RESEARCH ARTICLE

Bornological Semi Continuous Maps

Fatma Kamil AL-Basri[1,*]

[1] Department of Mathematics, College of Education, University of Al-Qadisiyah, Al-Diwaniyah 58001, Iraq

Abstract

In the current study, a new approach had been constructed to define new maps using the concept of bornological semi open and bornological semi closed sets, which includes sequential bornological semi continuous maps, bornological semi closed (open) maps, bornological strongly semi closed (open) maps, and bornological semi-irresolute closed (open) maps. We investigate and study the properties of these concepts.

Keywords: bornological semi open map, bornological semi closed map, bornological semi continuous map.

1 Introduction

Bornology on a set and convergence of sequences in bornological vector space have been studied by H. Hogbe-Nlend after he introduced b-closed set (b-open set) [1]. The concepts of a semi-bounded set, a semi-bounded linear map, semi-convergence, and the concept of a semi-unbounded linear map in bornological space and product space are also introduced by [2]. In [3, 4], Al-Basri studied the concepts of a bornological semi-convergent net in convex (bvs) space. Ameer [2] defined the concept of semi-compactness in bornological space. Semi-open, strongly semi-open, and semi-irresolute open sets play an important role in the study of continuity generalizations in topological spaces. By using these sets, many authors introduced and investigated various types of modifications to continuity. In 1963, Levine [5] introduced the notions of semi-open sets and semi-continuity in topological spaces. It is shown in [6] that semi-continuity is equivalent to quasi-continuity based on the work of Marcus [7]. The concepts of strongly continuous maps found in [8] and semi-generalized irresolute maps can be found in [9].

In this work, new types of maps in convex bornological vector space "cbvs" within bornological vector space E will be introduced. Further properties about sequentially bornological semi continuous "seq bs-cont" maps, bornological semi closed "bs-cl" maps, bornological strongly semi closed "bss-cl" maps, bornological semi irresolute closed "bsi-cl" maps, bornological semi open "bs-op" maps, bornological strongly semi open "bss-op" maps, and bornological semi irresolute open "bsi-op" maps have been introduced with their relationships.

2 Preliminaries

Definition 1 *Consider E as a bornological vector space (bvs). A subset A of E is said to be bornological semi-open (bs-op, in short) if for every sequence $\{x_n\}_{n\in\mathbb{N}} \subseteq E$, and $x_n \xrightarrow{s} x$ then $x \in A$ then $x_n \in A\ \forall n > n_o$.*

Definition 2 *Consider E as a bounded vector space (bvs). A subset A contained within E is termed bornological*

Submitted: 18 June 2025
Accepted: 11 July 2025
Published: 04 August 2025

Vol. 1, **No.** 2, 2025.
10.62762/JAM.2025.997630

***Corresponding author:**
Fatma Kamil AL-Basri
fatma.Albasri@qu.edu.iq

Citation
AL-Basri, F. K. (2025). Bornological Semi Continuous Maps. *ICCK Journal of Applied Mathematics*, 1(2), 62–65.

semi-closed or abbreviated as bs-cl if the following condition is satisfied: If you have a sequence $\{x_n\}$ indexed by natural numbers $\mathbb{N}$, where all the elements are part of A and these elements x_n tend to approach a point x in E according to the bornological semi convergence, then it must also be the case that x belongs to the set A.

Remark 1 *In a (bvs) every b-op $(b\text{-cl})$ set is bs-op $(bs\text{-cl})$, but the converse is not true in general.*

Proposition 1 *Take A be a b-op $(b\text{-cl})$ set in a (bvs) E. Then:*

(i) *If B is bs-op $(bs\text{-cl})$ set in E, then $B \bigcap A$ is bs-op $(bs\text{-cl})$ set in E.*

(ii) *If B is bs-op $(bs\text{-cl})$ set in E then $B \bigcap A$ is bs-open $(bs\text{-cl})$ set in A.*

Proof 1 (i) *Take A to be a b-op and take $\{x_n\}_{n\in\mathbb{N}} \subseteq E$ semi converging bornological to a point $x \in A \cap B$, then $x \in A$ and $x \in B$. If $x \in A$ and by Remark 1 A is bs-op then $x_n \in A\ \forall\ n > n_o$. If $x \in B$ and B is bs-op then $x_n \in B\ \forall\ n > n_o$, we have $x_n \in A \cap B\ \forall\ n > n_o$ then $A \cap B$ is bs-op.*

Now take A be a b-cl $\{x_n\}_{n\in\mathbb{N}} \subset A \cap B$ and $x_n \xrightarrow{s} x$ in E then $\{x_n\}_{n\in\mathbb{N}} \subseteq B$ and $\{x_n\}_{n\in\mathbb{N}} \subseteq A$. If $\{x_n\}_{n\in\mathbb{N}} \subseteq B$ and since B is bs-cl we have $x_n \xrightarrow{s} x$ then $x \in B$. If $\{x_n\}_{n\in\mathbb{N}} \subseteq A$ and since A is b-cl then by Remark 1 A is bs-cl we have $x_n \xrightarrow{s} x$ then $x \in A$, then $x \in A \cap B$.

