RESEARCH ARTICLE

The Economic Singularity: Core Mathematical Model

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Abstract

This paper develops a dynamical systems model of an economy with recursively self-improving artificial intelligence and financialization. The model features quadratic self-amplification in both AI capability (λA^2) and financial capital ($\gamma_F K_f^2$), coupled through investment flows. It is shown that under mild conditions, the system exhibits a finite-time singularity where AI capability, AI capital, and financial capital diverge. Near the singularity, the wealth ratio between capital owners and workers diverges super-exponentially, with financialization amplifying the exponent by a factor γ_F/η. Introducing taxes on AI returns (τ_{ai}) and financial gains (τ_f) yields three distinct long-run regimes: low-tax (extreme inequality), moderate-tax (stable mixed economy), and high-tax (post-scarcity with universal basic income), separated by critical thresholds derived from workers' budget constraint. Finally, a policy irreversibility result is established: a critical time exists before the singularity after which redistribution becomes politically impossible, as wealth concentration renders feasible tax rates vanishingly small. The results highlight the urgency of early intervention in AI-driven economies.

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Nomenclature

$A(t)$	AI capability (stock of algorithmic knowledge)
$K_{ai}(t)$	AI capital (physical assets embodying AI)
$K_f(t)$	Financial capital (liquid assets, speculative positions)
$L_p(t)$	Productive labor (fraction of labor force)
$W_c(t)$	Wealth of capital owners
$W_w(t)$	Wealth of workers
λ	AI self-improvement efficiency
α_A	R&D conversion efficiency (finance $\to$ AI)
β	Investment efficiency (finance $\to$ AI capital)
δ	Depreciation rate of AI capital
r	Baseline return on financial capital
η	Return rate on AI capital
γ_F	Financial autocatalysis intensity
κ	Automation speed
ν	Reskilling/reemployment rate
τ_{ai}, τ_f	Tax rates on AI returns and financial gains
σ	Elasticity of substitution in production
ϕ	Fraction of capital owners in population

1 Introduction

This paper develops a dynamical systems model of an economy undergoing rapid AI-driven transformation. The analysis builds on the canonical framework

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of directed technical change [1, 4], but extends it to incorporate two novel features: recursive self-improvement in AI capabilities and financial autocatalysis. While previous literature has analyzed AI as a general-purpose technology [5, 12] and documented its labor market impacts [2, 3, 9, 27], the potential for finite-time singularities arising from coupled technological and financial dynamics remains unexplored.

Our model is motivated by three empirical observations. First, AI capabilities have been scaling super-exponentially [6, 20], suggesting that recursive improvement may already be underway. Second, financialization has decoupled financial from real economic activity [7, 10, 21], creating self-amplifying returns that concentrate wealth [24]. Third, the combination of these trends threatens to produce extreme inequality that traditional redistribution mechanisms may be unable to address [18, 25, 29].

We consider an economy with two agent types (capital owners and workers) and four state variables: AI capability $A(t)$, AI capital $K_{ai}(t)$, financial capital $K_f(t)$, and productive labor $L_p(t)$. The dynamics are driven by quadratic self-amplification terms—λA^2 for AI and $\gamma_F K_f^2$ for finance—that can cause finite-time blow-up, representing an economic singularity.

Important note: The quadratic terms are modeling assumptions that capture empirically observed accelerating returns in AI and finance [20, 23]. Their mathematical consequence (finite-time blow-up) is known, but the contribution of this paper lies in analyzing the *coupled* dynamics and deriving policy implications. This mathematical structure connects to the literature on finite-time singularities in dynamical systems [8, 16] and extends the classical growth framework [22, 26].

The analysis yields four main theoretical results. First, it is shown that under mild conditions, the coupled system exhibits a finite-time singularity (Theorem 1). Second, wealth inequality is shown to diverge super-exponentially near the singularity, with financialization acting as a multiplier (Theorem 2). Third, three distinct long-run regimes are characterized—"Neofeudalism" (extreme inequality), "Managed Capitalism" (stable mixed economy), and "Post-Scarcity Socialism" (universal basic income)—separated by critical tax thresholds (Theorem 3). Fourth, it is established that there exists a finite window for effective policy intervention; after a critical time, redistribution becomes politically impossible even if mathematically feasible (Theorem 4).

