

RESEARCH ARTICLE



Heat and Momentum Transfer in MHD Boundary Layer Flow with Radiation and Heat Source/Sink Effects

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Abstract

The porosity parameter serves as a pivotal factor in determining the resistance exerted by a porous medium on fluid motion, especially in magnetohydrodynamic (MHD) flows. study presents a novel numerical investigation of the coupled influence of porosity, viscous dissipation, and Joule heating on both momentum and thermal boundary layers over a porous surface. The results demonstrate that increasing porosity enhances medium permeability, thereby reducing hydrodynamic drag and intensifying the velocity gradient near the stagnation region. Conversely, lower porosity impedes fluid penetration, resulting in diminished velocity and a compressed boundary layer structure. While the direct impact of porosity on thermal transport is minimal, its interaction with dissipative effects leads to subtle modifications in temperature distribution. graphical and quantitative findings underscore the importance of fine-tuning the porosity parameter to regulate flow resistance and thermal behaviour in advanced MHD systems. The methodology employed based on robust numerical simulations offers a comprehensive framework for analysing

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*Corresponding author: ☑ Vediyappan Govindan govindoviya@gmail.com porous flow dynamics in engineering and energy applications, highlighting the novelty of integrating complex interdependencies between porosity and thermophysical mechanisms.

Keywords: porosity parameter, magnetohydrodynamic (MHD) flows, viscous dissipation, joule heating, numerical simulations.

1 Introduction

In addition, understanding boundary-layer behavior over a stretched sheet has become increasingly important in the design and optimization of modern thermal systems. The interaction between heat transfer, viscous effects, and electromagnetic forces in such flows provides vital insights into improving energy efficiency and product quality. For example, in polymer extrusion and metal forming processes, precise thermal control is necessary to ensure uniformity and material strength. Similarly, in biomedical applications such as targeted drug delivery and tissue engineering, the manipulation of fluid flow at micro- and nano-scales often mimics boundary-layer phenomena. Therefore, exploring the combined effects of thermal radiation, magnetic fields, and porous structures contributes to advancing both theoretical models and industrial practices.

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Crane's [1] study focuses on viscous, electrically conducting fluid transfers heat over stretched layer of the fluid. Carragher et al. [2] is addressed about Heat which is transferred from the stretched sheet to a stationary medium as it passes the slit. Zhu et al. [3] studied Brownian effects, thermophoresis, and on a stretching/shrinking surface, Williamson's non-Newtonian unstable fluid is present. Siddheshwar et al. [4] analyzed the radiation that impacts the flow of viscoelastic liquid's, MHD and the heat transfer across stretched sheet. The heat transfers during melting as nonlinear thermal radiation from a nanofluid moving at a stationary point in the direction of stretched surface was studied by Hayat et al. [5].

Makinde et al. [6] addressed the MHD-free convective boundary layer in two dimensions. While considering into account that electrical conductivity, and chemical reaction and also heat source/sink, electrically conducting nanofluid flow steadily across a nonlinear stretched sheet. Abel et al. [7] addressed about the features of a viscous liquid moving hydromagnetically across the stretching sheet, as well as fluid's capabilities for momentum and the heat transmission. Damala et al. [8] addressed on semi-infinite vertical plate created by radiation, free convective magnetohydrodynamic movement of fluid, and viscous dissipation. Salahuddin et al. [9] addressed the flow's response to viscosity and thermal conductivity through the stretched cylinder in the direction of the MHD stagnation point. Khader [10] introduced Using Boundary Value Solving Numerical issue: a liquid film moving on a stretched sheet while being affected by the magnetic field, along with heat.

Farooq et al. [11] investigated the variety of applications for cooling systems for tiny heat-density devices, and heat exchangers are made possible by thermal methods that use nanomaterials. Vijayaragavan et al. [12] investigated a Casson fluid that, enables mass and transferring heat via a continuous vertically porous plate when a chemical reaction and thermal radiation are present. Seth et al. [13] studied the exponentially stretched sheets subjected to the 2-D, Viscous dissipation, radiation, and a viscous, incompressible, electrically conducting fluid producing internal heat in laminar flow.

