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RESEARCH ARTICLE

New Exponential Function Based Fixed/Preassigned-Time Synchronization for A Kind of Neural Networks with Time-Varying Delays

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Abstract

This paper investigates a class of neural networks (NNs) with time-varying delays. At first, based on the general exponential function and by using inequality techniques, we establish novel lemmas addressing fixed/preassigned-time synchronization for such NNs. Then, by employing these derived lemmas and designing two effective feedback controllers, we systematically study the fixed-time synchronization (FTS) and preassigned-time synchronization (PRTS) problems of delayed NNs. In addition, the settling-time estimation in our fixed-time stability lemma expresses superior accuracy compared to existing results in previous related works, which can all be viewed as special cases of this paper. Finally, numerical simulations demonstrate the validity and practicality of the theoretical findings.

Keywords: neural networks (NNs), fixed-time synchronization (FTS), preassigned-time synchronization (PRTS), feedback control, time-varying delays.

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1 Introduction

In recent years, dynamic behaviors of neural networks (NNs) have drawn widespread interests from researchers because of their crucial applications in areas such as image processing [1], information prediction [2, 3], and fault-tolerant control [4]. Among them, stability and synchronization of NNs have been given more pages and many interesting works have been reported, e.g., see [5–7]. In 2020, Arik [8] studied stability of neutral-type NNs with multiple delays, and by using continuous/periodic sampling algorithm, Wang *et al.* [9] discussed synchronization of delayed memristive NNs. In 2021, by designing event-triggered impulsive control, Chen *et al.* [10] got the synchronization of NNs with disconnected switching topology, Zhang *et al.* [11] achieved the synchronization of delayed second-order fuzzy memristive NNs, Fu *et al.* [12] through pinning impulsive control got exponential synchronization of memristive inertial NNs. In 2022, Shen *et al.* [13] derived synchronization of complex-valued NNs, Wang *et al.* [14] studied finite-time synchronization of delayed inertial NNs via feedback control. In 2023, Zhang *et al.* [15] investigated the stability of state-dependent switching inertial NNs. Recently, Duan *et al.* [16]discussed finite-time synchronization of complex-valued BAM inertial NNs.

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Noteworthy, the discussions in the aforementioned papers are all focused on asymptotic/finite-time stability and synchronization, which convergence time is too long or depends on initial conditions of systems, and these limitations restrict the practical implementation of the theoretical findings in real-world engineering applications. In 2012, Polyakov [17] proposed the fixed-time stability definition, which convergence time, that is, called the settling-time, can be estimated and do not connect with the initial conditions of systems. Then, many valuable results about fixed-time synchronization (FTS) and stability of NNs are reached, e.g., see [18–23]. In 2017, Hu *et al.* [18] discussed FTS of discontinuous NNs. In 2021, Kong *et al.* [19] investigated FTS for fuzzy inertial NNs, Aouiti *et al.* [20] got fixed-time stabilization of neutral-type inertial NNs. In 2022, Gan *et al.* [21] derived FTS of memristive complex-valued NNs. In 2023, by using aperiodically switching control, Hu *et al.* [22] given fixed-time stabilization of coefficients discontinuous spatiotemporal NNs. In 2024, Zhang *et al.* [23] expressed new approximate results of fixed-time stabilization for delayed second-order NNs.

Although the upper-bound for the settling-time function of fixed-time stability is a positive constant that independent of initial values, but it cannot be preassigned according to practical requirements. In 2017, Jiménez-Rodríuez *et al.* [24] showed results about predefined-time stabilization of systems, which settling-time can be preassigned at first and do not related with any parameters of the systems. So, preassigned-time synchronization (PRTS) has more advantages than FTS. In these years, some results on PRTS of NNs have been discussed in [25–29]. In 2021, Chen *et al.* [25] showed PRTS of a class of competitive NNs, Hu *et al.* [26] got PRTS of complex networks with an improved fixed-time stability. In 2022, Han *et al.* [27] discussed the preassigned-time anti-synchronization of delayed fuzzy inertial NNs. In 2023, by using non-reduced order approach, Zhang *et al.* [28] expressed PRTS of delayed fuzzy inertial NNs, Wang *et al.* [29] investigated PRTS of delayed fractional-order memristive NNs.

