RESEARCH ARTICLE

CRITIC-EDAS Method for Linguistic Picture Fuzzy Soft Sets and Its Application in Decision Making Problem

Md. Yasin Ali[1,*] **and Md. Ashraful Islam**[2]

[1] Faculty of Science and Engineering, University of Information Technology and Sciences, Dhaka 1212, Bangladesh
[2] Department of Computer Science and Engineering, Military Institute of Science and Technology, Dhaka 1216, Bangladesh

Abstract

Linguistic picture fuzzy soft sets (LPFSSs) provide a powerful tool for handling uncertainties in decision-making problems, particularly when multiple parameters are involved. By integrating the advantages of linguistic picture fuzzy sets (LPFSs) and soft sets (SSs), LPFSSs prove especially effective in situations involving imprecise and ambiguous information. This study extends the conventional CRITIC (Criteria Importance Through Inter-criteria Correlation) and EDAS (Evaluation based on Distance from Average Solution) methods to the LPFSS environment. First, the definitions of LPFSSs and linguistic picture fuzzy soft numbers (LPFSNs) are introduced along with their key properties. Subsequently, a CRITIC-EDAS framework is developed under the linguistic picture fuzzy soft setting. Finally, a numerical example concerning the selection of the most suitable organic fertilizer is presented to illustrate the effectiveness of the proposed approach.

Keywords: linguistic picture fuzzy set, linguistic picture fuzzy soft set, CRITIC method, EDAS method.

1 Introduction

In daily life, we frequently encounter various complicated problems that are filled with uncertainty, vagueness etc. Such uncertainties may be unachievable to model using conventional mathematical approaches. To handle like this uncertainties, fuzzy set (FS) [1] theory came into continuation wherein a membership degree is assigned to each element. But, in various realistic decision-making problems, due to the intricacy of the decision-making atmosphere, the decision makers may desire to use fuzzy linguistic variables [2–4] to capture the nuance of human opinion. Fuzzy linguistic term sets which are the extension of fuzzy linguistic variables have been studied and applied to many fields [5–10] etc..

However, in representing decision makers' opinions in qualitative assessments concerning multiple responses: yes, abstain, no, refusal, the concept of linguistic picture fuzzy set (LPFS) was introduced by Qiyas et al. [11]. They also developed weighted averaging and weighted geometric operators and discuss their properties. They also proposed an approach to deal with a multiple-attribute group decision-making (MAGDM) problem based on the

Submitted: 13 January 2026
Revised: 13 February 2026
Accepted: 16 February 2026
Published: 16 March 2026

Vol. 2, **No.** 1, 2026.
10.62762/JAM.2026.193939

***Corresponding author:**
✉ Md. Yasin Ali
ali.mdyasin56@gmail.com

Citation
Ali, M. Y., & Islam, M. A. (2026). CRITIC-EDAS Method for Linguistic Picture Fuzzy Soft Sets and Its Application in Decision Making Problem. *ICCK Journal of Applied Mathematics*, 2(1), 111–119.

© 2026 by the Authors. Published by Institute of Central Computation and Knowledge. This is an open access article under the CC BY license (https://creativecommons.org/licenses/by/4.0/).

developed aggregation operators. Liu et al. [12] considers the multi-criteria decision-making problem based on linguistic picture fuzzy information, where they used the TOPSIS and TODIM Methods to rank of the alternatives and entropy method to calculate the weight of criteria.

Recently, Soft sets [13] have become a flexible mathematical structure to model and handle uncertainty, vagueness, and incomplete information in complex systems. This makes them valuable for a wide range of real-world applications, especially in decision-making, data analysis, and various fields like engineering and medical diagnosis. This theory has been extended and combined with other concepts to create new mathematical tools, such as fuzzy soft sets [14] and intuitionistic soft set (ISS) theory [15], further enhancing its applicability.

The criteria weight determination method is a technique used in multi-criteria decision-making to assign a numerical weight to each criterion based on its importance for a problem. These weights are crucial for ranking and selecting the best alternative, with methods falling into two main categories: subjective, objective. Subjective methods for determining criteria weights rely on human judgment, expert opinion, and decision-maker preferences rather than objective data. Some popular subjective weighting methods are pairwise-comparison-based methods [16, 17], KEMIRA [18], P-SWING [19], PIPRECIA [20], FUCOM [21], and so on. Objective methods for determining criteria weights use computational processes based on the data itself, rather than subjective expert opinions. Some common objective weighting methods are entropy-based methods [22], CRITIC method [23], CILOS and IDOCRIW methods [24] etc. Based on our literature review, it appears that CRITIC are the most appropriate objective methods for the weighting of criteria, because it evaluates the criteria by considering both their contrast intensity (standard deviation of normalized values) and conflicting relationships (correlation coefficients between criteria) within a decision matrix. By analyzing how much each criterion varies and how it conflicts with others, the method calculates weights that reflect the true informational content of the data, leading to more accurate and reliable decision-making.

On the other hand the EDAS technique was initiated by Keshavarz et al. [25] in 2015 has become an essential tool in DM situations in diverse portions of our real living such as agriculture [26–28], business [29–31], construction [32–34], energy [35–37], healthcare management [38, 39], engineering and technology [40] etc..

In this article, we introduce the concept of linguistic picture fuzzy soft sets and discuss some of their properties. Then, we combine the CRITIC method and EDAS method under the linguistic picture fuzzy soft environment. Afterword, we illustrate a MCDM problem on the basis of our proposed CRITIC-EDAS method.

The paper comprises the following parts: Section 2 provides a brief overview of Linguistic Term Set, Linguistic Picture Fuzzy Sets, and their related properties. In Section 3, the definition of Linguistic Picture Fuzzy Soft Set is given and described some of their related properties. Section 4 detail description CRITIC-EDAS method is illustrated. Section 5 an illustrative example is given to show the reliability and effectiveness of our proffered approach. Section 6, sensitivity analysis is given of our proposed method. Section 7 concludes the paper.

