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Homotopic Computations for Non-Darcian Modified EyringPowell Incompressible Liquid Configured by Varying Thickened Surface

Muhammad Zubair[1,*], Muhammad Waqas[2], Muhammad Zeshan[2] and Sami Ullah Khan[3]

[1] School of Qilu Transportation, Shandong University, Jinan 250061, China
[2] NUTECH School of Applied Sciences and Humanities, National University of Technology, Islamabad 44000, Pakistan
[3] Department of Mathematics, Namal University, Mianwali 42250, Pakistan

Abstract

The simulations for non-Fourier heat transfer model has been performed for modified Eyring-Powell fluid, comprising the vairbale visocity. The interia outcomes for thermal problem are evaluated by interpreating the Darcy-Forchheimer features. The investigation for visualzing the heat transfer aspects is subject to variable thermal conductivty. The induction of flow is nonlinear moving surface. After developing the governing expressions, analytical treamtnet of problem is presented. The results are graphically presented to endorse physical aspects of current model.

Keywords: variable sheet thickness, non-linear stretching sheet, non-Fourier heat flux, modified eyring powell liquid, variable thermal conductivity.

1 Introduction

Fourier heat conduction model is the basic key for thermoelectric transport models [1]. However, a fundamental drawback of the widely used Fourier's law is that it describes a parabolic energy equation, which is valid for instantaneous propagation of thermal disturbances and consequently, the energy transport is anisotropic and assessment of unrealistics infinite heat propagation speed. To address this deficiency, a general structure known as Fourier's law with thermal relaxation was developed by Cattaneo [2] where a thermal relaxation time, representing the finite thermal conduction speed, was assigned. Exploiting this idea, Christov [3] goes a step further and improves the model by framing it in a frame-indifferent way using Oldroyd's theory of upper-convected derivatives, so that compatibility with the tenets of rational thermodynamics is achieved. The earlier work of Christov [3] initiated a series of studies of heat transfer with non-Fourier effects. Straughan [4] deduced the variation in thermal results subect to natural convective flow by improsing the non-tradtional Fourier's approach.

At the same time, there is a great interest in investigating different types of non-Newtonian fluids, ranging from everyday products to industrial liquids such as blood, ketchup, cheese, drilling mud, and colloidal suspensions. Of a variety of constitutive models, the 1944 Eyring–Powell fluid, originally proposed by Powell and Eyring, has been recognized for its ability to replicate Newtonian-like behavior at both low and high shear rates. Its empirical nature provides a more realistic description of complex rheological behavior, particularly for shear thickening and thinning phenomena. Several studies, including those by [5–7], have validated the versatility of the Eyring–Powell model in diverse flow configurations. [8] further advanced this field by incorporating ferromagnetic effects into the Eyring–Powell model while utilizing the Cattaneo-Christov (CC) heat flux theory. Their work highlights the necessity of coupling advanced heat conduction models with non-Newtonian flow theory to achieve accurate predictions of coupled heat and momentum transport. Recent progress in non-Newtonian fluid mechanics has expanded these insights: [9] investigated viscoelastic nanofluid transport under temperature gradients, while [10] analyzed Casson polymeric flows with radiative and non-Fourier effects. For bioconvective applications, [11] examined Eyring–Powell nanofluids over Riga surfaces, and [12] explored hybrid nanofluids with Hall current effects. Additionally, [13] developed numerical methods for fractional advection-diffusion systems, and [14, 15] addressed chemically reactive flows and double-diffusion phenomena in micropolar nanofluids.