However, fixed/preassigned-time stability Lemmas used in most of the previous works are mainly based on the inequality $\frac{\mathrm{d}\mathcal{V}(t)}{\mathrm{d}t} \leqslant -\alpha\mathcal{V}^p(t) - \beta\mathcal{V}^q(t)$, $(\alpha, \beta > 0, 0 \leqslant p \leqslant 1, q > 1)$, which has two exponential variables that lead to certain difficulties in the practical design of the controllers. Now, an interesting question comes up, that is , whether we can only use one exponential variable to realize FTS and PRTS of NNs. So, in this paper, we will use general exponential function that contains only one exponential variable to derive our FTS and PRTS lemmas, and based on these proposed lemmas, some new results on FTS and PRTS of a class of delayed NNs will be studied. This study makes the following novel contributions:

Firstly, a new FTS lemma for a delayed NNs is built, which settling-time is more precise compared to existing results [24, 25]. And the new lemma can be used to study other nonlinear dynamic systems.

Secondly, a new lemma on PRTS of the delayed NNs is constructed, and the preassigned-time of the PRTS do not depend on any parameters of the delayed NNs.

Thirdly, two new effective feedback controllers are designed to realize FTS and PRTS of the delayed NNs, respectively, and some comparisions between related previous works with this paper are given in the simulations.

This paper is organized as follows: Section 2 presents the preliminaries. In Section 3, new criteria for FTS and PRTS of the delayed NNs are given. Section 4 shows the simulation and comparison results. Finally, conclusions are displayed.

2 Preliminaries

Notations: Let $\mathcal{N} = \{1, 2, ..., v\}$, $\mathbf{R}^v$ is the Euclidean space with v-dimensional. $(\cdot)^T$ is the transpose of $(\cdot)$, $\dot{p}(t)$ stands for derivative of $p(t)$. For $\forall y = (y_1, y_2, ..., y_v)^T \in \mathbf{R}^v$, which norm is $\|y\| = \sum_{k=1}^{v} |y_k|$. $\vartheta_k = \max\{|\vartheta_k^-|, |\vartheta_k^+|\}$. And $\mathbf{C}([-\mu, 0], \mathbf{R}^v)$ denotes all continuous function from $[-\mu, 0]$ to $\mathbf{R}^v$, where $\mu = \max_{1\leqslant k\leqslant v}\{\mu_k\}, \mu_k = \max_{t\geqslant 0}\{\mu_k(t)\}, k \in \mathcal{N}$.

2.1 Delayed NNs model

In this paper, we consider the delayed NNs as follows:

$$\begin{aligned}\frac{\mathrm{d}m_k(t)}{\mathrm{d}t} = -a_k m_k(t) + \sum_{j=1}^{v} b_{kj}\mathcal{F}_j(m_j(t)) \\ + \sum_{j=1}^{v} c_{kj}\mathcal{F}_j(m_j(t-\mu_j(t))) \\ + \delta_k, t \geqslant 0, \qquad (1)\end{aligned}$$

where $m_k(t)$ is the k-th neural state, $a_k > 0$, b_{kj}, c_{kj} are all connect weights, the feedback function $\mathcal{F}_j(\cdot)$ is continuous, $\mu_j(t) > 0$, which is time delay and satisfy $\mu_j(t) \leqslant \mu_j$, δ_k is external input. Let the initial values of NNs (1) are $m_k(s) = \urcorner_k(s), \urcorner_k(s) \in \mathbf{C}([-\mu, 0], \mathbf{R}), k \in \mathcal{N}$.

Assumption 1. For NNs (1), which feedback function $\mathcal{F}_k(\cdot)$ is continuous, $\mathcal{F}_k(0)=0$ and for $\forall\, y_1, y_2 \in \mathbf{R}$, has

$$\vartheta_k^- \leqslant \frac{\mathcal{F}_k(y_1)-\mathcal{F}_k(y_2)}{y_1-y_2} \leqslant \vartheta_k^+, |\mathcal{F}_k(y_1)| \leqslant L_k, \qquad (2)$$

in which, $\vartheta_k^-, \vartheta_k^+ \in \mathbf{R}, y_1 \neq y_2, L_k > 0, k \in \mathcal{N}$.

Remark 1. Under Assumption 1, one can easy find that NNs (1) exists solution $m_k(t)$ with starting value $m_k(s) = \daleth_k(s), \daleth_k(s) \in \mathbf{C}([-\mu, 0], \mathbf{R})$, and $t \geqslant 0, k \in \mathcal{N}$.