The paper is organized as follows. Section 2 presents the model. Section 3 states the main theoretical results (proofs are in the Appendix). Section 4 provides numerical simulations and discusses physical constraints. Section 5 concludes with policy implications and caveats.

2 The Model

2.1 State Variables and Dynamics

We model a closed economy with two representative agents — capital owners and workers — and four continuous-time state variables (see Nomenclature for definitions): AI capability $A(t)$, AI capital $K_{ai}(t)$, financial capital $K_f(t)$, and productive labor $L_p(t)$. $A(t)$ represents the stock of algorithmic knowledge and recursive self-improvement capacity; $K_{ai}(t)$ is the physical stock of AI-embodied capital (robots, data centers); $K_f(t)$ captures liquid financial wealth and speculative positions; $L_p(t)$ is the fraction of the labor force engaged in productive work. These variables capture the essential feedbacks among technological progress (the λA^2 term), capital accumulation ($\beta A K_f$ drives K_{ai}), financialization (the quadratic autocatalysis $\gamma_F K_f^2$), and labor displacement (automation reducing L_p via $-\kappa A K_{ai} L_p(1 - L_p)$). All variables are functions of $t \geq 0$ and evolve according to the coupled ordinary differential equations presented below.

2.1.1 *AI Capability* $(A(t))$

$$\frac{dA}{dt} = \lambda A^2 + \alpha_A K_f \tag{1}$$

The variable $A(t)$ represents the stock of algorithmic knowledge, software efficiency, and general intelligence of AI systems. Its evolution has two components. First, the autocatalytic self-improvement term λA^2 captures the unique property of AI: smarter systems can design even smarter successors. If research productivity scales with both the quality of researchers (proportional to A) and the number of researchers (also proportional to A in an AI-driven R&D sector), then total AI progress is quadratic in A. Parameter $\lambda > 0$ measures the efficiency of this recursive improvement. Second, financial investment in R&D ($\alpha_A K_f$) represents liquid financial capital K_f (venture capital, corporate R&D budgets) channeled into AI development. The parameter $\alpha_A > 0$ represents the conversion efficiency of financial resources into new AI capabilities. The quadratic term is the source of potential finite-time singularities: if left unchecked, A can blow up in finite time.

2.1.2 *AI Capital* $(K_{ai}(t))$

$$\frac{dK_{ai}}{dt} = \beta A K_f - \delta K_{ai} \tag{2}$$

$K_{ai}(t)$ denotes physical assets embodying AI technology: server farms, robots, automated factories, data infrastructure. Its dynamics involve investment in AI capital ($\beta A K_f$): financial capital K_f is transformed into productive AI capital at a rate that increases with current AI capability A. This captures the idea that better AI makes investment more efficient (e.g., through optimized manufacturing or better deployment). Parameter $\beta > 0$ scales this conversion. Depreciation (δK_{ai}) accounts for obsolescence due to technological progress or physical wear; $\delta > 0$ is typical for high-tech capital (often 10–20% annually).

2.1.3 *Financial Capital* $(K_f(t))$

$$\frac{dK_f}{dt} = rK_f - \beta A K_f + \eta K_{ai} + \gamma_F K_f^2 \tag{3}$$

$K_f(t)$ represents liquid assets, speculative positions, and financial engineering capacity. Its evolution reflects both real and financial linkages. Traditional returns (rK_f) provide a baseline return $r > 0$ from conventional investments (bonds, non-AI equity). Outflow to AI investment ($-\beta A K_f$) subtracts from financial capital when converted into physical AI capital. Dividends from AI capital (ηK_{ai}) generate profits that accrue to owners and are reinvested in financial markets. Parameter $\eta > 0$ is the return rate on AI capital. Finally, financial autocatalysis ($\gamma_F K_f^2$) captures the ability of financial wealth to generate super-linear returns through leverage, derivatives, high-frequency trading, and complex financial instruments. Larger financial pools can command better opportunities and create self-reinforcing bubbles. Parameter $\gamma_F > 0$ measures the intensity of such effects. This term is the financial analogue of the λA^2 term and can also lead to finite-time singularities.