2 Preliminary Concepts

2.1 Linguistic Term Set

According to Herrera and Herrera-Viedma, let $\mathscr{L} = \{l_h \mid h = 0, 1, 2, \ldots, g\}$ be a linguistic term set (LTS) with odd cardinality, where l_h represents a possible value of a linguistic variable. For example, consider the linguistic variable "intelligence" and define its associated term set consisting of five linguistic terms as follows:

$$\begin{aligned}\mathscr{L} = \{&l_0 = \text{Very Low (VL)},\ l_1 = \text{Low (L)},\\ &l_2 = \text{Medium (M)},\ l_3 = \text{Very Medium (VM)},\\ &l_4 = \text{High (H)}\}.\end{aligned}$$

In this exposition, it is essential that these linguistic terms possess the following characteristics.

1. If $l_m, l_n \in \mathscr{L}$ and $m < n$, then $l_m < l_n$;
2. Negation Operator: $\text{Neg}(l_m) = l_n$, where $m = l - n$;
3. $Max(l_m, l_n) = l_m$, if $l_m \geq l_n$;
4. $Min(l_m, l_n) = l_m$, if $l_m \leq_n$

Besides this, to get original decision information, the discrete term set $\mathscr{L}$ is expanded to continuous form [6] as $\mathscr{L} = \{l_h | h = [0, g]\}$, where h is adequately large positive integer.

2.2 Linguistic Picture Fuzzy Sets

According to Qiyas et al. [11], a linguistic picture fuzzy set (LPFS) $\mathscr{A}$ on the universe $\mathscr{U}$ is defined as

$$\mathscr{A} = (\mathscr{Y}, \mathscr{A}_\alpha(\mathscr{Y}), \mathscr{A}_\gamma(\mathscr{Y}), \mathscr{A}_\beta(\mathscr{Y}) : \mathscr{Y} \in \mathscr{U}),$$

where $\mathscr{A}_\alpha(\mathscr{Y}), \mathscr{A}_\gamma(\mathscr{Y})$, and $\mathscr{A}_\beta(\mathscr{Y})$ represent, respectively, linguistic positive membership degree, linguistic neutral membership degree and linguistic negative membership degree of the element $\mathscr{Y}$ to the set $\mathscr{A}$.

For any element $\mathscr{Y} \in \mathscr{U}$, the condition $\mathscr{A}_\alpha(\mathscr{Y}), \mathscr{A}_\gamma(\mathscr{Y}), \mathscr{A}_\beta(\mathscr{Y}) \in \mathscr{L}_{[0,g]}$ and $0 \leq \alpha + \gamma + \beta \leq [0, g]$ are always accepted. The linguistic refusal membership degree is $\mathscr{A}_{g-\alpha-\gamma-\beta}(\mathscr{Y})$. We will denote the triple $(\mathscr{A}_\alpha(\mathscr{Y}), \mathscr{A}_\gamma(\mathscr{Y}), \mathscr{A}_\beta(\mathscr{Y}))$ as $\mathscr{A} = (\mathscr{A}_\alpha, \mathscr{A}_\gamma, \mathscr{A}_\beta)$ indicated to linguistic picture fuzzy number (LPFN).

3 Linguistic Picture Fuzzy Soft Sets

Suppose that $\mathscr{U}$ be a collection of all LPFSs and $\mathscr{E}$ be the set of parameters in relation to $\mathscr{U}$. Let $P(\mathscr{U})$ denote the power set of $\mathscr{U}$ and $\mathscr{P} \subseteq \mathscr{E}$.

Definition 3.1 A pair $(\mathscr{F}, \mathscr{P})$ is called a LPFSS over $\mathscr{U}$, where $\mathscr{F}$ is a mapping given by $\mathscr{F} : \mathscr{P} \to P(\mathscr{U})$.

For any parameter $\epsilon \in \mathscr{P}$, $\mathscr{F}(\epsilon)$ is a LPFSS given by:

$$\mathscr{F}(\epsilon) = \{u_i, \mathscr{F}(\epsilon)_\alpha(u_i), \mathscr{F}(\epsilon)_\gamma(u_i), \mathscr{F}(\epsilon)_\beta(u_i) : \mathscr{Y} \in \mathscr{U}\}$$

where $\mathscr{F}(\epsilon)_\alpha(u_i), \mathscr{F}(\epsilon)_\gamma(u_i)$ and $\mathscr{F}(\epsilon)_\beta(u_i)$ represent, respectively, linguistic positive membership degree, linguistic neutral membership degree and linguistic negative membership degree with the condition:

$$0 \leq \alpha + \gamma + \beta \leq [0, g]$$

and the refusal degree is $\mathscr{F}(\epsilon)_{(l-\alpha+\gamma+\beta)}(u_i)$.

Example 3.2 *Let $U = \{\mathscr{H}_1, \mathscr{H}_2, \mathscr{H}_3, \mathscr{H}_4, \mathscr{H}_5\}$ be a set of houses that are considered for rent. Let $\mathscr{P} = \{\epsilon_1, \epsilon_2, \epsilon_3\}$ be a set of parameters, where $\epsilon_1 =$ expensive , $\epsilon_1 =$ beautiful and $\epsilon_3 =$ good location. Then, the inducement of the houses can be depicted, as a LPFSS delineated by:*

$$(\mathscr{F}, \mathscr{P}) = \{\mathscr{F}(\epsilon_1), \mathscr{F}(\epsilon_2), \mathscr{F}(\epsilon_3)\}$$

where,

$$\begin{aligned}
\mathscr{F}(\epsilon_1) &= \{(\mathscr{H}_1, (l_6, l_1, l_1), (\mathscr{H}_2, (l_4, l_2, l_1), (\mathscr{H}_3, (l_5, l_1, l_1), \\
&\quad (\mathscr{H}_4, (l_3, l_2, l_2), (\mathscr{H}_5, (l_6, l_0, l_1)\}; \\
\mathscr{F}(\epsilon_2) &= \{(\mathscr{H}_1, (l_4, l_2, l_2), (\mathscr{H}_2, (l_3, l_2, l_2), (\mathscr{H}_3, (l_6, l_1, l_1), \\
&\quad (\mathscr{H}_4, (l_3, l_3, l_2), (\mathscr{H}_5, (l_5, l_0, l_1)\}; \\
\mathscr{F}(\epsilon_3) &= \{(\mathscr{H}_1, (l_4, l_2, l_2), (\mathscr{H}_2, (l_4, l_0, l_1), (\mathscr{H}_3, (l_5, l_0, l_1), \\
&\quad (\mathscr{H}_4, (l_6, l_2, l_0), (\mathscr{H}_5, (l_6, l_0, l_1)\}\},
\end{aligned} \tag{1}$$

for $0 \leq \alpha + \gamma + \beta \leq [0, 8]$.