Let NNs (1) as the drive system, which response system is

$$\begin{aligned}\frac{\mathrm{d}n_k(t)}{\mathrm{d}t} &= -a_k n_k(t) + \sum_{j=1}^{v} b_{kj}\mathcal{F}_j(n_j(t)) \\ &\quad + \sum_{j=1}^{v} c_{kj}\mathcal{F}_j(n_j(t-\mu_j(t))) \\ &\quad + \delta_k + u_k(t), \qquad (3)\end{aligned}$$

in which $u_k(t)$ is control input, and other parameters are same as in NNs (1). Let initial values of NNs (2) are $n_k(s) = \beth_k(s), \beth_k(s) \in \mathbf{C}([-\mu, 0], \mathbf{R}), k \in \mathcal{N}$.

Let $w_k(t) = n_k(t) - m_k(t), k \in \mathcal{N}$, then, the synchronization error system is

$$\begin{aligned}\frac{\mathrm{d}w_k(t)}{\mathrm{d}t} &= -a_k w_k(t) + \sum_{j=1}^{v} b_{kj}\mathcal{G}_j(w_j(t)) \\ &\quad + \sum_{j=1}^{v} c_{kj}\mathcal{G}_j(w_j(t-\mu_j(t))) \\ &\quad + u_k(t), t \geqslant 0, \qquad (4)\end{aligned}$$

in which, $\mathcal{G}_j(w_j(t)) = \mathcal{F}_j(n_j(t)) - \mathcal{F}_j(m_j(t)), k \in \mathcal{N}$.

2.2 Definitions and Lemmas

Definition 1 ([28]). Drive-response NNs (1) and (3) achieved FTS and the corresponding error system (4) gets fixed-time stable, if origin of system (4) is stable, and there is a positive constant $\mathbf{T}^{\max}$ such that the settling function $\mathbf{T}(\daleth(0), \beth(0)) \leqslant \mathbf{T}^{\max}$, and $\lim_{t\to\mathbf{T}^{\max}} \|w(t)\| = \lim_{t\to\mathbf{T}^{\max}} \|n(t)-m(t)\| = 0$ as well as $\|w(t)\| = \|n(t)-m(t)\| = 0$ when $t \geqslant \mathbf{T}^{\max}$, where $w(t) = n(t) - m(t) = (n_1(t) - m_1(t), n_2(t) - m_2(t), ..., n_v(t) - m_v(t))^T, t \geqslant 0$ and $\mathbf{T}^{\max}$ is named settling-time.

Definition 2 ([28]). Drive-response NNs (1) and (3) get PRTS and the error system (4) achieves preassigned-time stable, if system (4) is fixed-time stable, and for a preassigned positive constant $\mathbf{T}^p$, which do not depend on any parameters and initial values of NNs (1) and (3), such that the settling function $\mathbf{T}(\daleth(0), \beth(0)) \leqslant \mathbf{T}^p$, and $\lim_{t\to\mathbf{T}^p} \|w(t)\| = \lim_{t\to\mathbf{T}^p} \|n(t)-m(t)\| = 0$ as well as $\|w(t)\| = \|n(t) - m(t)\| = 0$ when $t \geqslant \mathbf{T}^p$, where $t \geqslant 0, \mathbf{T}^p$ is called preassigned-time.

Lemma 1. Suppose that there is a continuous regular function $\mathbf{V}(\cdot)$, which is positive definite and radially-unbounded, and follow with solutions of (4) that satisfy

$$\frac{\mathrm{d}\mathbf{V}(t)}{\mathrm{d}t} \leqslant -c\, b^{\mathbf{V}^q(t)}\mathbf{V}^{1-q}(t) - \sigma, \qquad (5)$$

in which, $b > 1, c > 0, \sigma \geqslant 0, 0 < q \leqslant 1$, then, system (4) achieves fixed-time stable as well as the drive-response NNs (1) and (3) get FTS. If $\sigma = 0$, the settling-time is $\mathbf{T}_1^{\max} = \frac{1}{qc\ln b}$, and if $\sigma > 0$, settling-time is

$$\mathbf{T}_2^{\max} = \frac{1}{q\ln b}\left[\frac{1}{\sigma}\ln\left(\frac{b(c+\sigma)}{bc+\sigma}\right) + \frac{1}{bc}\right]. \qquad (6)$$