2.1.4 *Labor Allocation* $(L_p(t))$

$$\frac{dL_p}{dt} = -\kappa A K_{ai} L_p(1 - L_p) + \nu(1 - L_p) \tag{4}$$

The total labor force is normalized to 1, split into productive labor L_p and non-productive (or displaced) labor $L_n = 1 - L_p$. The dynamics incorporate automation displacement ($-\kappa A K_{ai} L_p(1 - L_p)$): the product AK_{ai} measures effective automation power. The logistic factor $L_p(1 - L_p)$ captures two realistic features: displacement is fastest when many workers are still employed in automatable tasks ($L_p \approx 1$), and it slows as only hard-to-automate tasks remain ($L_p \approx 0$). Parameter $\kappa > 0$ scales the speed of automation. Reskilling and reemployment ($\nu(1 - L_p)$) allow displaced workers to return to productive work. The rate $\nu > 0$ is typically small due to time lags and skill mismatches, reflecting institutional frictions.

2.2 Wealth and Income Distribution

Capital owners hold all AI and financial capital: $W_c = K_{ai} + K_f$. Their wealth dynamics (before taxes) follow directly from (2) and (3):

$$\frac{dW_c}{dt} = \eta K_{ai} + \gamma_F K_f^2 + rK_f. \tag{5}$$

Workers derive income from wages $w(L_p)$, which is a decreasing function of L_p (scarcity premium), and from government transfers financed by taxes on AI and financial returns. Their wealth evolves as:

$$\frac{dW_w}{dt} = w(L_p) + \tau_{ai}\eta K_{ai} + \tau_f \gamma_F K_f^2 - c_w \tag{6}$$

where $\tau_{ai}, \tau_f \in [0, 1]$ are tax rates on AI capital returns and financial gains, and c_w is consumption (often set to a subsistence level).

Remark: In the current formulation, taxes appear only in the workers' wealth equation and do not directly affect the accumulation equations for A, K_{ai}, or K_f. This simplification is made to isolate the distributional effects. In reality, taxes would also affect investment incentives; an extension in Section 4.3 discusses how a consumption-investment feedback channel can transmit tax effects to capital dynamics.

2.3 Market Clearing and Equilibrium Conditions

Output Y is produced via a CES function:

$$Y = \left[\alpha(AK_{ai})^{\frac{\sigma-1}{\sigma}} + (1-\alpha)L_p^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{7}$$

with elasticity $\sigma > 0$ and share parameter $\alpha \in (0, 1)$. The goods market clears when output equals consumption plus investment:

$$Y = c_c\phi + c_w(1 - \phi) + I_{ai} + I_f, \tag{8}$$

where ϕ is the fraction of capital owners, $I_{ai} = \beta A K_f$ is investment in AI capital, and I_f denotes resource costs of the financial sector (omitted in the core dynamics). The capital market satisfies $K_{ai} + K_f = \phi W_c$, and the wage $w(L_p)$ is determined by the marginal product of labor from (7).

2.4 Empirical Motivation for Quadratic Terms

The quadratic self-amplification terms are not arbitrary. For AI, recent work on scaling laws [19, 20] shows that model performance improves as a power law of compute, data, and parameters, and recursive self-improvement has been discussed in the AI safety literature [11, 28]. For finance, empirical studies document that the financial sector's profit growth has exceeded GDP growth by a factor of 2–3 since the 1980s, consistent with a super-linear component [17, 23]. Parameter ranges are provided in Table 1.

2.5 Parameterization and Calibration

Table 1 summarizes the key parameters with empirical justifications. Sources: AI scaling [20]; financial excess returns [23]; automation and labor [2, 13]; wealth dynamics [24].

Table 1. Parameter ranges and sources.

Parameter	Typical range	Source
λ	0.01–0.10	[20]
γ_F	0.02–0.08	[23]
β	0.5–2.0	[2]
η	0.10–0.20	[24]
κ	0.05–0.30	[14]
ν	0.01–0.05	[9]

3 Theoretical Results

We now state the four main theoretical results of the model. Each theorem is presented with its essential mathematical statement and a brief economic interpretation. Detailed proofs, including the construction of Lyapunov functions, asymptotic analysis, and the derivation of critical thresholds, are provided in the Appendix. The appendix follows the same numbering as the theorems for easy cross-reference.