Recent developments in fluid dynamics have emphasized the importance of varying sheet thickness on the behaviour of non-Newtonian fluids especially in industrial processes involving calendering and coating. Zahid et al. [16] studied the calendering of non-isothermal visco-elastic materials and reported that temperature dependent viscosity has a major influence on the variable sheet thickness and final properties of the material in the process. Similarly, Anwar et al. [17] studied non-Newtonian flow of fluids over stretching sheets in porous space and stressed on the effect of thermal conductivity and magnetic fields in changing the pattern of flow and heat transfer rates. Abbas et al. [18] realized the curved surface flow with chemical specieis for non-Newtonian fluids. Nonlinear stretching surfaces play an important impact in transport phenomenon, especially for

complex materials. It is reported through some investigative studies that magnetic field, buoyancy and convective boundary conditions play a vital role in flow features and heat transfer characteristics in such systems [19–22]. These observations are also useful for the industrial processes regarding the nonlinear stretching surface. The Modified Eyring–Powell model is particularly useful in replicating shear-thinning behaviour without developing singularities at zero shear rate as seen in models such as power-law or Bingham. Modified Eyring–Powell model has recently been utilized to consider many engineering problems, such as temperature dependent viscosity in web coating process, non-similar boundary layer flow and heat transfer over a wedge, and chemically reacting nanofluid flow over wedges. These applications demonstrate the model's practical applicability and accuracy in describing the rheological response of the non-Newtonian fluid in various engineering and biomedical practices (see [23–25]).

Motivated by the above scenario, the present research is pursued to address the impacts of non-Fourier heat flux in Darcy–Forchheimer flow of a Modified Eyring–Powell fluid towards a nonlinear stretching surface with variable thickness. Thermal results are claimed under the assumptions of temperature dependent thermal conductivity. The problem is simplified into dimensionless forms. The homotopy analysis scheme, with excellent accuracy is followed to entertain the simulations.

2 Mathematical Modeling

We investigate a two-dimensional Darcy–Forchheimer flow of a modified Eyring–Powell fluid on a non-linear stretching sheet with variable thickness. The stretching surface starts with a slit placed at the origin with x-axis placed along the sheet and y-axis perpendicular to it. The thermal conductivity is considered as temperature-dependent, and heat diffusion is described using a non-Fourier (Cattaneo–Christov) heat flux model. The surface profile is $y = A(x+b)^{\frac{1-n}{2}}$. The boundary layer assumptions are introduced and the equations of continuity, momentum, and energy written out in the boundary layer coordinate system.

$$\partial_x u + \partial_y v = 0 \tag{1}$$

$$u\partial_x u + v\partial_y u = \frac{\mu}{\rho}\partial_{yy}u + \frac{q}{\rho}\left(\frac{1}{\rho BC}\right)^q\left(1 + \frac{q+2}{6c^2}(\partial_y u)^2\right)$$

$$\times (\partial_y u)^{q-1}\partial_{yy}u - \frac{\mu}{\rho k^*}u - \frac{C_b}{\sqrt{k^*}}u^2$$

$$\tag{2}$$

$$u\partial_x T + v\partial_y T + \lambda \{(u(\partial_x u)(\partial_x T) + v(\partial_y u)(\partial_y T))$$
$$+ v\partial_y u(\partial_x T) + 2uv\partial_{xy}T + u^2\partial_{xx}T + v^2\partial_{yy}T\}$$
$$= -\frac{1}{\rho c_p}k(T)\partial_{yy}T \tag{3}$$

with conditions

$$u = U_w(x) = U_0(x+b)^n, \quad v = 0,$$
$$T = T_w \text{ at } y = A(x+b)^{\frac{1-n}{2}}, \tag{4}$$
$$u \to 0, \quad T \to T_\infty \text{ as } y \to \infty$$