Proof. From (5), one can find that $\frac{\mathrm{d}\mathbf{V}(t)}{\mathrm{d}t} < 0$, because $\mathbf{V}(\cdot)$ is positive definite and $\mathbf{V}(w(t)) \to +\infty\, (w(t) \to +\infty)$. Therefore, system (4) is global asymptotic stable. And from (5), we get the settling-time function is

$$\mathbf{T}(w(0)) = \int_0^{\mathbf{V}(w(0))} \frac{\mathrm{d}p}{cb^{p^q}p^{1-q} + \sigma}. \qquad (7)$$

Now, we derive the estimate value of (7). If $\sigma = 0$, then,

$$\begin{aligned}\mathbf{T}(w(0)) &\leqslant \int_0^{+\infty} \frac{\mathrm{d}p}{cb^{p^q}p^{1-q}} = \frac{1}{qc}\int_0^{+\infty}\frac{\mathrm{d}s}{b^s} \\ &= -\frac{1}{qc}\int_0^{+\infty} b^{-s}\mathrm{d}(-s) \\ &= -\frac{1}{qc}\int_0^{-\infty} b^x\mathrm{d}x = \frac{1}{qc}\int_{-\infty}^{0} b^x\mathrm{d}x \\ &= \frac{b^x}{qc\ln b}\Big|_{-\infty}^{0} = \frac{1}{qc\ln b} = \mathbf{T}_1^{\max}. \qquad (8)\end{aligned}$$

And if $\sigma > 0$, we have

$$\begin{aligned}\mathbf{T}(w(0)) &\leqslant \int_0^{+\infty} \frac{\mathrm{d}p}{cb^{p^q}p^{1-q}+\sigma} \\ &= \int_0^1 \frac{\mathrm{d}p}{cb^{p^q}p^{1-q}+\sigma} + \int_1^{+\infty} \frac{\mathrm{d}p}{cb^{p^q}p^{1-q}+\sigma} \\ &\leqslant \int_0^1 \frac{\mathrm{d}p}{cb^{p^q}p^{1-q}+\sigma p^{1-q}} \\ &\quad + \int_1^{+\infty} \frac{\mathrm{d}p}{cb^{p^q}p^{1-q}+\sigma}. \end{aligned} \tag{9}$$

Noted that,

$$\begin{aligned}\int_0^1 \frac{\mathrm{d}p}{cb^{p^q}p^{1-q}+\sigma p^{1-q}} &= \int_0^1 \frac{\mathrm{d}p}{(cb^{p^q}+\sigma)p^{1-q}} \\ &= \frac{1}{q}\int_0^1 \frac{\mathrm{d}s}{(cb^s+\sigma)} \\ &= \frac{1}{q}\int_0^1 \frac{b^{-s}\mathrm{d}s}{(c+\sigma b^{-s})}. \end{aligned} \tag{10}$$

Let $y = c + \sigma b^{-s}$, we get $\mathrm{d}y = -\sigma \ln b\, b^{-s}\mathrm{d}s$, then, $b^{-s}\mathrm{d}s = -\frac{1}{\sigma \ln b}\mathrm{d}y$. Now, from (10), one obtains

$$\begin{aligned}\frac{1}{q}\int_0^1 \frac{b^{-s}\mathrm{d}s}{(c+\sigma b^{-s})} &= -\frac{1}{q\sigma \ln b}\int_{c+\sigma}^{c+\frac{\sigma}{b}} \frac{\mathrm{d}y}{y} \\ &= \frac{1}{q\sigma \ln b}\int_{c+\frac{\sigma}{b}}^{c+\sigma} \frac{\mathrm{d}y}{y} \\ &= \frac{\ln y}{q\sigma \ln b}\Bigg|_{c+\frac{\sigma}{b}}^{c+\sigma} \\ &= \frac{1}{q\sigma \ln b}\ln\left(\frac{b(c+\sigma)}{bc+\sigma}\right). \end{aligned} \tag{11}$$