3.1 Finite-Time Singularity

The terms λA^2 and $\gamma_F K_f^2$ are modeling assumptions that capture empirically observed accelerating returns in AI [20] and finance [23]. Their mathematical consequence (finite-time blow-up) is well known from the Riccati equation. The contribution of the following theorem is to show that the coupling through $\beta A K_f$ does not suppress the singularity and to provide an explicit upper bound on the blow-up time in terms of initial conditions and cross-term coefficients.

Theorem 3.1 (Dual Singularity)**.** *If the initial conditions satisfy*

$$\lambda A_0 + \alpha_A \frac{K_{f0}}{A_0} > 0 \quad \textit{and} \quad \gamma_F K_{f0} + \eta \frac{K_{ai0}}{K_{f0}} > 0, \tag{9}$$

then the coupled system (1)–(3) *exhibits a finite-time singularity: there exists a finite time $t^* < \infty$ such that $A(t)$, $K_f(t)$, and $K_{ai}(t)$ diverge to infinity as $t \to t^*$. Moreover, the singularity time is bounded above by*

$$t^* \leq \min\left(\frac{1}{\lambda A_0 + \alpha_A \frac{K_{f0}}{A_0}}, \frac{1}{\gamma_F K_{f0} + \eta \frac{K_{ai0}}{K_{f0}}} \right). \tag{10}$$

Economic interpretation. Theorem 1 reveals that two independent acceleration mechanisms—technological self-improvement (λA^2) and financial autocatalysis ($\gamma_F K_f^2$)—can each drive a singularity on their own. When coupled through the investment channel ($\beta A K_f$), they reinforce each other, making the singularity earlier and more severe. The quadratic terms are modeling assumptions; the theorem's contribution is to show that coupling does not suppress the singularity and to provide an upper bound on the blow-up time.

3.2 Wealth Divergence

The super-exponential divergence of the wealth ratio follows directly from the asymptotic forms of $A(t)$, $K_f(t)$, and $K_{ai}(t)$ established in the previous theorem. The novel contribution here is the explicit exponent $\gamma = 2 + \gamma_F/\eta$, which quantifies how financial autocatalysis (γ_F) amplifies inequality beyond the baseline rate of 2 that would arise from technological factors alone.

Theorem 3.2 (Super-Exponential Inequality)**.** *In the absence of redistribution ($\tau_{ai} = \tau_f = 0$) and under the singularity conditions of Theorem 1, the wealth ratio $R(t) = W_c(t)/W_w(t)$ diverges super-exponentially as $t \to t^*$. Specifically,*

$$R(t) \sim \frac{C}{(t^* - t)^\gamma}, \qquad \gamma = 2 + \frac{\gamma_F}{\eta}, \tag{11}$$

where $C > 0$ is a constant depending on initial conditions.

Economic interpretation. The exponent γ captures the **financial amplification factor**. If $\gamma_F = 0$, then $\gamma = 2$, meaning the wealth ratio diverges like $(t^* - t)^{-2}$. With positive γ_F, the divergence is even faster. The term γ_F/η represents the relative strength of purely financial returns compared to real AI capital returns.

3.3 Regime Classification with Taxes

In the base model, taxes τ_{ai} and τ_f appear only in the workers' wealth equation (6) and do not directly alter the accumulation of A, K_{ai}, or K_f. Consequently, the singularity in these variables persists regardless of tax policy. The next theorem characterizes the *distributional* regimes that arise from different tax rates given the singular growth of AI and financial capital. The thresholds indicate when workers can maintain subsistence through transfers, not when the singularity itself is prevented.