(ii) *Same approach as above proof.*

Proposition 2 *Take E be a (bvs) and $B \subseteq A \subseteq E$. Then:*

(i) *If B is bs-op $(bs\text{-cl})$ set in A and A bs-op $(bs\text{-cl})$ set in E then B is bs-op $(bs\text{-cl})$ set in E.*

(ii) *If B is bs-op $(bs\text{-cl})$ set in E then B is bs-op $(bs\text{-cl})$ in A.*

Proof 2 (i) *Let $\{x_n\}_{n\in\mathbb{N}} \subseteq E$ and $x_n \xrightarrow{s} x$ such that $x \in B$, thus $x \in B \subseteq A$ we have $x \in A$, since A is bs-op set in E and $\{x_n\}_{n\in\mathbb{N}} \subseteq A$, $x_n \xrightarrow{s} x$ such that $x \in B$, since B is bs-op in A then $\{x_n\}_{n\in\mathbb{N}} \subseteq B$, we have B is bs-op in E.*

Definition 3 *If we take E to be a bounded vector space (bvs), then the bornological semi-closure of a subset A in E, denoted as bs-clr(A), is the intersection of all bs-cl subsets of E that contain the set A.*

3 Some Types of Bornological Semi Maps

This section includes some basic properties about sequentially bornological semi continuous map is denoted by seq bs-con map, bs-cl(bs-op) map, bss-cl(bss-op) map, bsi-cl(op) map and relationships among their maps are investigated.

Definition 4 *Let E, F are (bvs) and let f be a mapping from E into F. We say that f is a seq bs-con map at a point x if for any sequence $\{x_n\}_{n\in\mathbb{N}}$ in E, $x_n \xrightarrow{s} x$ then $f(x_n) \xrightarrow{s} f(x)$ in F. If f is a seq bs-con map at every point x in E, then f is called seq bs-con map.*

Example 1 *Inclusion map $i : A \to F$ in a bornological vector space (bvs) is sequential bornological semi continuous map $(bs\text{-con})$ map if and only if A is closed set in E.*

Definition 5 *Let E and F are (bvs)s, a map $f : E \to F$ is called:*

(i) *bs-cl map if $\forall$ b-cl set A in X, $f(A)$ is bs-cl set in F*

(ii) *bss-cl map if $\forall$ bs-cl set A in E, $f(A)$ is b-cl set in F*

(iii) *bsi-cl map if $\forall$ bs-cl set A in E, $f(A)$ is bs-cl set in F*

Definition 6 *Let E and F are bornological vector spaces (bvs)s, a map $f : E \to F$ is called:*

(i) *bs-op map if $\forall$ b-op set A in E, $f(A)$ is bs-op set in F*

(ii) *bss-op map if $\forall$ bs-op set A in E, $f(A)$ b-op set in F*

(iii) *bsi-op map if $\forall$ bs-op set A in E, $f(A)$ is bs-op set in F*

Remark 2 *Every b-cl $(b\text{-op})$ map is bs-cl $(bs\text{-op})$ map [10].*

Theorem 1 *Let E, F and H are (bvs), $f : E \to F$, $g : F \to H$, then, if f is b-cl map and g is bs-cl map, then $g \circ f$ is bs-cl.*

Proof 3 *Let $A \subseteq E$ is a b-cl set, since f b-cl map then $f(A)$ b-cl set in F, by g bs-cl map we have $g(f(A))$ is bs-cl set in H, then $g \circ f$ is bs-cl map.*

Theorem 2 *Let E, F and H are (bvs)s, $f : E \to F$, $g : F \to H$*

(i) *If f is bss-cl map and g is bss-cl map then $g \circ f$ is bss-cl map.*

(ii) *If f is bsi-cl map and g is bsi-cl map then $g \circ f$ is bsi-cl map.*

Proof 4 (i) *Let $A \subseteq E$ is a bs-cl set, since f bss-cl map then $f(A)$ is b-cl set by Remark 2 then $f(A)$ is bs-cl in F, by g bss-cl map we have $g(f(A))$ is b-cl set in H, then $g \circ f$ is bss-cl map.*