A LPFSS set can be represented in a matrix format for easier storage and access on a computer.

$$(\mathscr{F}, \mathscr{P}) = \begin{array}{c} \\ \mathscr{H}_1 \\ \mathscr{H}_2 \\ \mathscr{H}_3 \\ \mathscr{H}_4 \\ \mathscr{H}_5 \end{array} \begin{array}{c} \begin{array}{ccc} \epsilon_1 & \epsilon_2 & \epsilon_3 \end{array} \\ \begin{bmatrix} (l_6, l_1, l_1) & (l_4, l_2, l_2) & (l_4, l_2, l_2) \\ (l_4, l_2, l_1) & (l_3, l_2, l_2) & (l_4, l_0, l_1) \\ (l_5, l_1, l_1) & (l_6, l_1, l_1) & (l_5, l_0, l_1) \\ (l_3, l_2, l_2) & (l_3, l_3, l_2) & (l_6, l_2, l_0) \\ (l_6, l_0, l_1) & (l_5, l_0, l_1) & (l_6, l_0, l_1) \end{bmatrix} \end{array}$$

In addition, for convenience and necessary computations, LPFSS can also be treated as $\mathscr{F}_\mathscr{P} = (l_\alpha, l_\gamma, l_\beta)$ and called as LPFSN, where $\mathscr{P}$ is referential subscript used for establishing a link between alternatives and parameters in computational examples. For application purposes, to rank these numbers, the score and accuracy functions for the LPFSN are interpreted as follows:

Definition 3.3 Let $\mathscr{F}_\mathscr{P} = (l_\alpha, l_\gamma, l_\beta)$ be a LPFSN, the score function and accuracy function of $\mathscr{F}_\mathscr{P}$ are defined as

$$\mathscr{S}(\mathscr{F}_\mathscr{P}) = \alpha - (\gamma + \beta) \tag{2}$$

$$\mathscr{H}(\mathscr{F}_\mathscr{P}) = \alpha + \gamma + \beta \tag{3}$$

Definition 3.4 Let $\mathscr{F}_\mathscr{P} = (l_{\alpha_1}, l_{\gamma_1}, l_{\beta_1})$ and $\mathscr{F}_\mathscr{Q} = (l_{\alpha_2}, l_{\gamma_2}, l_{\beta_2})$ be two LPFSNs. Then $\mathscr{F}_\mathscr{P}$ and $\mathscr{F}_\mathscr{Q}$ can be compared by the following rule.

1. If $\mathscr{S}(\mathscr{F}_\mathscr{P}) > \mathscr{S}(\mathscr{F}_\mathscr{Q})$, subsequently $\mathscr{F}_\mathscr{P} > \mathscr{F}_\mathscr{Q}$.
2. If $\mathscr{S}(\mathscr{F}_\mathscr{P}) = \mathscr{S}(\mathscr{F}_\mathscr{Q})$, subsequently
   (a) If $\mathscr{H}(\mathscr{F}_\mathscr{P}) > \mathscr{H}(\mathscr{F}_\mathscr{Q})$, subsequently $\mathscr{F}_\mathscr{P} > \mathscr{F}_\mathscr{Q}$
   (b) I If $\mathscr{H}(\mathscr{F}_\mathscr{P}) = \mathscr{H}(\mathscr{F}_\mathscr{Q})$, subsequently $\mathscr{F}_\mathscr{P} = \mathscr{F}_\mathscr{Q}$.

4 CRITIC-EDAS Method for Linguistic Picture Fuzzy Soft Sets

4.1 Problem Description

In this section, we proposed the CRITIC-EDAS method to deal with MCDM in the LPFSNs. Supposing that a linguistic picture fuzzy soft MCDM has m alternatives $\mathscr{M} = \{\mathscr{M}_1, \mathscr{M}_2, \ldots, \mathscr{M}_m\}$, each alternative is assessed on the basis of n criterion $\mathscr{C} = \{\mathscr{C}_1, \mathscr{C}_2, \ldots, \mathscr{C}_n\}$ and $\mathscr{W} = \{\mathscr{W}_1, \mathscr{W}_2, \ldots, \mathscr{W}_j\}$ is the corresponding weight vector of criterion $\mathscr{C}_j (j = 1, 2, \ldots n)$ with $\sum_{j=1}^{n} \mathscr{W}_j = 1 (0 \leq \mathscr{W}_j \leq 1)$. The assessment of alternatives given by the expert is characterized by LPFSNs $\mathscr{N}_{ij} = \left(l_{\alpha_{ij}}, l_{\gamma_{ij}}, l_{\beta_{ij}}\right) (i = 1, 2 \ldots, m; j = 1, 2 \ldots, n)$ and the linguistic picture fuzzy soft information matrix $\mathscr{N}$ is given as follows:

$$\mathscr{N} = [\mathscr{N}_{ij}]_{m\times n} = \begin{array}{c} \\ \mathscr{M}_1 \\ \mathscr{M}_2 \\ \\ \mathscr{M}_m \end{array} \begin{array}{c} \begin{array}{cccc} \mathscr{C}_1 & \mathscr{C}_2 & \cdots & \mathscr{C}_n \end{array} \\ \begin{bmatrix} (l_{\alpha_{11}}, l_{\gamma_{11}}, l_{\beta_{11}}) & (l_{\alpha_{12}}, l_{\gamma_{12}}, l_{\beta_{12}}) & \cdots & (l_{\alpha_{1n}}, l_{\gamma_{1n}}, l_{\beta_{1n}}) \\ (l_{\alpha_{21}}, l_{\gamma_{21}}, l_{\beta_{21}}) & (l_{\alpha_{22}}, l_{\gamma_{22}}, l_{\beta_{22}}) & \cdots & (l_{\alpha_{2n}}, l_{\gamma_{2n}}, l_{\alpha_{2n}}) \\ \vdots & \vdots & \ddots & \vdots \\ (l_{\alpha_{m1}}, l_{\gamma_{m1}}, l_{\beta_{m1}}) & (l_{\alpha_{m2}}, l_{\gamma_{m2}}, l_{\beta_{m2}}) & \cdots & (l_{\alpha_{mn}}, l_{\gamma_{mn}}, l_{\beta_{mn}}) \end{bmatrix} \end{array} \tag{4}$$

In the following, the CRITIC-EDAS method under linguistic picture fuzzy soft decision environment is outlined by the following computational steps.