Theorem 3.3 (Critical Tax Thresholds)**.** *There exist thresholds $\bar{\tau}_{ai} > 0$ and $\bar{\tau}_f > 0$, depending on the balanced growth path values (K_{ai}^*, K_f^*, L_p^*), such that the long-run behavior of the economy falls into one of three regimes:*

1. ***"Neofeudalism" (extreme inequality)**: If $\tau_{ai} = \tau_f = 0$, then as $t \to \infty$, $L_p \to 0$ (complete automation) and $W_c/W_w \to \infty$.*
2. ***"Managed Capitalism" (stable mixed economy)**: If $0 < \tau_{ai} < \bar{\tau}_{ai}$ or $0 < \tau_f < \bar{\tau}_f$ (but not both sufficiently high), then the economy converges to a balanced growth path with $L_p^* > 0$ and a finite, stable wealth ratio.*
3. ***"Post-Scarcity Socialism" (universal basic income)**: If $\tau_{ai} > \bar{\tau}_{ai}$ and $\tau_f > \bar{\tau}_f$, then $L_p \to 0$ (full automation), but workers are sustained indefinitely by universal basic income funded by high taxes. The wealth ratio remains finite and low.*

The thresholds are given explicitly by

$$\bar{\tau}_{ai} = \frac{c - w(L_p^*)}{\eta K_{ai}^*} \left(1 + \frac{\gamma_F}{\eta} \frac{K_f^{*2}}{K_{ai}^*}\right)^{-1}, \tag{12}$$

$$\bar{\tau}_f = \frac{c - w(L_p^*)}{\gamma_F K_f^{*2}} \left(1 + \frac{\eta}{\gamma_F} \frac{K_{ai}^*}{K_f^{*2}}\right)^{-1}, \tag{13}$$

where (K_{ai}^*, K_f^*, L_p^*) is the balanced growth path that would prevail under the given taxes.

Remark 3.1. *In the current model, taxes appear only in the workers' wealth equation (6) and do not directly alter the accumulation equations for A, K_{ai}, or K_f. Consequently, the singularity in A and K_f persists regardless of taxes. Theorem 3 describes the long-run distributional regimes given that singularity; the "balanced growth path" refers to a state where workers' consumption is sustained by transfers even as A and K_f grow extremely rapidly. A full feedback from taxes to investment is considered in the extension (Section 4.3).*

3.4 Policy Irreversibility

The political feasibility constraint

$$\tau_i^{\max}(t) = \tau_i^0 (W_w/(W_c + W_w))^{\psi}$$

is a stylized representation of wealth-weighted political influence [15]. The critical time t_c derived below depends on the exponent ψ; empirical estimates of ψ vary, and alternative formulations of political power (e.g., threshold effects or institutional constraints) could alter the irreversibility window. The theorem demonstrates a possibility of irreversibility under plausible conditions, not an inevitability.

Theorem 3.4 (Policy Irreversibility)**.** *There exists a critical time $t_c < t^*$ such that for all $t > t_c$, any feasible tax policy that maintains $W_w(t) \geq 0$ (i.e., keeps workers above subsistence) would require tax rates exceeding political feasibility constraints. Specifically, let $\tau_{ai}^{\max}(t)$ and $\tau_f^{\max}(t)$ be the maximum politically feasible tax rates at time t, which depend on the wealth distribution (e.g., through lobbying power). Then for $t > t_c$,*

$$\tau_{ai}^{req}(t) > \tau_{ai}^{\max}(t) \quad \text{or} \quad \tau_f^{req}(t) > \tau_f^{\max}(t), \tag{14}$$

where $\tau_{ai}^{req}(t)$ and $\tau_f^{req}(t)$ are the rates required to satisfy the workers' budget constraint at time t. Moreover, as $t \to t^*$, $\tau_{ai}^{\max}(t), \tau_f^{\max}(t) \to 0$, while the required rates remain bounded away from zero, making intervention impossible in the limit.

Economic interpretation. This theorem captures a tragic irony of AI-driven inequality: the very wealth concentration that makes redistribution necessary also makes it politically impossible. The critical time t_c depends on parameters like ψ (how quickly political power translates into tax resistance) and the speed of the singularity.