The thermal conductivity is assumed to be $k(T) = k_\infty(1 + \epsilon\Theta)$, where k_∞ is the thermal conductivity of the ambient fluid, ϵ symbolizes a small coefficient for thermal conductivity, and Θ is dimensionless temperature. The constants (b, c, d) have no dimension, and μ is the dynamic viscosity and ρ the fluid density. The factors B, C, and q are related to the Modified Eyring–Powell (MEP) fluid model. Where c_p is the specific heat, T is the fluid temperature, T_∞ the ambient temperature, and u, v are the velocity components in the x, y direction, respectively. Following quantities are adopted for simplification:

$$\chi = \sqrt{\frac{2}{n+1}}U_0\nu(x+b)^{n+1}F(\xi),$$
$$\xi = \sqrt{\frac{n+1}{2}}\left(\frac{U_0}{\nu}\right)(x+b)^{n-1},$$
$$u = U_0(x+b)^n F'(\xi),$$
$$v = -\sqrt{\frac{n+1}{2}}U_0\nu(x+b)^{n-1}\left(F(\xi) + \xi\frac{n-1}{n-1}F'(\xi)\right),$$
$$\Theta(\xi) = \frac{T - T_\infty}{T_w - T_\infty} \tag{5}$$

By the use of Eq. (5) into the Eqs.(2-4), we get

$$\left(1 + Kq\sqrt{\frac{n+1}{2}}\delta F''\right)^{q-1}\left(1 - \frac{q+2}{3}\delta(F'')^2\right)F'''$$
$$+ FF'' - \frac{2n}{n+1}(F')^2 - \frac{2}{n+1}\beta F' - \frac{2}{n+1}fr(F')^2 = 0 \tag{6}$$
$$(1 + \epsilon\Theta)\Theta'' + \epsilon\Theta'^2 + Pr(F\Theta'$$
$$+ \gamma\left(\frac{n-3}{2}F\Theta'F' - \frac{n+1}{2}F^2\Theta''\right)) = 0 \tag{7}$$

with

$$F(0) = 1, \quad F(0) = \frac{1-n}{1+n}, \tag{8}$$
$$\Theta(0) = 1, \quad F'(\infty) = 0, \quad \Theta(\infty) = 0$$

Now we have $F(\xi) = f(\xi - \alpha) = f(\eta)$ and $\Theta(\xi) = \theta(\xi - \alpha) = \theta(\eta)$, then Eq. (7-9) becomes

$$\left(1 + Kq\sqrt{\frac{n+1}{2}}\delta f''\right)^{q-1}\left(1 - \frac{q+2}{3}\delta(f'')^2\right)f''' + ff''$$
$$- \frac{2n}{n+1}(f'')^2 - \frac{2}{n+1}\beta f' - \frac{2}{n+1}fr(f')^2 = 0 \tag{9}$$
$$(1 + \epsilon\Theta)\Theta'' + \epsilon\Theta'^2 + Pr(f\Theta'$$
$$+ \gamma\left(\frac{n-3}{2}f\Theta'f' - \frac{n+1}{2}f^2\Theta''\right)) = 0 \tag{10}$$

with

$$f(0) = 1, \quad f(0) = \frac{1-n}{1+n}, \tag{11}$$
$$\Theta(0) = 1, \quad f'(\infty) = 0, \quad \Theta(\infty) = 0$$

Here $K\left(= \frac{1}{\mu BC}\right)$ and $\delta\left(= \frac{U_0^3(x+b)^{3n-1}}{2\nu c^2}\right)$ illustrating the fluid parameters, $fr\left(= \frac{2c_b(x+b)}{\sqrt{k^*(n+1)}}\right)$, $\beta\left(= \frac{2\nu}{k^*(n+1)U_0(x+b)^{n-1}}\right)$, γ the thermal relaxation parameter and $Pr\left(= \frac{\nu}{\alpha}\right)$. The wall drag force is:

$$C_f = \frac{2\tau_w}{\rho U_w^2} \tag{12}$$

In which

$$\tau_w = \mu\partial_y u + \frac{1}{BC}\partial_y u - \frac{1}{6BC^3}(\partial_y u)^3 \quad \text{at} \quad y = A(x+b)^{\frac{1-n}{2}} \tag{13}$$

The dimensionless form is

$$\frac{\sqrt{Re_x}C_f}{2} = \sqrt{\frac{n+1}{2}}\left(1 + K - \frac{n+1}{2}\frac{K\delta}{3}f''(0)\right)f''(0) \tag{14}$$

where $Re_x = \frac{U_w(x+b)}{\nu}$ the local Reynolds number.