4.2 CRITIC Method

Step 1. Determine the score function of $\mathscr{S} = [\mathscr{S}_{ij}]_{m\times n}$ by using the equation 1.

Step 2. Evaluate the standard deviations of the criteria by applying equation 4.

$$\sigma_j = \sqrt{\frac{\sum_{i=1}^{m} \left(\mathscr{S}_{ij} - \overline{\mathscr{S}}_j\right)^2}{m}} \tag{5}$$

where $\overline{\mathscr{S}}_j = \frac{\sum_{i=1}^{m} \mathscr{S}_{ij}}{m}$

Step 3. Find out the correlation between pairs of criteria by using equation 5 for computing analysis.

$$\rho_{jk} = \frac{\sum_{i=1}^{m} \left(\mathscr{S}_{ij} - \overline{\mathscr{S}}_j\right)\left(\mathscr{S}_{ij} - \overline{\mathscr{S}}_k\right)}{\sqrt{\sum_{i=1}^{m} \left(\mathscr{S}_{ij} - \overline{\mathscr{S}}_j\right)^2 \sum_{i=1}^{m} \left(\mathscr{S}_{ij} - \overline{\mathscr{S}}_k\right)^2}} \tag{6}$$

Step 4. Calculate the quantity of information of each criteria as follows:

$$c_j = \sigma_j \sum_{k=1}^{n} \left(1 - \rho_{jk}\right) \tag{7}$$

Step 5. Assess the objective weight of each criteria,

$$\mathscr{W}_j = \frac{c_j}{\sum_{j=1}^{n} c_j} \tag{8}$$

4.3 EDAS Method

Step 1. Choose the inherent parameters and probable alternatives, and accomplish the linguistic picture fuzzy soft information matrix $\mathscr{N}$ according to the equation equation 3.

Step 2. Determine the value of the average solution $(\mathscr{AV})$ by analyzing all the proposed criteria.

$$\begin{aligned} \mathscr{AV} &= \left[\mathscr{AV}_j\right]_{1\times n} = \left[\frac{\sum_{i=1}^{m} \left(l_{\alpha_{ij}}, l_{\gamma_{ij}}, l_{\beta_{ij}}\right)}{m}\right]_{1\times n} \\ &= \left[l_{\left(g - \prod_{i=1}^{m} (g - \alpha_{ij})^{\frac{1}{m}}\right)}, l_{\prod_{i=1}^{m} (\gamma_{ij})^{\frac{1}{m}}}, l_{\prod_{i=1}^{m} (\beta_{ij})^{\frac{1}{m}}}\right] \end{aligned} \tag{9}$$

Step 3. Based on the $\mathscr{AV}$ results, the positive distance from average $(\mathscr{PDA})$ and negative distance from average $(\mathscr{NDA})$ can be computed as:

$$\begin{aligned} \mathscr{PDA}_{ij} &= \left[\mathscr{PDA}_{ij}\right]_{m\times n} \\ &= \frac{max\left(0, \left(\mathscr{S}\left(\mathscr{N}_{ij}\right) - \mathscr{S}\left(\mathscr{AV}_j\right)\right)\right)}{\mathscr{S}\left(\mathscr{AV}_j\right)} \end{aligned} \tag{10}$$

$$\begin{aligned} \mathscr{NDA}_{ij} &= \left[\mathscr{NDA}_{ij}\right]_{m\times n} \\ &= \frac{max\left(0, \left(\mathscr{S}\left(\mathscr{AV}_j\right) - \mathscr{S}\left(\mathscr{N}_{ij}\right)\right)\right)}{\mathscr{S}\left(\mathscr{AV}_j\right)} \end{aligned} \tag{11}$$

where $\mathscr{S}\left(\mathscr{AV}_j\right)$ and $\mathscr{S}\left(\mathscr{N}_{ij}\right)$ are score function of $\mathscr{AV}_k$ and $\mathscr{N}_{ij}$ respectively.

Step 4. Determine the values of $\mathscr{SP}_i$ and $\mathscr{SN}_i$ which indicate the weighted sum of $\mathscr{PDA}$ and $\mathscr{NDA}$.

$$\mathscr{SP}_i = \sum_{j=1}^{n} \mathscr{W}_j \cdot \mathscr{PDA}_{ij}, \ \mathscr{SN}_i = \sum_{j=1}^{n} \mathscr{W}_j \cdot \mathscr{NDA}_{ij} \tag{12}$$

Step 5. Determine the normalized values of $\mathscr{SP}_i$ and $\mathscr{SN}_i$ as follows:

$$\mathscr{NSP}_i = \frac{\mathscr{SP}_i}{\max_i(\mathscr{SP}_i)}, \ \mathscr{NSN}_i = g - \frac{\mathscr{SN}_i}{\max_i(\mathscr{SN}_i)} \tag{13}$$

Step 6. Determine the assessment score $(\mathscr{MS}_j)$ for all alternatives, shown as follows:

$$\mathscr{MS}_i = \frac{1}{2}\left(\mathscr{NSP}_i + \mathscr{NSN}_i\right) \tag{14}$$

Table 1. Linguistic picture fuzzy soft information matrix.