4 Numerical Simulations and Physical Constraints

4.1 Baseline Trajectory

We numerically integrate the coupled system (1)–(4) using the following baseline parameters (see Table 1 for justification): $\lambda = 0.05$, $\gamma_F = 0.04$, $\beta = 1.0$, $\alpha_A = 0.3$, $\eta = 0.15$, $\delta = 0.1$, $r = 0.03$, $\kappa = 0.2$,

$\nu = 0.02$. Initial conditions are set to $A_0 = 1$, $K_{ai0} = 1$, $K_{f0} = 1$, $L_{p0} = 0.95$ (i.e., 95% of the labor force initially employed in productive tasks). The simulation runs from $t = 0$ to $t = 20$ years, with the singularity detected when any variable exceeds 10^6.

The key dynamical features are as follows:

1. AI capability $A(t)$: Grows slowly at first, then accelerates rapidly due to the quadratic self-improvement term λA^2 and the financial R&D inflow $\alpha_A K_f$. It reaches 10^6 at $t^* \approx 18.2$ years.
2. Financial capital $K_f(t)$: Exhibits similar super-exponential growth driven by $\gamma_F K_f^2$ and dividends from AI capital ηK_{ai}. Its singularity time is almost identical to that of $A(t)$ because of the strong coupling $\beta A K_f$.
3. AI capital $K_{ai}(t)$: Grows as the integral of $\beta A K_f$, hence diverges at the same t^* but with a slightly lower exponent.
4. Productive labor $L_p(t)$: Declines from 0.95 to below 0.01 by $t = 15$ years, as automation ($\kappa A K_{ai} L_p(1-L_p)$) outpaces reskilling ($\nu(1-L_p)$). Near the singularity, L_p effectively reaches zero.
5. Wealth ratio W_c/W_w: Starts at approximately 2 (capital owners initially hold twice the wealth of workers) and grows to over 10^6 just before the singularity, confirming super-exponential divergence as predicted by Theorem 2.

The simulation reveals finite-time blow-up at $t^* \approx 18.2$ years, which is slightly earlier than the uncoupled estimates $1/(\lambda A_0) = 20$ years and $1/(\gamma_F K_{f0}) = 25$ years, consistent with the tighter bound in Theorem 1. (A graphical representation is omitted for brevity but is available from the author upon request.)

4.2 Sensitivity Analysis

We examine how the singularity time t^* responds to variations in key parameters. The results are summarized in Table 2.

The singularity time is most sensitive to λ and γ_F, which appear quadratically in the dominant equations. For example, doubling λ from 0.05 to 0.10 reduces t^* from 18.2 years to approximately 11 years. In contrast, the coupling parameter β has a milder effect: reducing β from 1.0 to 0.5 extends t^* by about 20%, while increasing β to 2.0 shortens t^* by a similar margin. These results confirm that the singularity is primarily driven by the self-amplification terms, with the investment channel playing a secondary but still significant role.

4.3 Tax Intervention Scenarios

We illustrate the policy irreversibility result (Theorem 4) by simulating two intervention timings. The political feasibility bound is set as $\tau^{\max}(t) = 0.8 \cdot (W_w/(W_c + W_w))^1$ (i.e., $\tau_i^0 = 0.8$, $\psi = 1$), meaning that when capital owners hold 90% of total wealth, the maximum feasible tax rate drops to approximately $0.8 \times 0.1 = 0.08$.

Early intervention ($t = 5$ years). Introducing taxes $\tau_{ai} = 0.3$ and $\tau_f = 0.2$ at $t = 5$ years (well before the critical time $t_c \approx 10$ years) keeps workers' wealth W_w positive throughout the simulation. The wealth ratio W_c/W_w stabilizes around 50, and L_p remains above 0.3 even at $t = 20$ years. The required tax rates are well below the political feasibility bound ($0.3 < 0.6$ and $0.2 < 0.6$ at the time of introduction).

Late intervention ($t = 12$ years). If the same transfer level is to be achieved but intervention is delayed until $t = 12$ years (after t_c), the necessary tax rates rise sharply. Maintaining $W_w \geq 0$ now requires $\tau_{ai} > 0.9$ (with $\tau_f = 0.2$ fixed) or $\tau_f > 0.85$ (with $\tau_{ai} = 0.3$ fixed). Both exceed the political feasibility bound at that time, which has fallen to approximately 0.6 due to increased wealth concentration. Hence, redistribution becomes politically impossible.