3 Homotopic Procedure

The convergent solutions of Eqs. (10) and (11) are solved using the HAM, in the presence of the boundary conditions (5)-(7), (12). The solutions are given in series, consistent with the nature of the problem. The initial guesses and associated linear operators are given by:

$$f_0(\eta) = 1 - e^{-\eta}, \quad \theta_0(\eta) = e^{-\eta},$$
$$L_f = f''' - f', \quad L_\theta = \theta'' - \theta' \tag{15}$$

Such that

$$L_f\left[C_1 + C_2 e^\eta + C_3 e^{-\eta}\right] = 0,$$
$$L_\theta\left[C_4 e^\eta + C_5 e^{-\eta}\right] = 0. \tag{16}$$

in which C_i $(i = 1 - 5)$ elucidate the arbitrary constants. An important part of the HAM method is the H-curve, which presents the convergence of the Eqs. (10) and (11) subject to Eqs. (12). The optimal ranges for the convergence-control parameters are $-1.85 \leq h_f \leq -0.15$ and $-1.6 \leq h_\theta \leq -0.4$. The H-curve derived is shown in Figure 1. Table 1 presents accuracy of HAM simulations. The illustration of residual error for HAM has been plotted in Figure 2.
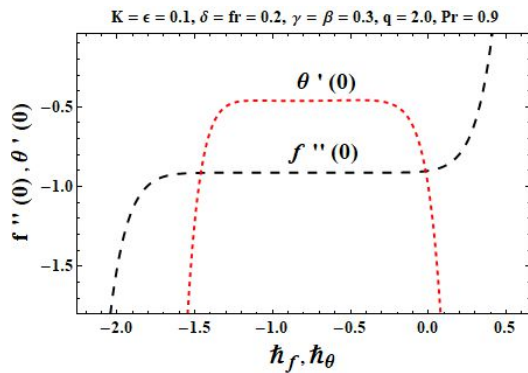

Figure 1. H-Curve for h_f and h_θ.

Table 1. The Homotopic solutions for convergence when $K = \epsilon = 0.1, \delta = fr = 0.2, \gamma = \beta = 0.3, q = 2.0$ and $Pr = 0.9$.

Approximation Order	$-f''(0)$	$-\theta(0)$
1	0.7182	0.7264
6	0.7223	0.5678
12	0.7223	0.5691
18	0.7223	0.5710
24	0.7223	0.5705
30	0.7223	0.5705
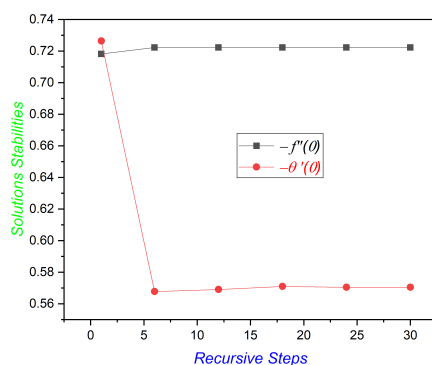
Figure 2. Residual error for HAM.
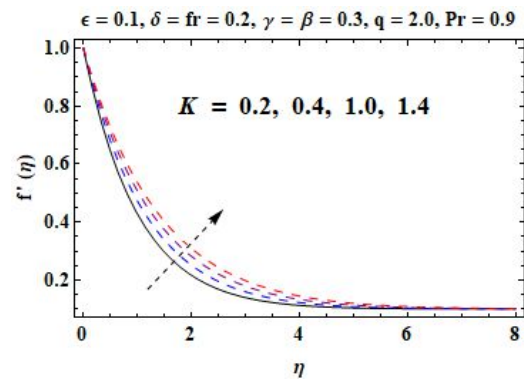
Figure 3. K via f'.