And

$$\begin{aligned}\int_1^{+\infty} \frac{\mathrm{d}p}{cb^{p^q}p^{1-q}+\sigma} &\leqslant \int_1^{+\infty} \frac{\mathrm{d}p}{cb^{p^q}p^{1-q}} = \frac{1}{qc}\int_1^{+\infty} \frac{\mathrm{d}s}{b^s} \\ &= -\frac{1}{qc}\int_1^{+\infty} b^{-s}\mathrm{d}(-s) \\ &= \frac{1}{qc}\int_{-\infty}^{-1} b^x \mathrm{d}(x) \\ &= \frac{b^x}{qc\ln b}\Bigg|_{-\infty}^{-1} = \frac{1}{qbc\ln b}. \end{aligned} \tag{12}$$

Now, from (8)-(10), if $\sigma > 0$, we get the settling-time is

$$\mathbf{T}(w(0)) \leqslant \frac{1}{q\ln b}\left[\frac{1}{\sigma}\ln\left(\frac{b(c+\sigma)}{bc+\sigma}\right) + \frac{1}{bc}\right] = \mathbf{T}_2^{\max}.$$

The proof of Lemma 1 is completed.

Remark 2. From (8), (9) and (10), one can easily find that $\mathbf{T}_2^{\max} < \mathbf{T}_1^{\max}$.

Remark 3. If $b = e, \sigma = 0$, then, the inequality (5) become the one used in [24, 25]. From Remark 2, of course, we can get the settling-time $\mathbf{T}_2^{\max}$ of this paper is more precise than the earlier works [24, 25]. And if $c = 0$ in inequality (5), one can get the finite-time stable of NNs discussed in [30]. Therefore, the Lemma 1 of this paper are more general, which can be used to discuss FTS for other more complex nonlinear systems.

Lemma 2. Let $\mathbf{T}^p$ is a preassigned positive constant, and suppose that the continuous regular function $\mathbf{V}(\cdot)$ is positive definite and radially-unbounded, and follow with solutions of (4) that satisfy

$$\frac{\mathrm{d}\mathbf{V}(t)}{\mathrm{d}t} \leqslant -\frac{\mathbf{T}^{\max}}{\mathbf{T}^p}\Big(c\, b^{\mathbf{V}^q(t)}\mathbf{V}^{1-q}(t) + \sigma\Big), \tag{13}$$

in which, $b > 1, c > 0, 0 < q \leqslant 1$, if $\sigma = 0, \mathbf{T}^{\max} = \mathbf{T}_1^{\max}$, and if $\sigma > 0, \mathbf{T}^{\max} = \mathbf{T}_2^{\max}$, then, system (4) achieves preassigned-time stable as well as the drive-response NNs (1) and (3) get PRTS, and $\mathbf{T}^p$ is named preassigned-time.

Proof. From (13), we get the settling-time function is

$$\mathbf{T}(w(0)) = \frac{\mathbf{T}^p}{\mathbf{T}^{\max}}\int_0^{\mathbf{V}(w(0))} \frac{\mathrm{d}p}{c\, b^{p^q}p^{1-q}+\sigma}. \tag{14}$$

Now, from (8)-(12), one can easily get that $\mathbf{T}(w(0)) \leqslant \mathbf{T}^p$. By using Definition 2, one can get Lemma 2 hold. This proof is finished.

Remark 4. The inequalities (5) and (13) depend on the function $\mathbf{V}(t)$. And inequalities (5) and (13) only have one exponential variable q that satisfies $0 < q \leqslant 1$, which is unlike the previous works [17–23] that based on inequality $\frac{\mathrm{d}\mathbf{V}(t)}{\mathrm{d}t} \leqslant -\alpha\mathbf{V}^p(t) - \beta\mathbf{V}^q(t)$ to get the FTS results, where $\alpha, \beta > 0, 0 \leqslant p \leqslant 1, q > 1$. So, our results are more straightforward and simplify the designing structure of the fixed-time controllers.

3 FTS and PRTS of the delayed NNs

3.1 FTS of the Delayed NNs (1) and (3)

If $w_k(t) \neq 0, k \in \mathcal{N}$, in order to get FTS of NNs (1) and (3), the control scheme in error system (4) is designed as follows:

$$\begin{aligned}u_k(t) = &-\xi_k w_k(t) - \eta_k\, b^{\mathbf{V}^q(t)}\mathbf{V}^{-q}(t)w_k(t) \\ &-\gamma_k \mathrm{sign}(w_k(t)), \end{aligned} \tag{15}$$

where $\xi_k \geqslant 0, \eta_k > 0, b > 1, \gamma_k > 0, 0 < q \leqslant 1, \mathbf{V}(t) = \sum_{k=1}^{v} |w_k(t)|$.