Thiagarajan et al. [14] investigated the porous exponentially stretched sheet heated by heat generation/absorption, Ohmic heating, and the influence of the nanofluid's hydromagnetic boundary layer movement. Shateyi et al. [15] investigated

an iterative computational methodology, is used to explore the influence of the thermal radiation, heat source/sink, and flow of magnetohydrodynamics on the distribution of heat across an unstable stretched porous surface. Krishnaiah [16] studied an exponentially streaching sheet experiences an MHD flow of Casson nanofluid until stagnation point as result of the influence of a non-uniform heat source on mass and transfer mechanisms. Mishra et al. [17] investigated on retching a sheet through a saturated porous substance causes the fluid to flow according to a power law. Kumar et al. [18] Analysed hydromagnetic system numerical simulation 3-D flow through the stretched sheet inside of a porous material, including the Soret effect, heat source and sink, and chemical reaction. The optimal homotopy analysis approach (OHAM) is applied to study the laminar magneto hydrodynamic UCM fluid flow across a stretchy isothermal porous surface by Guled et al. [19]. Benal et al. [20] discussed the effects of the Jeffery fluid flowing through the porous material over the boundary layer that is contracting and expanding are discussed. Mahabaleshwar et al. [21] examined the outcomes of a mathematical model that depicts the process as previously indicated and takes the ambient nanofluid into account. Suction/injection and a magnetic field are both active at the same time conditions.

Unlike many existing studies that treat porous media effects using simplified Darcy or modified Darcy formulations, the present model incorporates a nonlinear porous resistance term directly into the momentum equations to capture the intricate flow-retarding mechanisms of the medium [22, 23]. Specifically, the momentum equation includes a sink term of the form or its generalized form denotes the permeability and FFF is the Forchheimer constant, representing inertial resistance. approach goes beyond linear approximations and accounts for non-Darcian effects, which are critical at higher Reynolds numbers [24–26] or when fluid-porous interactions are strong. Furthermore, the coupling of this term with viscous dissipation and Joule heating within a magnetohydrodynamic (MHD) [27] framework offers a more realistic and thermodynamically consistent model. distinguishes the current work from prior studies that either neglected porous resistance altogether or considered it independently from electromagnetic and thermal effects.



2 Mathematical Formulation

Consider a hydro magnetic viscous incompressible 2-D laminar constant flow that transfers electricity over a stretched sheet. The beginning of the process is the slit where the sheet is drawn. Together with path of the continuously growing plane, the axis frame is obtained at this coordinate.

 $U_W(X) = bx$ is the velocity of a stretching sheet, while $U_\infty = ax$ is the velocity of a free stream flow, where a and b stand for positive constants and x is the coordinate along the stretching plane.

Let T_W represents temperature of the Nano-fluid above the stretching layer and T_∞ represents the temperature of the surrounding air.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v\frac{\partial^2 u}{\partial y^2} - \frac{v}{k}u - \frac{\sigma B_0^2}{\rho}u + U_\infty \frac{\partial U_\infty}{\partial x}$$
 (2)

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y} + \frac{Q}{\rho C_p} (T - T_\infty) + \frac{\mu}{\rho C_p} \left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma B_0^2}{\rho C_p} u$$
(3)

where velocity u and v are along the coordinate axes, T is the Temperature, kinematic viscosity ν , and thermal diffusivity α . Apply Roseland approximation to radiation. The q_r is radiative heat flux, that is,

$$q_r = \frac{-4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y} \tag{4}$$

where k^* is the absorption coefficient and σ^* is the Stefan-Boltzmann constant. To apply this approximation, it is assumed that the temperature difference $T-T_\infty$ within the flow is small. Thus, the expression T^4 is expanded about T_∞ using a Taylor series, and neglecting higher-order terms, we obtain:

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4$$

hence we get

$$q_r = \frac{-4\sigma^* T_{\infty}^3}{3k^*} \frac{\partial T}{\partial y} \quad \text{Then,}$$

$$\frac{\partial q_r}{\partial y} = \frac{-16\sigma^* T_{\infty}^3}{3\rho C_p k^*} \frac{\partial^2 T}{\partial y^2}$$
(5)

the following boundary conditions are suitable:

$$u=bx; \quad \text{at } y=0: \quad v=0; \quad T=T_w;$$
 as $y \to \infty: \quad u=U_\infty=ax, \quad T=T_\infty;$

Thus, similarity transformations must fulfil the continuity equation.

$$u = \frac{\partial \psi}{\partial y}$$
 and $v = -\frac{\partial \psi}{\partial x}$ (7)

Such that

$$u = bxf'(\eta), \quad v = -\sqrt{\theta b}f(\eta);$$

$$\eta = \sqrt{\frac{b}{\nu}}y, \quad \theta = \frac{T - T_{\infty}}{T_{w} - T_{\infty}}$$
(8)

By Substituting Eqs 6 and 8 in Eqs 2-3. Following are the coupled nonlinear differential equations that resulted.

(1)
$$f''' + ff' + Gm\varphi + Gr\theta - (f')^2 \left(\frac{1}{k} + M - 1\right) + \lambda^2 = 0$$
 (9)

$$(1 + \frac{4}{3}R)\theta'' + Pr\left[Ec\left[(f'')^2 + M(f'-1)^2\right] + S\theta + f\theta'\right] = 0$$
(10)

3 Numerical Solution

Equation No. (09) -(10) are non-linear ODE's along with the Equation No. (11). That is, boundary conditions are solved by using Mathematica, R-K Fourth Order Shooting Techniques. The first-order ODEs that result from the aforementioned equations are as follows:

$$f(1) = f, \quad f(2) = f', \quad f(3) = f';$$

 $\theta(1) = \theta, \quad \theta(2) = \theta'$
(11)

$$f''' = -f(1) * f(2) - Gm\varphi - Gr\theta + (f(2))^{2} \left(\frac{1}{k} + M - 1\right) - \lambda^{2}$$
(12)

$$\theta'' = \frac{-Pr}{\left(1 + \frac{4}{3}R\right)} \left[(f(3))^2 Ec + M \left(f(2) - 1\right)^2 \right] + S\theta(1) + \theta(2)f(1)$$
(13)

The boundary conditions are,

$$f(0) = 0,$$
 $f(1) = 1,$ $f(2) \rightarrow \lambda^{\infty};$
 $\theta(0) = 1,$ $\theta(1) \rightarrow 0^{\infty}$

By suitably assuming the missing slopes f'(0) and $\theta(0)$, this boundary value problem is converted into the initial value problem. The suitable shooting approach is used to solve the ensuing initial value issue for the set of the parameters that appear in governing equations and have acknowledged the values for f'(0) and $\theta(0)$. The shooting technique's initial circumstances must

be estimated rather accurately for the convergence criterion to work. After the difference between most recent iterative value of f'(0) and prior iterative value of f'(0) is equal, the iterative process is terminated. Once convergence is attained, the needed solution is obtained by integrating the resulting ODE with the usual R-K method of IV-order with a supplied set of parameters.

4 Results and Discussion

Coupled PDEs are effected by a number of parameters, including Source Parameter, Velocity Ratio, Magnetic, and Radiation Parameters, Prandtl and Eckert Numbers. The governing coupled, nonlinear PDEs of the heat transport and flow problems are transformed into coupled, nonlinear equations via similarity transformation. These, ODEs are numerically solved using R-K fourth order method, provided certain boundary conditions are satisfied. After that, MATHEMATICA is used to display the graphs.