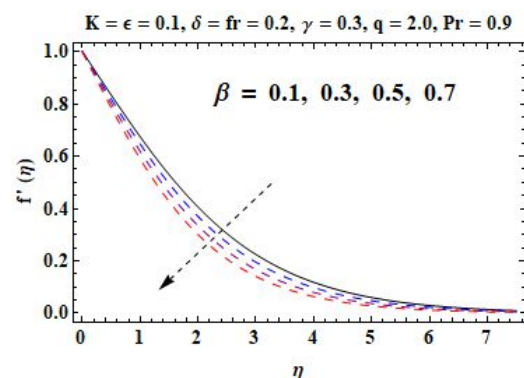**Figure 4.** θ via f'.
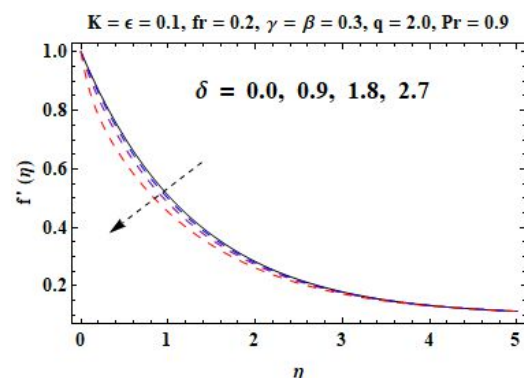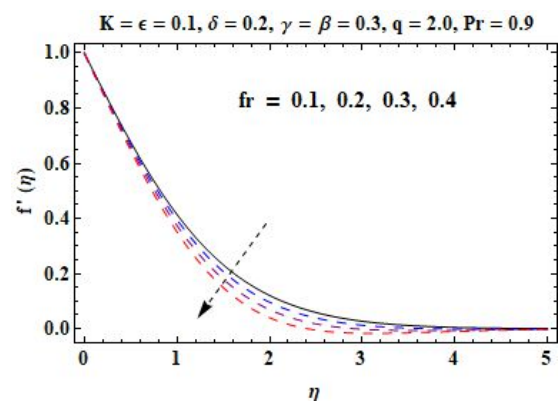

Figure 5. β via f'.

Figure 6. fr via f'.

4 Analysis of results

In this section an investigation is made regarding the effects of various parameters on skin friction, velocity
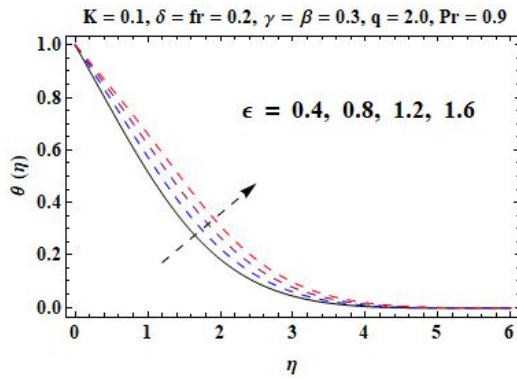
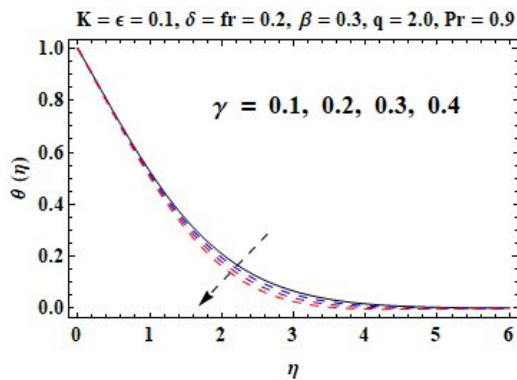**Figure 7.** ϵ via θ.

Figure 8. γ via θ.