Theorem 1. Under Assumption 1 and the control scheme (15), and if the following conditions hold

$$a_k + \xi_k - \sum_{j=1}^{v} |b_{jk}|\vartheta_k > 0, \tag{16}$$

$$\gamma_k - \sum_{j=1}^{v} |c_{kj}|L_j > 0, \tag{17}$$

then, drive-response NNs (1) and (3) achieve FTS, which settling-time is $\mathbf{T}_2^{\max}$.

Proof. Now, let us consider the following C-*regular* function

$$\mathbf{V}(t) = \sum_{k=1}^{v} |w_k(t)|. \tag{18}$$

By using the chain-rule given in [31], one can get the derivative of $\mathbf{V}(t)$ follow with solutions of (4) as follows:

$$\begin{aligned}
\frac{\mathrm{d}\mathbf{V}(t)}{\mathrm{d}t}\bigg|_{(4)} &= \sum_{k=1}^{v} \dot{w}_k(t) \cdot \mathrm{sign}(w_k(t)) \\
&= \sum_{k=1}^{v} \mathrm{sign}(w_k(t)) \cdot \Bigg(- a_k w_k(t) \\
&\quad + \sum_{j=1}^{v} b_{kj}\mathcal{G}_j(w_j(t)) + u_k(t) \\
&\quad + \sum_{j=1}^{v} c_{kj}\mathcal{G}_j(w_j(t-\mu_j(t)))\Bigg) \\
&= \sum_{k=1}^{v} \mathrm{sign}(w_k(t)) \cdot \Bigg(- (a_k + \xi_k) w_k(t) \\
&\quad + \sum_{j=1}^{v} b_{kj}\mathcal{G}_j(w_j(t)) + \sum_{j=1}^{v} c_{kj} \\
&\quad \times \mathcal{G}_j(w_j(t-\mu_j(t))) - \eta_k\, b^{\mathbf{V}^q(t)}\mathbf{V}^{-q}(t) \\
&\quad \times w_k(t) - \gamma_k \mathrm{sign}(w_k(t))\Bigg),
\end{aligned}$$

under Assumption 1, we get $|\mathcal{G}_j(w_j(t))| \leqslant \vartheta_j |w_j(t)|$, and

$$\begin{aligned}
\sum_{k=1}^{v} \mathrm{sign}(w_k(t)) \sum_{j=1}^{v} b_{kj}\mathcal{G}_j(w_j(t)) &\leqslant \sum_{k=1}^{v} \sum_{j=1}^{v} |b_{kj}\mathcal{G}_j(w_j(t))| \\
&\leqslant \sum_{k=1}^{v} \sum_{j=1}^{v} |b_{kj}|\vartheta_j |w_j(t)| \\
&= \sum_{k=1}^{v} \sum_{j=1}^{v} |b_{jk}|\vartheta_k |w_k(t)|,
\end{aligned}$$

therefore,

$$\begin{aligned}
\frac{\mathrm{d}\mathbf{V}(t)}{\mathrm{d}t}\bigg|_{(4)} &\leqslant - \sum_{k=1}^{v} \Big(a_k + \xi_k - \sum_{j=1}^{v} |b_{jk}|\vartheta_k\Big)|w_k(t)| \\
&\quad - \sum_{k=1}^{v} \eta_k\, b^{\mathbf{V}^q(t)}\mathbf{V}^{-q}(t)|w_k(t)| \\
&\quad - \sum_{k=1}^{v} \Big(\gamma_k - \sum_{j=1}^{v} |c_{kj}|L_j\Big) \\
&\leqslant - \sum_{k=1}^{v} \Big(a_k + \xi_k - \sum_{j=1}^{v} |b_{jk}|\vartheta_k\Big)|w_k(t)| \\
&\quad - b^{\mathbf{V}^q(t)}\mathbf{V}^{-q}(t) \sum_{k=1}^{v} \eta_k |w_k(t)| \\
&\quad - \sum_{k=1}^{v} \Big(\gamma_k - \sum_{j=1}^{v} |c_{kj}|L_j\Big).
\end{aligned} \tag{19}$$

Let $c = \min_{1\leqslant k\leqslant v}\{\eta_k\}, \sigma = \sum_{k=1}^{v}\Big(\gamma_k - \sum_{j=1}^{v} |c_{kj}|L_j\Big)$. Based on (16), (17) and (19), one gets

$$\frac{\mathrm{d}\mathbf{V}(t)}{\mathrm{d}t} \leqslant -c\, b^{\mathbf{V}^q(t)}\mathbf{V}^{1-q}(t) - \sigma. \tag{20}$$

Now, by using the Lemma 1, we obtain error system (4) reaches fixed-time stable, the drive-response system (1) and (3) achieve FTS under control (15), and the settling-time is $\mathbf{T}_2^{\max}$. The proof is completed.