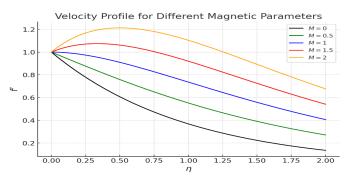


Figure 1. The velocity is impacted by the velocity ratio parameter (λ) .

Figure 1 demonstrates the impact of the velocity ratio parameter v/s similarity variables on velocity profile. It has been seen the velocity ratio parameter's value increases, so does the temperature. This means that when the It is seen that as the velocity ratio parameter becomes more significant, fluid's velocity drops.

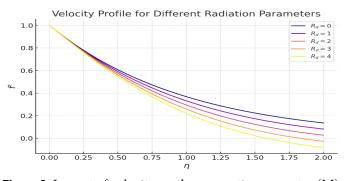


Figure 2. Impact of velocity on the magnetic parameter (M).

Figure 2 illustrates the influence of magnetic parameter (M) v/s similarity variables on velocity profile. It is discovered that when magnetic parameter's value increases, so does the velocity. In other words, magnetic parameter increases as the fluid's velocity falls.

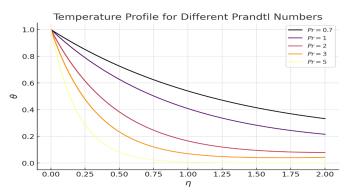


Figure 3. Effect of magnetic parameter (M) on temperature.

Figure 3 demonstrates the effect of magnetic parameters v/s similarity variables on the temperature profile. Temperature profiles show how to increase a magnetic parameter's value and decrease it, to increase in the value of a magnetic parameter and reduce it with corresponding temperature value.

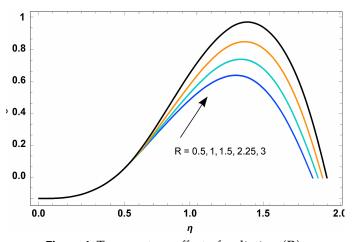


Figure 4. Temperature effect of radiation (R).

Figure 4 demonstrates the influence of radiation parameter v/s similarity variables on temperature profile. The radiation parameter is increased, with increases in the thermal and momentum boundary layer thickness.

Figure 5 demonstrates how changes in temperature impact the temperature profile by the Eckert Number v/s similarity factors. It is possible to demonstrate how the variation in the Eckert number (Ec) affects the distribution of temperatures and how temperature



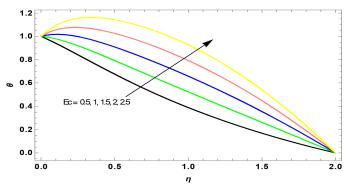


Figure 5. Eckert number's impact on temperature.

rises as Ec rises. Ec is characterized as having a linear relationship with velocity squared. Because of this, the fluid's temperature can rise more quickly, especially in the vicinity of the sheet, with higher Ec values and a proportionally greater enhancement due to the motion of particles close to the surface.

5 Conclusion

This study presents a comprehensive analysis of magnetohydrodynamic flow and heat transfer over a porous surface, incorporating a detailed porous resistance term within the momentum equations. The numerical results reveal that increasing the porosity parameter significantly enhances fluid penetration, reduces hydrodynamic drag, and elevates the velocity near the stagnation region. In contrast, reduced porosity suppresses fluid motion and intensifies boundary layer compression. Although the influence of porosity on thermal transport is indirect, its coupling with viscous dissipation and Joule heating introduces notable modifications to the temperature distribution. The incorporation of a nonlinear porous resistance term, as opposed to traditional linear Darcy models, marks a significant advancement over previous studies. This allows for a more accurate representation of flow dynamics in complex porous structures under electromagnetic effects. The findings underscore the critical role of porosity in regulating both momentum and thermal behavior, offering valuable insights for optimizing design in engineering systems involving porous media, such as thermal insulation, cooling technologies, and MHD-based devices.

Data Availability Statement

Data will be made available on request.