Figure 9. Pr via θ.

and temperature in Figures 3, 4, 5, 6, 7, 8, 9 and 10. Figure 3 shows that when the material parameter K becomes larger, the velocity f' gets increased and the momentum boundary layer thickness becomes thick due to reduction in effective viscosity. In contrast, Figure 4, an increase in the sheet thickness parameter δ causes a decrease in velocity and the boundary layer thickness. From Figure 5, it can be observed that, with an ascent rate of the porosity parameter β, the velocity decreases because of higher resistance in the porous medium. Similarly, Figure 6 demonstrates that at a higher Forchheimer number fr, inertial effects become more pronounced, which leads to slower flow velocity. Notice from Figure 7 that, temperature θ increases with
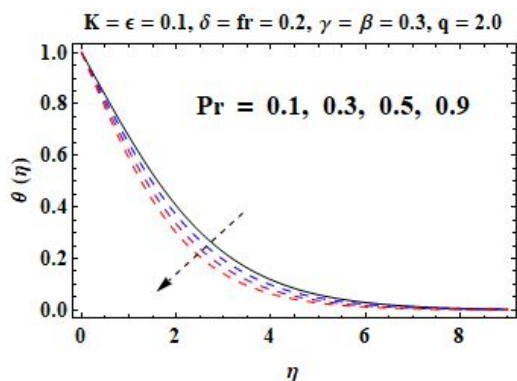
increasing thermal conductivity parameter ϵ because the higher the value of the thermal conductivity, the more heat is moved from the sheet to the fluid. In Figure 8, the temperature θ decreases as ν increases with the maximum heat transfer performance when $\gamma = 0$. It is evident from Figure 9 that, Pr inhibits the thermal boundary layer and temperature profile considerably because of smaller thermal diffusivity. Finally, Figure 10 indicates that skin friction gets enhanced with respect to both K and δ showing stronger shearing effect with the surface as compared to the high value of both these parameters. The homotopic solutions for exhibiting convergence can be seen in Table 1 and the stability of these solutions is shown in Figure 2. A comparison of the skin friction coefficient is given in Table 2, which shows good agreement with available data.
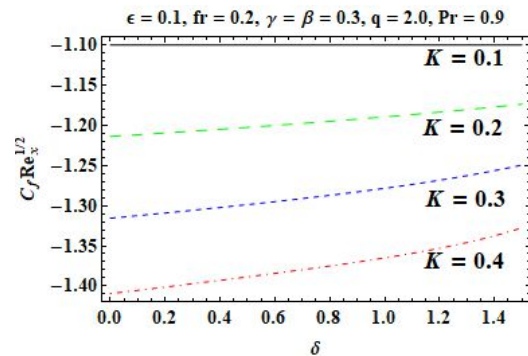

Figure 10. Skin Friction on K and δ.

Table 2. The comparative analysis of skin friction coefficient.

	$-f''(0)$	$-f'''(0)$
β	Turkyilmazoglu [26]	Present
0.0	1.00000	1.00000
0.5	1.22474487	1.01980
1.0	1.41421356	1.11803

5 Conclusion

This work aims to provide a comprehensive analysis of the Darcy–Forchheimer flow of a Modified Eyring–Powell material. Updated heat flux approach is followed to model the problem. The noteworthy observations are:

- The velocity decreases as the porosity parameter and Forchheimer number are increased.

- Larger material parameter lead to increasing the fluid velocity.

- The velocity and corresponding layer thickness decrease with increasing δ.

- Peak variation in temperature rate is assessed due to variable thermal conductivity.

- A decrement in wal shear force is observed due to sheet thickness parameter.

Data Availability Statement

Data will be made available on request.

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Conflicts of Interest

The authors declare no conflicts of interest.

Ethical Approval and Consent to Participate

Not applicable.

References

[1] Fourier, J. B. J. (1822). *Théorie analytique de la chaleur*. Paris: Firmin Didot.

[2] Cattaneo, C. (1948). Sulla conduzione del calore [On heat conduction]. *Atti del Seminario Matematico e Fisico dell'Università di Modena e Reggio Emilia, 3*, 83–101.