3.2 PRTS of the Delayed NNs (1) and (3)

Now, we design the following control to get PRTS of NNs (1) and (3)

$$\begin{aligned}
u_k^*(t) = &- \xi_k w_k(t) - \frac{\mathbf{T}^{\max}}{\mathbf{T}^p}\Big(\eta_k\, b^{\mathbf{V}^q(t)}\mathbf{V}^{-q}(t) w_k(t) \\
&+ \gamma_k \mathrm{sign}(w_k(t))\Big) + \Big(\frac{\mathbf{T}^{\max}}{\mathbf{T}^p} - 1\Big)\zeta_k,
\end{aligned} \tag{21}$$

where $\zeta_k = \sum_{j=1}^{v} |c_{kj}|L_j, k \in \mathcal{N}$, and the other parameters are given same as in (15).

Theorem 2. Choose a preassigned positive constant $\mathbf{T}^p$, under Assumption 1 and with the control scheme (21), if the conditions (16) and (17) hold, then, the drive-response NNs (1) and (3) achieve PRTS, which settling-time is $\mathbf{T}^p$.

Proof. From the proof of Theorem 1 and under control (21), one can get

$$\frac{\mathrm{d}\mathbf{V}(t)}{\mathrm{d}t}\bigg|_{(4)} = \sum_{k=1}^{v} \dot{w}_k(t) \cdot \mathrm{sign}(w_k(t))$$

$$
\begin{aligned}
&= \sum\nolimits_{k=1}^{v} \mathrm{sign}(w_k(t)) \cdot \Bigg(- a_k w_k(t) \\
&\quad + \sum_{j=1}^{v} b_{kj}\mathcal{G}_j(w_j(t)) + u_k^*(t) \\
&\quad + \sum_{j=1}^{v} c_{kj}\mathcal{G}_j(w_j(t-\mu_j(t)))\Bigg) \\
&= \sum\nolimits_{k=1}^{v} \mathrm{sign}(w_k(t)) \cdot \Bigg(- (a_k+\xi_k) w_k(t) \\
&\quad + \sum_{j=1}^{v} b_{kj}\mathcal{G}_j(w_j(t)) + \sum_{j=1}^{v} c_{kj} \\
&\quad \times \mathcal{G}_j(w_j(t-\mu_j(t))) \\
&\quad - \frac{\mathbf{T}^{\max}}{\mathbf{T}^{p}}\Big(\eta_k\, b^{\mathbf{V}^q(t)}\mathbf{V}^{-q}(t) w_k(t) \\
&\quad + \gamma_k \mathrm{sign}(w_k(t))\Big) \\
&\quad + \zeta_k\Big(\frac{\mathbf{T}^{\max}}{\mathbf{T}^{p}} - 1\Big)\mathrm{sign}(w_k(t))\Bigg) \\
&\leqslant - \sum\nolimits_{k=1}^{v}\Big(a_k + \xi_k - \sum_{j=1}^{v}|b_{jk}|\vartheta_k\Big)|w_k(t)| \\
&\quad - \frac{\mathbf{T}^{\max}}{\mathbf{T}^{p}} \sum\nolimits_{k=1}^{v} \eta_k\, b^{\mathbf{V}^q(t)}\mathbf{V}^{-q}(t)|w_k(t)| \\
&\quad - \frac{\mathbf{T}^{\max}}{\mathbf{T}^{p}} \sum\nolimits_{k=1}^{v}\Big(\gamma_k - \zeta_k\Big) \\
&\leqslant - \frac{\mathbf{T}^{\max}}{\mathbf{T}^{p}}\Big(c\, b^{\mathbf{V}^q(t)}\mathbf{V}^{1-q}(t) + \sigma\Big). \qquad (22)
\end{aligned}
$$

Now, by using the Lemma 2, we obtain error system (4) reaches preassigned-time stable, the drive-response system (1) and (3) achieve PRTS under control (21), and the settling-time is $\mathbf{T}^p$. The proof is completed.