	$\mathscr{C}_1$	$\mathscr{C}_2$	$\mathscr{C}_3$	$\mathscr{C}_4$	$\mathscr{C}_5$
$\mathscr{M}_1$	(l_4, l_1, l_0)	(l_5, l_1, l_0)	(l_4, l_1, l_1)	(l_4, l_1, l_1)	(l_5, l_0, l_1)
$\mathscr{M}_2$	(l_5, l_1, l_0)	(l_4, l_0, l_1)	(l_4, l_1, l_0)	(l_4, l_1, l_1)	(l_4, l_0, l_1)
$\mathscr{M}_3$	(l_3, l_1, l_1)	(l_4, l_1, l_1)	(l_4, l_0, l_1)	(l_4, l_1, l_1)	(l_5, l_1, l_0)
$\mathscr{M}_4$	(l_3, l_1, l_1)	(l_4, l_1, l_1)	(l_5, l_0, l_0)	(l_4, l_0, l_1)	(l_5, l_1, l_0)
$\mathscr{M}_5$	(l_4, l_1, l_1)	(l_3, l_0, l_0)	(l_4, l_1, l_1)	(l_5, l_0, l_1)	(l_4, l_2, l_0)

Step 7. Sort the alternative by priority from highest to lowest according to the results obtained from Step 6, when the highest value of $\mathscr{AS}_i$ has a higher priority.

5 Numerical Result

Currently, the population of Bangladesh is increasing significantly. In order to satisfy the growing demand for the food, the farming community needs to produce more food. To produce more food, a large land area is required, but it is impossible to bring more land for cultivation because the land area is limited in Bangladesh. To overcome this challenge, the farming community is trying to produce more food by applying a huge amount of pesticides and chemical fertilizers.

However, increasing application of pesticides and chemical fertilizers have detrimental effects, including human health problems such as neurological damage and organ damage, environmental pollution through contamination of soil, water and air with persistent residues, and harm to unintended organisms such as useful insects, birds, and fish. These chemicals can enter the food chain and waterways, causing issues such as nutrient enrichment in water bodies and carrying long-term risks of cancer and birth defects.

To overcome the harmful effects of chemical fertilizers, we can use organic fertilizers for improving long-term soil health by enhancing its structure, water retention, and microbial activity, while providing nutrients slowly and sustainably to plants and crops. Organic fertilizers support a healthy soil ecosystem by increasing organic matter, and their slow-release nature reduces the risk of over fertilizing and nutrient runoff, making them a more eco-friendly alternative to synthetic options.

All crops can benefit from organic fertilizers, but certain crops, such as vegetables, fruits (like tomatoes, cucumbers, and melons), corn, legumes, potatoes, and coffee, respond particularly well due to their increased nutrient demands and the soil-building properties of organic matter. The specific benefits often relate to the improved soil structure, water retention, and the supply of macro nutrients like Nitrogen (N), Phosphorus (P), and Potassium (K) that organic fertilizers provide.

In this part, we present a numerical example for selecting a suitable organic fertilizer for producing vegetables by using the Linguistic Picture Fuzzy Soft EDAS method. Based on the literature review associated to organic fertilizers selection, five parameters have been considered, which are Soil Characteristics ($\mathscr{P}_1$), Environmental and Management Factors ($\mathscr{P}_2$), Availability ($\mathscr{P}_3$), Long-Term Effects ($\mathscr{P}_4$), and Farmer's Knowledge and Access to Resources ($\mathscr{P}_5$).

After careful evaluation of the information, an expert set up the five organic fertilizers, namely Compost ($\mathscr{M}_1$), Vermicompost ($\mathscr{M}_2$), Cow Dung ($\mathscr{M}_3$), Poultry Manure ($\mathscr{M}_4$), and Green Manure ($\mathscr{M}_5$), to select the best organic fertilizer. The evaluation information of organic fertilizers $\mathscr{A}(i = 1, 2, 3, 4, 5)$ is represented in LPFN $(l_\alpha, l_\gamma, l_\beta) \in \mathscr{L}[0, g] = L[0, 6]$, where $\mathscr{L} = l_0 =$ Very Very Low (VVL), $l_1 =$ Low (L), $l_2 =$ Medium (M), $l_3 =$ Very Medium (VM), $l_4 =$ High (H), $l_5 =$ Very High (VH), $l_6 =$ Very Very High (VVH). The linguistic picture fuzzy soft information matrix $\mathscr{N}$ by the expert is given in Table 1.

In order to select best organic fertilizers, the steps of the proposed CRITIC-EDAS approach under linguistic picture fuzzy soft environment is presented as follows:

5.1 CRITIC Method

Step 1. According to the equation 1, we have the score values given in Table 2.

Table 2. The score values of $\mathscr{S}_{jk}$.

	$\mathscr{C}_1$	$\mathscr{C}_2$	$\mathscr{C}_3$	$\mathscr{C}_4$	$\mathscr{C}_5$
$\mathscr{M}_1$	3	4	2	2	4
$\mathscr{M}_2$	4	3	3	2	3
$\mathscr{M}_3$	1	2	3	2	4
$\mathscr{M}_4$	1	2	5	3	4
$\mathscr{M}_5$	2	3	2	4	2

Step 2. By applying the equation 4, 5 and 6 respectively, we obtain standard deviation values, correlation between criteria pairs and the quantity of information of each criteria which are listed in Tables 3, 4 and 5 respectively.

Table 3. Standard Deviations.

σ_1	σ_2	σ_3	σ_4	σ_5
1.3038	0.8366	0.8366	0.8944	0.8944

Table 4. Correlation coefficient values of criteria.

	$\mathscr{C}_1$	$\mathscr{C}_2$	$\mathscr{C}_3$	$\mathscr{C}_4$	$\mathscr{C}_5$
$\mathscr{C}_1$	1	0.7333	-0.4125	-0.3429	-0.3001
$\mathscr{C}_2$	0.7333	1	-0.7857	-0.1336	-0.2004
$\mathscr{C}_3$	-0.4125	-0.7857	1	-0.1336	0.4677
$\mathscr{C}_4$	-0.3429	-0.1336	-0.1336	1	-0.6875
$\mathscr{C}_5$	-0.3001	-0.2004	0.4677	-0.6875	1

Table 5. Quantity of information.

$\mathscr{C}_1$	$\mathscr{C}_2$	$\mathscr{C}_3$	$\mathscr{C}_4$	$\mathscr{C}_5$
5.7489	3.6699	4.0696	4.7384	4.2220

Step 3. Determine the weights of each criteria by the equation 7, given in Table 6.