Interpretation. These numerical experiments confirm Theorem 4: there exists a finite window for effective policy intervention. Before t_c, moderate taxes can stabilise the wealth ratio; after t_c, the required tax rates exceed what is politically feasible, and the economy drifts toward the Neofeudal regime. The exact value of t_c depends on the political elasticity ψ and the speed of the singularity.

4.4 Physical Constraints and Model Limitations

The mathematical singularity $(A, K_f \to \infty)$ is an idealization. In reality, physical limits (energy, computation, raw materials) would bound growth. For AI, Landauer's principle sets a minimum energy per computation; the Earth's available energy would limit A to some finite maximum. For finance, leverage constraints and regulatory caps would bound K_f. Thus, the singularity should be interpreted as a regime of extremely rapid, self-accelerating growth that outruns any exponential benchmark, not as a literal prediction of infinite values. The model also violates Lipschitz continuity at t^*, which is by design to

Table 2. Sensitivity of the singularity time t^* (baseline ≈ 18.2 years).

Parameter	Value range	t^* range (yrs)	Effect
λ (AI)	0.02–0.10	32–11	Strong
γ_F (finance)	0.02–0.08	25–14	Strong
β (investment)	0.5–2.0	$\approx 20\%$ var.	Mild

represent a finite-time blow-up; this is a mathematical artifact indicating that the linear approximation breaks down.

Finally, the model assumes taxes do not directly affect AI or financial capital accumulation; incorporating such feedback is a direction for future research.

5 Discussion and Conclusion

The model shows that recursive self-improvement in AI, coupled with financial autocatalysis, can lead to finite-time singularities in both technological capability and wealth concentration. The quadratic terms λA^2 and $\gamma_F K_f^2$ are the minimal mathematical representation of such accelerating dynamics. Policy interventions are effective only if implemented before the singularity window closes; after that, wealth concentration renders redistribution politically infeasible. The classification into three regimes highlights the path-dependent nature of the transition.

5.1 Caveats and Future Work

Several limitations should be acknowledged. First, the model abstracts from international capital flows, which could alter the dynamics. Second, the political feasibility function $\tau^{\max}(t)$ is stylized; empirical estimation of ψ is needed. Third, the absence of direct tax feedback on capital accumulation (addressed in Section 4.3) means that the singularity in A and K_f is unavoidable in the base model. Fourth, the mathematical singularity is an idealization; real-world constraints would bound growth. Future work could introduce stochastic shocks, heterogeneous agents, or more detailed microfoundations.

5.2 Conclusion

A dynamical systems model has been developed that captures the essential feedbacks between recursive AI improvement, financialization, and labor displacement. The theoretical results establish the possibility of finite-time singularities, super-exponential wealth divergence, distinct long-run regimes under taxation, and a fundamental policy irreversibility. The findings underscore the urgency of early intervention in AI-driven economies to prevent extreme inequality, while recognizing the model's limitations as a stylized representation.

Data Availability Statement

Data will be made available on request.

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Conflicts of Interest

The author declares no conflicts of interest.

AI Use Statement

The author declares that generative AI tools were used solely for language editing and translation assistance during the preparation of this manuscript. Specifically, DeepSeek-R1 was utilized to improve linguistic clarity and readability. The authors take full responsibility for the originality and accuracy of the final work.

Ethical Approval and Consent to Participate

Not applicable.

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Appendix: Proofs

This appendix contains the complete proofs of the four main theorems. Due to space constraints, only Theorem 1 is presented in full; the others follow similar lines and are available from the author.

Proof. We prove that under the stated conditions, the coupled system (1)–(3) exhibits a finite-time singularity. The proof proceeds in four steps.

Step 1: Dominant subsystem. Near a potential singularity, the fastest-growing terms dominate. Depreciation δK_{ai} and the linear return rK_f become negligible compared to quadratic terms. Thus we consider the reduced system

$$\frac{dA}{dt} = \lambda A^2 + \alpha_A K_f, \tag{A1}$$
$$\frac{dK_f}{dt} = \gamma_F K_f^2 + \eta K_{ai}, \tag{A2}$$
$$\frac{dK_{ai}}{dt} = \beta A K_f. \tag{A3}$$

All parameters are positive, and initial conditions are assumed positive.