(ii) *Let $A \subseteq E$ is a bs-cl set, since f bsi-cl map, then $f(A)$ is bsi-cl set in F. On the other hand, g is bsi-cl map, we have $g(f(A))$ is bsi-cl set in H, then $g \circ f$ is bsi-cl map.*

Example 2 *Let A be subset of a (bvs) E, then the inclusion map $i : A \to E$ is b-cl (bs-cl) iff A is b-cl set in E.*

Theorem 3 *Let E, F are (bvs)s. Assuming $f : E \to F$ and $g : E \to F$ seq bs-con map then:*

(i) *cf where $cf : E \to F$ seq bs-con map $\forall\, c \in K$*

(ii) *$f + g$ seq bs-con map*

(iii) *$f \cdot g$ is a seq bs-con map*

Proof 5 *(i) Let $x \in E$ and $\{x_n\}$ be a sequence in (bvs) E, such that $x_n \xrightarrow{s} x$. Since f is a seq bs-con at x, then $f(x_n) \xrightarrow{s} f(x)$ in F implies $cf(x_n) \xrightarrow{s} cf(x)\ \forall\, c \in K$. Then $\forall\, x$ we have cf seq bs-con.*

(ii) Let $x \in E$ and $\{x_n\}$ be a sequence in (bvs) E, such that $x_n \xrightarrow{s} x$. Since f, g seq bs-con map at x then $f(x_n) \xrightarrow{s} f(x)$ in F and $g(x_n) \xrightarrow{s} g(x)$ in F, then $f(x_n) + g(x_n) \xrightarrow{s} f(x) + g(x)$ in F. We have $(f + g)(x_n) \xrightarrow{s} (f + g)(x)$ in F, then $f + g$ is a seq bs-con map at every point x in E. Then f is seq bs-con map.

(iii) Let $x \in E$ and $\{x_n\}$ be a sequence in (bvs) E, such that $x_n \xrightarrow{s} x$. Since f, g seq bs-con map at x then $f(x_n) \xrightarrow{s} f(x)$ in F and $g(x) \xrightarrow{s} g(x)$ in F then $f(x_n) \cdot g(x_n) \xrightarrow{s} f(x) \cdot g(x)$ in F, hence $(f \cdot g)(x_n) \xrightarrow{s} (fg)(x)$ in F at every x in E then $f \cdot g$ is seq bs-con map.

Theorem 4 *Let E and F are (bvs)s and A a non-empty subset of E if $f : E \xrightarrow{s} F$ is a seq bs-con map then the restriction f_A is a seq bs-con map, where A has the relative bornology B_A.*

Proof 6 *Let $x \in A$ and $\{x_n\}_{n\in\mathbb{N}} \subseteq A$ such that $x_n \xrightarrow{s} x$ in A. Since $A \subseteq E$ then $x_n \xrightarrow{s} x$ in E. Since $f : E \to F$ is a seq bs-con map then if $x_n \xrightarrow{s} x$ we have $f(x_n) \xrightarrow{s} f(x)$ hence f_A is a seq bs-con map.*

Theorem 5 *A mapping f where $f : E \to F$ from a bornological vector spaces (bvs) E into (bvs) F is seq bs-con map, for every B is bs-cl set in F then $f^{-1}(B)$ is bs-cl set in E.*

Proof 7 *Let B is bs-cl set in F, if $f^{-1}(B) = \phi$ and thus the proof is complete. If $f^{-1}(B) \neq \phi$. Let $x_n \in f^{-1}(B)$ such that $x_n \xrightarrow{s} x$ in $f^{-1}(B) \subseteq E$, we have $f(x_n) \in B$. Since f is seq bs-con map then $f(x_n) \xrightarrow{s} f(x)$. Since B is bs-cl set then $f(x) \in B$. We have $x \in f^{-1}(B)$ then $f^{-1}(B)$ is bs-cl set in E.*