5.2 EDAS Method

Step 1. According to Table 8, we can get the value of average solution $(\mathscr{AV})$ based on all proposed parameter as:

$$\mathscr{AV}_{1\times 5} = \Big((l_{3.95}, l_1, l_0)(l_{4.11}, l_1, l_0) (l_{4.26}, l_0, l_0)(l_{4.48}, l_0, l_1)(l_{4.68}, l_0, l_0)\Big) \tag{15}$$

Step 2. According to the results of average solution $(\mathscr{AV})$, we can calculate the positive distance from average $(\mathscr{PDA})$ and negative distance from average $(\mathscr{NDA})$ by using the formula 9 and 10 which are listed in Tables 7, 8, and 9.

Step 4. Calculating the values of $\mathscr{SP}_j$ and $\mathscr{SN}_j$ using equation 11 and the weighting vector of the parameter which is obtain from CRITIC method (Table 6). $w = (0.2560, 0.1634, 0.1812, 0.2110, 0.1880)$, we can obtain the results as follows:

$$\begin{aligned}
&\mathscr{SP}_1 = 0.0128, \mathscr{SP}_2 = 0.2688, \mathscr{SP}_3 = 0,\\
&\mathscr{SP}_4 = 0, \mathscr{SP}_5 = 0.1097;\\
&\mathscr{SN}_1 = 0.8679, \mathscr{SN}_2 = 1.0382, \mathscr{SN}_3 = 1.5130,\\
&\mathscr{SN}_4 = 1.1206, \mathscr{SN}_5 = 1.3384
\end{aligned} \tag{16}$$

Table 6. Criteria weights.

$\mathscr{W}_1$	$\mathscr{W}_2$	$\mathscr{W}_3$	$\mathscr{W}_4$	$\mathscr{W}_5$
0.2560	0.1634	0.1812	0.2110	0.1880

Table 7. The score values of $\mathscr{S}_{jk}$ and $\mathscr{AV}_k$.

	$\mathscr{C}_1$	$\mathscr{C}_2$	$\mathscr{C}_3$	$\mathscr{C}_4$	$\mathscr{C}_5$
$\mathscr{M}_1$	3	4	2	2	4
$\mathscr{M}_2$	4	3	3	2	3
$\mathscr{M}_3$	1	2	3	2	4
$\mathscr{M}_4$	1	2	4	3	4
$\mathscr{M}_5$	2	3	2	4	2
$\mathscr{AV}$	2.95	4.11	4.26	3.48	4.68

Table 8. The results of $\mathscr{PDA}_{jk}$.

	$\mathscr{C}_1$	$\mathscr{C}_2$	$\mathscr{C}_3$	$\mathscr{C}_4$	$\mathscr{C}_5$
$\mathscr{M}_1$	0.05	0	0	0	0
$\mathscr{M}_2$	1.05	0	0	0	0
$\mathscr{M}_3$	0	0	0	0	0
$\mathscr{M}_4$	0	0	0	0	0
$\mathscr{M}_5$	0	0	0	0.52	0

Table 9. The results of $\mathscr{NDA}_{jk}$

	$\mathscr{C}_1$	$\mathscr{C}_2$	$\mathscr{C}_3$	$\mathscr{C}_4$	$\mathscr{C}_5$
$\mathscr{M}_1$	0	0.11	2.26	1.48	0.68
$\mathscr{M}_2$	0	1.11	1.26	1.48	1.68
$\mathscr{M}_3$	1.95	2.11	1.26	1.48	0.68
$\mathscr{M}_4$	1.95	2.11	0.26	0.48	0.68
$\mathscr{M}_5$	0.95	1.11	2.26	0	2.68

Step 5. The results of Step 4 can be normalized by formula 12 and the results are listed as:

$$\begin{aligned}
&\mathscr{NSP}_1 = 0.0047, \mathscr{NSP}_2 = 0.0999, \mathscr{NSP}_3 = 0,\\
&\mathscr{NSP}_4 = 0, \mathscr{NSP}_5 = 0.0408,\\
&\mathscr{NSN}_1 = 5.4263, \mathscr{NSN}_2 = 5.3137, \mathscr{NSN}_3 = 5,\\
&\mathscr{NSN}_4 = 5.2593, \mathscr{NSN}_5 = 5.1
\end{aligned} \tag{17}$$

Step 6. Based on each alternative's $\mathscr{NSP}_i$ and $\mathscr{NSN}_i$ to determine the values of $\mathscr{MS}$;

$$\begin{aligned}
\mathscr{MS}_1 = 2.7369, \mathscr{MS}_2 = 3.1569, \mathscr{MS}_3 = 2.5,\\
\mathscr{MS}_4 = 2.6296, \mathscr{MS}_5 = 2.7617
\end{aligned} \tag{18}$$

Step 7. According to the determining results of $\mathscr{AS}_i$, we can rank all the alternatives, the bigger value of $\mathscr{AS}_i$ is, the best alternative selected will be. Obviously, the rank of all alternatives is

$$\mathscr{M}_2 > \mathscr{M}_5 > \mathscr{M}_1 > \mathscr{M}_4 > \mathscr{M}_3. \tag{19}$$

Therefore, the Vermicompost is the best organic fertilizer for vegetables cultivation.

6 Sensitivity Analysis

To show the stability of the results, the aforementioned example is solved using various sets of parameter weights. Two alternative weight sets (S_1 and S_2) were compared with the actual conditions of the study.

S_1: The weights were calculated by entropy method.

S_2: The weights were taken equally for all parameters.

Figure 1 shows the weights in each set. Figure 2 shows the ranking of each alternative in each set of weights parameters. As can be seen in Figure 2, the ranks of all alternatives are relatively stable in diverse weights of parameter. For example, the best alternative is M_2 and the worst alternative is M_3. Therefore, we can say that the proposed method is proficient for ranking alternatives in an MCDM problems.

