is very less.

Table 1 shows the evaluation of the skin friction coefficient -f''(0) for  $A = B = Bi = R_d = 0$ , comparing the results from the present study with those of Sarah et al. [48], Nadeem et al. [49], Gupta and Sharma [50], and Ahmad et al. [51].

Table 2 presents a comparison of the skin friction coefficient  $-f''(\infty)$  for  $A = B = Bi = R_d = 0$ , comparing the results from the present study with those of Nadeem et al. [49].

### 4 Conclusion

A numerical analysis has been executed for the nonlinear thermal radiation effect on 3D MHD ("Magnetohydrodynamic") nanofluid flow via stretching/shrinking sheet with slip velocity. The main out comes of the present study are mentioned below:

• The nanofluid temperature enhances with enlarge values of nonlinear thermal radiation while opposite trend in concentration of the fluid flow.

• The heat transfer rate decline with increasing values of for both suction and injection cases because of higher thermal conduction in the flow of nanofluid on stretching/shirking surfaces.

#### Nomenclature

 $(x^*, y^*)$  Cartesian coordinate's

 $u_1, v_1, w_1$  velocity components along  $x^*, y^*, z^*$ -axis

A Velocity slip along x-axes  $\sqrt{\alpha \gamma_0} N_1$ 

B Velocity slip along y-axes  $\sqrt{\alpha \gamma_0} N_2$ 

 $C^*$  Concentration

 $C_f^*$  Skin friction coefficient

 $C_p^*$  Specific heat

 $C_{\infty}^*$  Uniform ambient concentration

 $D_B$  Brownian diffusion

 $D_T$  Thermophoresis diffusion

f Dimensionless stream function

f' Dimensionless velocity

S Constant mass flux parameter  $w_0/\sqrt{\alpha_1\gamma_0}$ 

*k*\* Thermal conductivity

Le Lewis number =  $\frac{\alpha_m^*}{D_B}$ 

M Magnetic field parameter =  $\frac{\sigma_1 B_0^2}{a_1 \rho_f}$ 

M Magnetic Hera parameter  $= \tau D_T \frac{T_w^* - T_\infty^*}{\alpha_m^* T_\infty^*}$   $N_t \quad \text{Thermophoresis parameter} = \tau D_T \frac{T_w^* - T_\infty^*}{\alpha_m^* T_\infty^*}$ 

 $N_b$  Brownian motion coefficient =  $D_B \tau \frac{(C_b)^2}{2}$ 

Pr Prandtl number =  $\frac{v_1}{\alpha^*}$ 

 $q_r$  Radiative heat flux

 $Re_x$  Reynolds number

 $R_d$  Radiation parameter =  $\frac{16\sigma^*T^{*3}}{3\alpha_m^*kk^*}$ 

 $T^*$  Temperature of the fluid

 $T_{\infty}^*$  fluid temperature far away from the surface

 $T_w^*$  Constant fluid temperature of the wall

 $U_w$  Stretching velocity

 $U_{\infty}$  Free stream velocity

## Greek symbols

 $\rho$  Density

 $\varphi$  Dimensionless concentration

 $\sigma_1$  Boltzmann constant

 $\lambda$  Constant stretching/shrinking parameter  $\frac{b_1}{a_1}$ 

au Ratio of the nanoparticle to the fluid  $\frac{(\rho c)_p}{(\rho c)_f}$ 

 $v_1$  Kinematic viscosity of the fluid

 $\sigma^*$  Electrical conductivity

 $\theta$  Dimensionless temperature

 $\alpha_m^*$  Thermal diffusivity  $=\frac{k}{(\rho c_p)_f}$ 

 $(\rho c_p)_f$  Heat capacity of the fluid

 $(\rho c_p)_p$  Heat capacity of the nanoparticle to the fluid  $\rho_f$  Fluid density

#### Subscripts

 $\infty$  condition at free stream

# **Data Availability Statement**

Data will be made available on request.

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## **Conflicts of Interest**

The authors declare no conflicts of interest.

# **Ethical Approval and Consent to Participate**

Not applicable.

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