Theorem 6 *Let E and F are (bvs)s, if $f : E \to F$ is seq bs-con map then:*

(i) *f(bs-clr A) $\subseteq$ bs-clr $f(A)$ for every $A \subseteq E$*

(ii) *bs-clr $f^{-1}(B) \subseteq f^{-1}$(bs-clr B) for every $B \subseteq F$*

Proof 8 *(i) Let f be seq bs-cont. Since bs-clr $f(A)$ is bs-cl in f^{-1}(bs-clr $f(A)$) is bs-cl set in E (Theorem 5) bs-clr $f^{-1}(f(A)) = f^{-1}$(bs-clr $f(A)$). Since $f(A) \subseteq$ bs-clr $f(A) \Rightarrow A \subseteq f^{-1}(f(A)) \subseteq f^{-1}$(bs-clr $f(A)$), then bs-clr $A \subseteq f^{-1}$(bs-clr $f(A)$) [since $f^{-1}(f(A))$ is bs-cl set in E and by Remark 3]. We have f(bs-clr A) $\subseteq f^{-1}$(bs-clr $f(A)$)*

(ii) Let f be seq bs-con map. Since bs-clr B is bs-cl set in F, then f^{-1}(bs-clr B) is bs-cl set in E (Theorem 5). f^{-1}(bs-clr B) $= f^{-1}(B)$. (1). Since $B \subset$ bs-clr B then f^{-1}(bs-clr B) $\subseteq f^{-1}(B)$. We have bs-clr $f^{-1}(B) =$ bs-clr f^{-1}(bs-clr B) $= f^{-1}$(bs-clr B) [by (1)].

Theorem 7 *Let E, F and H be (bvs)s and let $f : E \to F$ be a seq bs-cont map at a point x, $g : F \to H$ be seq bs-con at $f(x)$ then $g \circ f : E \to H$ be a seq bs-con map at a point x.*

Proof 9 *Let $\{x_n\}$ be a sequence in E such that $x_n \xrightarrow{s} x$ since f is a seq bs-con map at x then $f(x_n) \xrightarrow{s} f(x)$. Since $\{f(x_n)\}$ a sequence in F. Since g is a seq bs-con map at $f(x)$ then $g(f(x_n)) \xrightarrow{s} g(f(x))$. Then $(g \circ f)(x_n) \xrightarrow{s} (g \circ f)(x)$. Hence $g \circ f$ is a seq bs-con map at x.*

4 Conclusion

In this study, we introduced and explored a novel framework for defining new classes of maps grounded in the notions of bornological semi open and bornological semi closed sets. Through this approach, we established and analyzed several types of maps, including sequential bornological semi continuous maps, bornological semi closed (open) maps, bornological strongly semi closed (open) maps, and bornological semi-irresolute closed (open) maps. Our investigation provided insight into the structural properties and interrelationships among these mappings, offering a foundation for further development in the field of bornological topology and its applications in general topological structures.

Data Availability Statement

Not applicable.

Funding

This work was supported without any funding.

Conflicts of Interest

The author declares no conflicts of interest.

Ethical Approval and Consent to Participate

Not applicable.

References

[1] Hogbe-Nlend, H. (1977). *Bornologies and functional analysis: introductory course on the theory of duality topology-bornology and its use in functional analysis* (Vol. 26). Elsevier.

[2] Ameer, A. A., & Huda, A. (2019). Semi Compactness Space in Bornological Space. *Diyala Journal for Pure Science, 15*(4). [Crossref]

[3] Al-Basri, F. K. (2017). On Semi â€"Complete Bornological Vector Space. Journal of AL-Qadisiyah for computer science and mathematics, 9(1), 40-48.

[4] Al-Basri, F. K. M. (2014). The Relationship Between Boronological Convergence of Net and Topological Convergence of Net. *Journal of AL-Qadisiyah for computer science and mathematics, 6*(2), 65-76.

[5] Levine, N. (1963). Semi-open sets and semi-continuity in topological spaces. *The American mathematical monthly, 70*(1), 36-41. [Crossref]

[6] Neubrunnová, A. (1973). On certain generalizations of the notion of continuity. *Matematický časopis, 23*(4), 374-380.

[7] Marcus, S. (1961). Sur les fonctions quasicontinues au sens de S. Kempisty. In *Colloquium Mathematicae* (Vol. 8, No. 1, pp. 47-53).

[8] Balachandran, K. (1991). On generalized continuous maps in topological spaces. *Mem. Fac. Sci. Kochi Univ. Ser. A Math., 12*, 5-13.

[9] Cueva, M. C. (1995). Semi-generalized continuous maps in topological spaces. *Portugaliae Mathematica, 52*(4), 399-407.

[10] Al-Basri, F. K. (2018). Sequentially Bornological Compact Space. *AL-Qadisiyah Journal of pure Science, 23*(2). [Crossref]