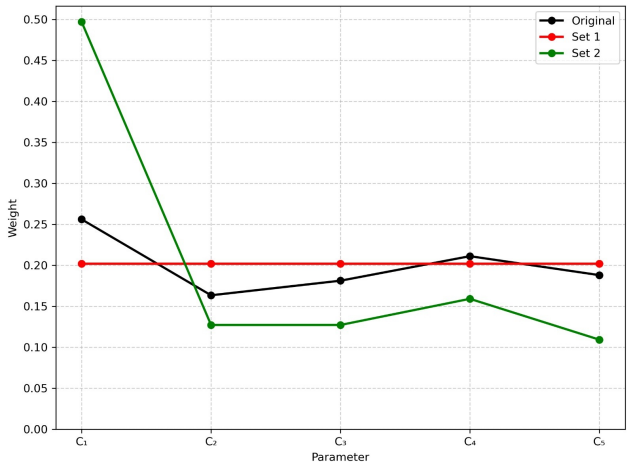


Figure 1. Weight Distribution across Parameter.

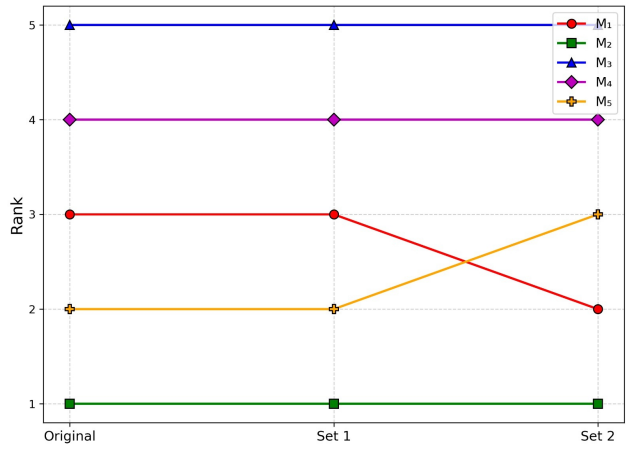


Figure 2. The rank of alternatives in different sets.

7 Conclusion

This article proposes a novel extension of the CRITIC-EDAS method in the linguistic picture fuzzy soft set environment. The proposed model is particularly suitable, as the CRITIC method objectively determines the criteria weights while the EDAS method ranks the alternatives. Both approaches involve straightforward computational steps and can be readily applied to various real-world problems. To demonstrate the validity and effectiveness of the proposed approach, a numerical example on the selection of the most suitable organic fertilizer for vegetable production is presented. The results indicate that Vermicompost ranks highest among the alternatives. In future work, we plan to further extend the EDAS method to other variants of linguistic picture fuzzy soft sets.

Data Availability Statement

Data will be made available on request.

Funding

This work was supported without any funding.

Conflicts of Interest

The authors declare no conflicts of interest.

AI Use Statement

The authors declare that no generative AI was used in the preparation of this manuscript.

Ethical Approval and Consent to Participate

Not applicable.

References

[1] Zadeh, L. A. (1965). Fuzzy sets.*Information and Control*, *8*(3), 338-353. [CrossRef]

[2] Zadeh, L. A. (1975). The concept of a linguistic variable and its application to approximate reasoning—I. *Information Sciences*, *8*(3), 199–249. [CrossRef]

[3] Zadeh, L. A. (1975). The concept of a linguistic variable and its application to approximate reasoning—II. *Information Sciences*, *8*(4), 301–357. [CrossRef]

[4] Zadeh, L. A. (1975). The concept of a linguistic variable and its application to approximate reasoning—III. *Information Sciences*, *9*(1), 43–80. [CrossRef]

[5] Herrera, F., & Herrera-Viedma, E. (2000). Linguistic decision analysis: Steps for solving decision problems under linguistic information. *Fuzzy Sets and Systems, 115*(1), 67–82. [CrossRef]

[6] Herrera, F., Herrera-Viedma, E., & Martinez, L. (2008). A fuzzy linguistic methodology to deal with unbalanced linguistic term sets. *IEEE Transactions on Fuzzy Systems, 16*(2), 354–370. [CrossRef]

[7] Martinez-Lopez, L., Ruan, D., Herrera, F., Herrera-Viedma, E., & Wang, P. P. (2009). Linguistic decision-making: Tools and applications. *Information Sciences, 179*(14), 2297–2298. [CrossRef]

[8] Rodríguez, R. M., Martínez, L., & Herrera, F. (2013). A group decision-making model dealing with comparative linguistic expressions based on hesitant fuzzy linguistic term sets. *Information Sciences, 241*, 28–42. [CrossRef]

[9] Liao, H., Xu, Z., Zeng, X. J., & Merigó, J. M. (2015). Qualitative decision-making with correlation coefficients of hesitant fuzzy linguistic term sets. *Knowledge-Based Systems, 76*, 127–138. [CrossRef]

[10] Chen, Z., Liu, P., & Pei, Z. (2015). An approach to multiple attribute group decision-making based on linguistic intuitionistic fuzzy numbers. *International Journal of Computational Intelligence Systems, 8*(4), 747–760. [CrossRef]

[11] Qiyas, M., Abdullah, S., Ashraf, S., & Aslam, M. (2020). Utilizing linguistic picture fuzzy aggregation operators for multiple-attribute decision-making problems. *International Journal of Fuzzy Systems, 22*(1), 310-320. [CrossRef]

[12] Liu, D., Luo, Y., & Liu, Z. (2020). The linguistic picture fuzzy set and its application in multi-criteria decision-making: an illustration to the TOPSIS and TODIM methods based on entropy weight. *Symmetry, 12*(7), 1170. [CrossRef]

[13] Molodtsov, D. (1999). Soft set theory—First results. *Computers and Mathematics with Applications, 37*(4-5), 19-31. [CrossRef]

[14] Cagman, N., Enginoglu, S., & Citak, F. (2011). Fuzzy soft set theory and its applications. *Iranian Journal of Fuzzy Systems, 8*(3), 137-147. [CrossRef]

[15] Xu, Y. J., Sun, Y. K., & Li, D. F. (2010, May). Intuitionistic fuzzy soft set. In *2010 2nd International Workshop on Intelligent Systems and Applications* (pp. 1-4). IEEE. [CrossRef]

[16] Hovanov, N. V., Kolari, J. W., & Sokolov, M. V. (2008). Deriving weights from general pairwise comparison matrices. *Mathematical Social Sciences, 55*(2), 205–220. [CrossRef]

[17] Golden, B. L., Wasil, E. A., & Harker, P. T. (1989). The analytic hierarchy process. *Applications and Studies, Berlin, Heidelberg, 2*(1), 1-273. [CrossRef]