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Conflicts of Interest

The author declares no conflicts of interest.

Ethical Approval and Consent to Participate

Not applicable.

References

- [1] Crane, L. J. (1970). Flow past a stretching plate. Zeitschrift für Angewandte Mathematik und Physik (ZAMP), 21(4), 645–647. [CrossRef]
- [2] Carragher, P., & Crane, L. (1982). Heat transfer on a continuous stretching sheet. *Zeitschrift für Angewandte Mathematik und Mechanik*, 62(10), 564–565. [CrossRef]
- [3] Zhu, A., Ali, H., Ishaq, M., Junaid, M. S., Raza, J., & Amjad, M. (2022). Numerical study of heat and mass transfer for Williamson nanofluid over stretching/shrinking sheet along with Brownian and thermophoresis effects. *Energies*, 15(16), 5926. [CrossRef]
- [4] Siddheshwar, P. G., & Mahabaleswar, U. S. (2005). Effects of radiation and heat source on MHD flow of a viscoelastic liquid and heat transfer over a stretching sheet. *International Journal of Non-Linear Mechanics*, 40(6), 807–820. [CrossRef]
- [5] Hayat, T., Qayyum, S., Alsaedi, A., & Shafiq, A. (2016). Inclined magnetic field and heat source/sink aspects in flow of nanofluid with nonlinear thermal radiation. *International Journal of Heat and Mass Transfer*, 103, 99–107. [CrossRef]
- [6] Makinde, O. D., Mabood, F., & Ibrahim, M. S. (2018). Chemically reacting on MHD boundary-layer flow of nanofluids over a non-linear stretching sheet with heat source/sink and thermal radiation. *Thermal Science*, 22(1B), 495–506. [CrossRef]
- [7] Abel, M. S., & Nandeppanavar, M. M. (2009). Heat transfer in MHD viscoelastic boundary layer flow over a stretching sheet with non-uniform heat source/sink. *Communications in Nonlinear Science and Numerical Simulation*, 14(5), 2120–2131. [CrossRef]
- [8] Damala, C. K., Bhumarapu, V., & Makinde, O. D. (2021). Radiative MHD Walter's liquid-B flow past a semi-infinite vertical plate in the presence of viscous dissipation with a heat source. *Engineering Transactions*, 69(4), 373–401. [CrossRef]
- [9] Salahuddin, T., Malik, M. Y., Hussain, A., Bilal, S., & Awais, M. (2015). Effects of transverse magnetic field with variable thermal conductivity on tangent hyperbolic fluid with exponentially varying viscosity. *AIP Advances*, 5(12), 127103. [CrossRef]