[3] Christov, C. I. (2009). On frame indifferent formulation of the Maxwell-Cattaneo model of finite-speed heat conduction. *Mechanics Research Communications, 36*(4), 481–486. [CrossRef]

[4] Straughan, B. (2010). Thermal convection with the Cattaneo-Christov model. *International Journal of Heat and Mass Transfer, 53*(1–3), 95–98. [CrossRef]

[5] Abbasi, F. M., Alsaedi, A., & Hayat, T. (2014). Peristaltic transport of Eyring-Powell fluid in a curved channel. *Journal of Aerospace Engineering, 27*(6), 04014037. [CrossRef]

[6] Rahimi, J., Ganji, D. D., Khaki, M., & Hosseinzadeh, K. (2017). Solution of the boundary layer flow of an Eyring-Powell non-Newtonian fluid over a linear stretching sheet by collocation method. *Alexandria Engineering Journal, 56*(4), 621-627. [CrossRef]

[7] Hayat, T., Zubair, M., Waqas, M., Alsaedi, A., & Ayub, M. (2017). On doubly stratified chemically reactive flow of Powell–Eyring liquid subject to non-Fourier heat flux theory. *Results in Physics, 7*, 99–106. [CrossRef]

[8] Zubair, M., Ijaz, M., Abbas, T., & Riaz, A. (2019). Analysis of modified Fourier law in flow of ferromagnetic Powell–Eyring fluid considering two equal magnetic dipoles. *Canadian Journal of Physics, 97*(8), 772–776. [CrossRef]

[9] Saif, R. S., Haneef, M., Nawaz, M., & Muhammad, T. (2023). Transport mechanism under temperature and concentration gradient for nano-sized species in Maxwell viscoelastic fluid over cylindrical object moving with non-uniform velocity. *Chemical Physics Letters, 813*, 140293. [CrossRef]

[10] Shahid, A., Wei, W., Bhatti, M. M., Bég, O. A., & Bég, T. A. (2023). Mixed convection Casson polymeric flow from a nonlinear stretching surface with radiative flux and non-Fourier thermal relaxation effects: Computation with CSNIS. *ZAMM-Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik, 103*(10), e202200519. [CrossRef]

[11] Usman, M., Khan, M. I., Shah, F., Khan, S. U., Ghaffari, A., & Chu, Y.-M. (2022). Heat and mass transfer analysis for bioconvective flow of Eyring Powell nanofluid over a Riga surface with nonlinear thermal features. *Numerical Methods for Partial Differential Equations, 38*(4), 777–793. [CrossRef]

[12] Kodi, R., Ravuri, M. R., Veeranna, V., Khan, M. I., Abdullaev, S., & Tamam, N. (2023). Hall current and thermal radiation effects of 3D rotating hybrid nanofluid reactive flow via stretched plate with internal heat absorption. *Results in Physics, 53*, 106915. [CrossRef]

[13] Rubbab, Q., Nazeer, M., Ahmad, F., Chu, Y.-M., Khan, M. I., & Kadry, S. (2021). Numerical simulation of advection–diffusion equation with Caputo-Fabrizio time fractional derivative in cylindrical domains: Applications of pseudo-spectral collocation method. *Alexandria Engineering Journal, 60*(1), 1731–1738. [CrossRef]

[14] Abbasi, A., Khan, S. U., Farooq, W., Mughal, F. M., Khan, M. I., Prasannakumara, B. C., ... & Galal, A. M. (2023). Peristaltic flow of chemically reactive Ellis fluid through an asymmetric channel: Heat and mass transfer analysis. *Ain Shams Engineering Journal, 14*(1), 101832. [CrossRef]

[15] Li, Y. X., Alqsair, U. F., Ramesh, K., Khan, S. U., & Khan, M. I. (2022). Nonlinear heat source/sink and activation energy assessment in double diffusion flow of micropolar (non-Newtonian) nanofluid with convective conditions. *Arabian Journal for Science and Engineering, 47*(1), 859–866. [CrossRef]