[18] Krylovas, A., Zavadskas, E. K., Kosareva, N., & Dadelo, S. (2014). New KEMIRA method for determining criteria priority and weights in solving MCDM problem. *International journal of information technology & decision making, 13*(06), 1119-1133. [CrossRef]

[19] Danielson, M., & Ekenberg, L. (2019). An improvement to swing techniques for elicitation in MCDM methods. *Knowledge-Based Systems, 168*, 70-79. [CrossRef]

[20] Stanujkic, D., Zavadskas, E. K., Karabasevic, D., Smarandache, F., & Turskis, Z. (2017). The use of the pivot pairwise relative criteria importance assessment method for determining the weights of criteria. *Romanian Journal of Economic Forecasting, 20*(4), 116-133. https://ipe.ro/rjef/rjef4_17/rjef4_2017p116-133.pdf

[21] Pamučar, D., Stević, Ž., & Sremac, S. (2018). A new model for determining weight coefficients of criteria in mcdm models: Full consistency method (fucom). *symmetry, 10*(9), 393. [CrossRef]

[22] Hwang, C. L., & Yoon, K. (2012). *Multiple attribute decision making: methods and applications a state-of-the-art survey*. Springer Science & Business Media. [CrossRef]

[23] Diakoulaki, D., Mavrotas, G., & Papayannakis, L. (1995). Determining objective weights in multiple criteria problems: The CRITIC method. *Computers & Operations Research, 22*(7), 763–770. [CrossRef]

[24] Zavadskas, E. K., & Podvezko, V. (2016). Integrated determination of objective criteria weights in MCDM. *International Journal of Information Technology & Decision Making, 15*(02), 267–283. [CrossRef]

[25] Keshavarz Ghorabaee, M., Zavadskas, E. K., Olfat, L., & Turskis, Z. (2015). Multi-criteria inventory classification using a new method of evaluation based on distance from average solution (EDAS). *Informatica, 26*(3), 435-451. [CrossRef]

[26] Dhumras, H., & Bajaj, R. K. (2023). Modified EDAS method for MCDM in robotic agrifarming with picture fuzzy soft Dombi aggregation operators. *Soft Computing, 27*(8), 5077-5098. [CrossRef]

[27] Akram, M., Ramzan, N., & Deveci, M. (2023). Linguistic Pythagorean fuzzy CRITIC-EDAS method for multiple-attribute group decision analysis. *Engineering Applications of Artificial Intelligence, 119*, 105777. [CrossRef]

[28] Peng, X., Krishankumar, R., & Ravichandran, K. S. (2021). A novel interval-valued fuzzy soft decision-making method based on CoCoSo and CRITIC for intelligent healthcare management evaluation. *Soft Computing, 25*(6), 4213-4241. [CrossRef]

[29] Zhang, S., Wei, G., Gao, H., Wei, C., & Wei, Y. (2019). EDAS method for multiple criteria group decision making with picture fuzzy information and its application to green suppliers selections. *Technological and Economic Development of Economy, 25*(6), 1123-1138. [CrossRef]

[30] Alshammari, I., Parimala, M., Ozel, C., Riaz, M., & Kammoun, R. (2022). New MCDM algorithms

with linear Diophantine fuzzy soft TOPSIS, VIKOR and aggregation operators. *Mathematics, 10*(17), 3080. [CrossRef]

[31] Ali, S., Ali, A., Azim, A. B., ALoqaily, A., & Mlaiki, N. (2023). Averaging aggregation operators under the environment of q-rung orthopair picture fuzzy soft sets and their applications in MADM problems. *AIMS Mathematics, 8*(4), 9027-9053. [CrossRef]

[32] Akram, M., & Shumaiza. (2021). Multi-criteria decision-making methods based on q-rung picture fuzzy information. *Journal of Intelligent & Fuzzy Systems, 40*(5), 10017-10042. [CrossRef]

[33] Yuan, J., Chen, Z., & Wu, M. (2023). A novel distance measure and CRADIS method in picture fuzzy environment. *International Journal of Computational Intelligence Systems, 16*(1), 186. [CrossRef]

[34] Yang, X., Mahmood, T., & Ahmmad, J. (2023). Picture fuzzy soft Bonferroni mean aggregation operators and their applications. *Heliyon, 9*(6). [CrossRef]

[35] Agbakwuru, J. A., Nwaoha, T. C., & Udosoh, N. E. (2023). Application of CRITIC–EDAS-based approach in structural health monitoring and maintenance of offshore wind turbine systems. *Journal of Marine Science and Application, 22*(3), 545–555. [CrossRef]

[36] Akram, M., Noreen, U., & Deveci, M. (2023). Enhanced ELECTRE II method with 2-tuple linguistic m-polar fuzzy sets for multi-criteria group decision making. *Expert Systems with Applications, 213*, 119237. [CrossRef]

[37] Haktanır, E., & Kahraman, C. (2022). A novel picture fuzzy CRITIC & REGIME methodology: Wearable health technology application. *Engineering Applications of Artificial Intelligence, 113*, 104942. [CrossRef]

[38] Chellamani, P., Ajay, D., Broumi, S., & Ligori, T. A. A. (2022). An approach to decision-making via picture fuzzy soft graphs. *Granular computing, 7*(3), 527-548. [CrossRef]

[39] Yang, X., Mahmood, T., Ahmmad, J., & Hayat, K. (2023). A novel study of spherical fuzzy soft Dombi aggregation operators and their applications to multicriteria decision making. *Heliyon, 9*(6). [CrossRef]

[40] Rouyendegh, B. D., & Savalan, Ş. (2022). An integrated fuzzy MCDM hybrid methodology to analyze agricultural production. *Sustainability, 14*(8), 4835. [CrossRef]



Md. Yasin Ali received the PhD. degree in Fuzzy Mathematics from Jahangirnagar University in 2021. His research interests include Fuzzy sets, Soft sets and Picture fuzzy sets. (Email: ali.mdyasin56@gmail.com)



Md. Ashraful Islam completed his PhD degree under the Faculty of Science and Technology at Bangladesh University of Professionals in 2020. (Email: ashraful47@yahoo.com)