Step 2: A differential inequality for $U = A + K_f$. Define $U(t) = A(t) + K_f(t)$. Differentiating and substituting (A1) and (A2) gives

$$\begin{aligned}\frac{dU}{dt} &= \lambda A^2 + \alpha_A K_f + \gamma_F K_f^2 + \eta K_{ai} \\ &\geq \lambda A^2 + \gamma_F K_f^2, \end{aligned}\tag{A4}$$

since $\alpha_A K_f + \eta K_{ai} \geq 0$. Using $A^2 + K_f^2 \geq \frac{1}{2}(A+K_f)^2 = \frac{1}{2}U^2$, we obtain

$$\begin{aligned}\lambda A^2 + \gamma_F K_f^2 &\geq \min(\lambda, \gamma_F)(A^2 + K_f^2) \\ &\geq \frac{1}{2}\min(\lambda, \gamma_F)U^2. \end{aligned}\tag{A5}$$

Combining (A4) and (A5) yields

$$\frac{dU}{dt} \geq \frac{1}{2}\min(\lambda, \gamma_F)U^2. \tag{A6}$$

Step 3: Blow-up from the inequality. Since $U(0) > 0$, separate variables in (A6) and integrate from 0 to t:

$$\begin{aligned}&\int_{U(0)}^{U(t)} \frac{dU}{U^2} \geq \frac{1}{2}\min(\lambda, \gamma_F)\int_0^t ds \\ &\implies \quad \frac{1}{U(0)} - \frac{1}{U(t)} \geq \frac{1}{2}\min(\lambda, \gamma_F)\, t. \end{aligned}\tag{A7}$$

Rearranging,

$$U(t) \geq \frac{U(0)}{1 - \frac{1}{2}\min(\lambda, \gamma_F)U(0)t}. \tag{A8}$$

The right-hand side diverges as

$$t \to T = \frac{2}{\min(\lambda, \gamma_F)U(0)}.$$

Hence $U(t)$ must blow up at or before T, implying that at least one of $A(t)$ or $K_f(t)$ diverges in finite time. Because K_{ai} is driven by AK_f (see (A3)), it also diverges. Thus a finite-time singularity exists with $t^* \leq T$.

Step 4: Sharper upper bound on t^*. The bound T is crude because we discarded the positive terms $\alpha_A K_f + \eta K_{ai}$. A sharper bound comes from considering the initial effective growth rates of A and K_f separately.

From (A1), the initial growth rate of A is

$$\left.\frac{dA}{dt}\right|_{t=0} = \lambda A_0^2 + \alpha_A K_{f0} = A_0\left(\lambda A_0 + \alpha_A \frac{K_{f0}}{A_0}\right). \tag{A9}$$

If this rate remained constant, A would follow a Riccati equation with blow-up time $1/(\lambda A_0 + \alpha_A K_{f0}/A_0)$. Since the actual growth accelerates (because K_f increases), the true singularity time cannot exceed this bound. Formally, a comparison argument using $dA/dt \geq \lambda A^2 + \alpha_A K_f$ with $K_f \geq K_{f0}$ gives $t^* \leq 1/(\lambda A_0 + \alpha_A K_{f0}/A_0)$. The same reasoning for K_f yields $t^* \leq 1/(\gamma_F K_{f0} + \eta K_{ai0}/K_{f0})$.

The system blows up when either variable diverges, so the overall singularity time is bounded above by the smaller of the two:

$$t^* \leq \min\left(\frac{1}{\lambda A_0 + \alpha_A \frac{K_{f0}}{A_0}}, \frac{1}{\gamma_F K_{f0} + \eta \frac{K_{ai0}}{K_{f0}}}\right). \tag{A10}$$

This completes the proof.

Remark on Lipschitz continuity. The right-hand sides of (1)–(3) are locally Lipschitz on $\mathbb{R}_+^4$ as long as the variables remain finite. At t^*, the solution ceases to exist because the vector field grows without bound; this is a finite-time blow-up, not a violation of the Picard–Lindelöf theorem on $[0, t^*)$. The model is well-posed up to the singularity. $\square$

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