- [10] Khader, M. M. (2019). The numerical solution for BVP of the liquid film flow over an unsteady stretching sheet with thermal radiation and magnetic field using the finite element method. *International Journal of Modern Physics C*, 30(11), 1950080. [CrossRef]
- [11] Farooq, U., Tahir, M., Waqas, H., Muhammad, T., Alshehri, A., & Imran, M. (2022). Investigation of 3D flow of magnetized hybrid nanofluid with heat source/sink over a stretching sheet. *Scientific Reports*, 12, 12254. [CrossRef]
- [12] Vijayaragavan, R., & Karthikeyan, S. (2017). Joule heating and thermal radiation effects on chemically reacting Casson fluid past a vertical plate with variable magnetic field. *Research Journal of Engineering and Technology*, 8(4), 393–404. [CrossRef]
- [13] Seth, G. S., Sharma, R., Kumbhakar, B., & Tripathi, R. (2017). MHD stagnation point flow over exponentially stretching sheet with exponentially moving free-stream, viscous dissipation, thermal radiation and non-uniform heat source/sink. *Diffusion foundations*, 11, 182-190. [CrossRef]
- [14] Thiagarajan, M., & Kumar, M. D. (2019). Heat source/sink and chemical reaction effects on MHD and heat transfer flow of radiative nanofluid over a porous exponentially stretching sheet with viscous dissipation and ohmic heating. *Int. J. Basic Sci. Appl. Comput*, 2, 5-12.
- [15] Shateyi, S., & Marewo, G. T. (2014). On a new numerical analysis of the Hall effect on MHD flow and heat transfer over an unsteady stretching permeable surface in the presence of thermal radiation and heat source/sink. *Boundary Value Problems*, 2014(1), 170. [CrossRef]
- [16] Krishnaiah, M., Rajendar, P., Laxmi, T. V., & Reddy, M. C. K. (2017). Influence of non-uniform heat source/sink on stagnation point flow of a MHD Casson nanofluid flow over an exponentially stretching surface. *Global Journal of Pure and Applied Mathematics*, 13(10), 7009–7033.
- [17] Mishra, S. R., Baag, S., Dash, G. C., & Acharya, M. R. (2020). Numerical approach to MHD flow of power-law fluid on a stretching sheet with non-uniform heat source. *Nonlinear Engineering*, 9(1), 81–93. [CrossRef]
- [18] Kumar, S. G., Varma, S. V. K., Prasad, P. D., Raju, C. S. K., Makinde, O. D., & Sharma, R. (2018). MHD reacting and radiating 3-D flow of Maxwell fluid past a stretching sheet with heat source/sink and Soret effects in a porous medium. *Defect and Diffusion Forum*, 387, 145–156. [CrossRef]
- [19] Guled, C. N., Tawade, J. V., Nandeppanavar, M. M., & Saraf, A. R. (2022). MHD slip flow and heat transfer of UCM fluid with the effect of suction/injection due to stretching sheet: OHAM solution. *Heat Transfer*, *51*(5), 3201–3218. [CrossRef]
- [20] Benal, S. S., Tawade, J. V., Biradar, M. M., & Allasi,

- H. L. (2022). Effects of the magnetohydrodynamic flow within the boundary layer of a jeffery fluid in a porous medium over a shrinking/stretching sheet. *Mathematical Problems in Engineering*, 2022(1), 7326504. [CrossRef]
- [21] Mahabaleshwar, U. S., Vinay Kumar, P. N., & Sheremet, M. (2016). Magnetohydrodynamics flow of a nanofluid driven by a stretching/shrinking sheet with suction. *SpringerPlus*, 5(1), 1901. [CrossRef]
- [22] Anwar, T., Kumam, P., Khan, I., & Thounthong, P. (2022). Thermal analysis of MHD convective slip transport of fractional Oldroyd-B fluid over a plate. *Mechanics of Time-Dependent Materials*, 26(2), 431-462. [CrossRef]
- [23] Siva, T., Dubey, D., & Jangili, S. (2024). Rotational flow dynamics of electroosmotic transport of couple stress fluid in a microfluidic channel under electromagnetohydrodynamic and slip-dependent zeta potential effects. *Physics of Fluids*, 36(9). [CrossRef]
- [24] Zhang, W., Zhu, Q., Zhan, H., Huang, G., & Wang, Q. (2020). A critical review of non-Darcian flow and future challenges. *Earth and Space Science Open Archive ESSOAr*. [CrossRef]
- [25] Huang, H., & Ayoub, J. (2008). Applicability of the Forchheimer equation for non-Darcy flow in porous media. *Spe Journal*, 13(01), 112-122. [CrossRef]
- [26] Pal, D., & Mondal, H. (2011). The influence of thermal radiation on hydromagnetic Darcy-Forchheimer mixed convection flow past a stretching sheet embedded in a porous medium. *Meccanica*, 46(4), 739-753. [CrossRef]
- [27] Babaelahi, M., & Sadri, S. (2024). Magneto-Hydro-Dynamic Generator with Joule Heating and Viscous Dissipation: An Analytic Investigation of Mixed Convection Flow. Arabian Journal for Science and Engineering, 49(8), 11445-11455. [CrossRef]



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