[16] Zahid, M., Ali, F., Souayeh, B., & Khan, M. T. (2024). Influence of variable viscosity on existing sheet thickness in the calendering of non-isothermal viscoelastic materials. *Open Physics, 22*(1), 20240023. [CrossRef]

[17] Anwar, M. S., Irfan, M., & Muhammad, T. (2024). Non-Newtonian fluid flow over a stretching sheet in a porous medium with variable thermal conductivity under magnetohydrodynamics influence. *ZAMM-Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik, 104*(12), e202301048. [CrossRef]

[18] Abbas, M. S., Shaheen, A., Abbas, N., & Shatanawi,

W. (2024). Dynamic analysis of radiative chemical species with non-Newtonian fluid flow over an exponentially curved stretching sheet. *Proceedings of the Institution of Mechanical Engineers, Part E: Journal of Process Mechanical Engineering*, 09544089241271866. [CrossRef]

[19] Ghosh, S., Chatterjee, S., & Mondal, H. (2025). Entropy generation over nonlinear thermally radiative and mixed convective nanofluid flow. In *EPJ Web of Conferences* (Vol. 325, p. 01016). EDP Sciences. [CrossRef]

[20] Kumar, R. M., Raju, R. S., Mebarek-Oudina, F., Kumar, M. A., & Narla, V. K. (2024). Cross-diffusion effects on an MHD Williamson nanofluid flow past a nonlinear stretching sheet immersed in a permeable medium. *Frontiers in Heat and Mass Transfer, 22*(1), 15-34. [CrossRef]

[21] Nagendra, V. B., Narasimhamurthy, S., Shankar, S., & Munivenkatappa, U. (2024). Mixed Convection of Heat and Mass Transfer of Casson Fluid Flow Over a Moving Horizontal Plate with Convective Boundary Conditions.

[22] Ram Sharma, K., & Jain, S. (2024). A numerical study of MHD nonlinear mixed convection flow over a nonlinear vertical stretching sheet with the buoyancy and suction/injection effects. *Numerical Heat Transfer, Part B: Fundamentals*, 1-30. [CrossRef]

[23] Akolade, M. T., Idowu, A. S., Oyekunle, T. L., Agunbiade, S. A., Momoh, H. O., Dada, M. S., & Olotu, O. T. (2025). Preserving the rheological equation of Eyring-Powell fluid through non-similar approach: A numerical analysis by BSCM. *The European Physical Journal Plus, 140*(2), 1–12. [CrossRef]

[24] Peter, F., Sambath, P., & Dhanasekaran, S. (2024). Analyzing the MHD bioconvective Eyring–Powell fluid flow over an upright cone/plate surface in a porous medium with activation energy and viscous dissipation. *Computation, 12*(3), 48. [CrossRef]

[25] Gundagani, M., Babu, N. V. N., Gadially, D., Bhati, S. M., Ch, S., & Nirmala Kasturi, V. (2024). Study of Nano-Powell-Erying fluid flow past a porous stretching sheet by the effects of MHD, thermal and mass convective boundary conditions. *Journal of Umm Al-Qura University for Engineering and Architecture, 15*(3), 271-281. [CrossRef]

[26] Turkyilmazoglu, M. (2013). The analytical solution of mixed convection heat transfer and fluid flow of a MHD viscoelastic fluid over a permeable stretching surface. *International Journal of Mechanical Sciences, 77*, 263–268. [CrossRef]

Dr. Muhammad Zubair is working as a Postdoc fellow in School of Qilu Transportation, Shandong University, Jinan, 250061, China. He is working mainly in the field of Newtonian, non-Newtonian fluid mechanics, Porous Media, Heat and mass transfer, stretching sheet, stretching cylinders, curved surfaces etc. (Email: mzubair@sdu.edu.cn)