Remark 5. Because controllers (15) and (21) contain sign$(\cdot)$, so, $\tanh(r\cdot(\cdot)), r > 0$ can be used in the simulation to avoid chattering phenomenon.

Remark 6. In this paper, the Lyapunov function in (16) is given in 1-norm, and we can also use z-norm ($z \geqslant 2$) to construct the Lyapunov function. On the other hand, we consider the delayed NNs with continuous right-hand-side, in fact, the results of this paper also hold for delayed NNs with discontinuous right hand side, we think these issues are interesting and worthy our work together to discuss them in the future.

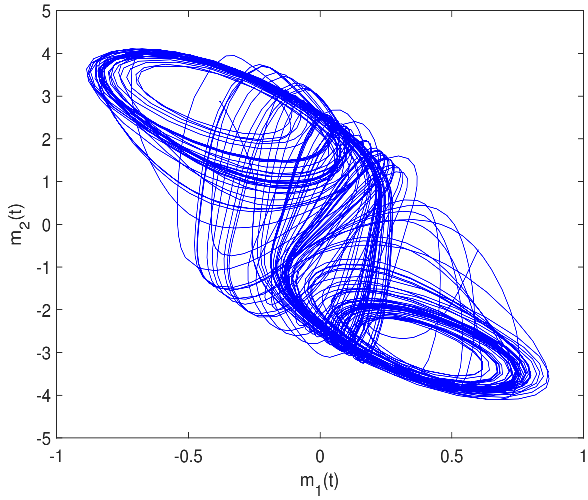

Figure 1. The delayed NNs (23) exists chaotic behaviors.

4 Example simulations

Example. We consider the following delayed NNs with two-dimensional

$$
\begin{aligned}
\frac{\mathrm{d}m_k(t)}{\mathrm{d}t} &= -a_k m_k(t) + \sum\nolimits_{j=1}^{2} b_{kj}\mathcal{F}_j(m_j(t)) \\
&\quad + \sum\nolimits_{j=1}^{2} c_{kj}\mathcal{F}_j(m_j(t-\mu_j(t))) \\
&\quad + \delta_k, \qquad (23)
\end{aligned}
$$

where $a_1 = a_2 = 1, b_{11} = 2, b_{12} = -0.1, b_{21} = -5, b_{22} = 3, c_{11} = -1.5, c_{12} = -0.1, c_{21} = -0.2, c_{22} = -2, \delta_1 = \delta_2 = 0, \mu_1(t) = \mu_2(t) = \frac{\exp(t)}{1+\exp(t)}$. Feedback function $\mathcal{F}_j(m_j(t)) = \tanh(m_j(t)), k, j = 1, 2$. Let the initial values of the delayed NNs (23) are $\daleth_1(s) = 0.6, \daleth_2(s) = -0.6, \forall s \in [-1, 0)$, and NNs (23) exist chaotic behaviors, which is shown in Figure 1. And the state trajectories $m_1(t)$ and $m_2(t)$ of NNs (23) are, respectively, shown in Figures 2 and 3.

Now, let the delayed NNs (23) as the derive system, and the response system as follows:

$$
\begin{aligned}
\frac{\mathrm{d}n_k(t)}{\mathrm{d}t} &= -a_k n_k(t) + \sum\nolimits_{j=1}^{2} b_{kj}\mathcal{F}_j(n_j(t)) \\
&\quad + \sum\nolimits_{j=1}^{2} c_{kj}\mathcal{F}_j(n_j(t-\mu_j(t))) \\
&\quad + \delta_k + u_k(t), k = 1, 2, \qquad (24)
\end{aligned}
$$

Let the initial values of the delayed NNs (24) are $\beth_1(s) = -0.4, \beth_2(s) = 0.4, \forall s \in [-1, 0)$. Then, the error state trajectories without control between drive-response systems (23) and (24) are, respectively, shown in Figures 4 and 5.

Now, we choose values of the parameters: $\xi_1 = 15, \xi_2 = 10, \gamma_1 = 2, \gamma_2 = 0.28, b = 2.5, q =$

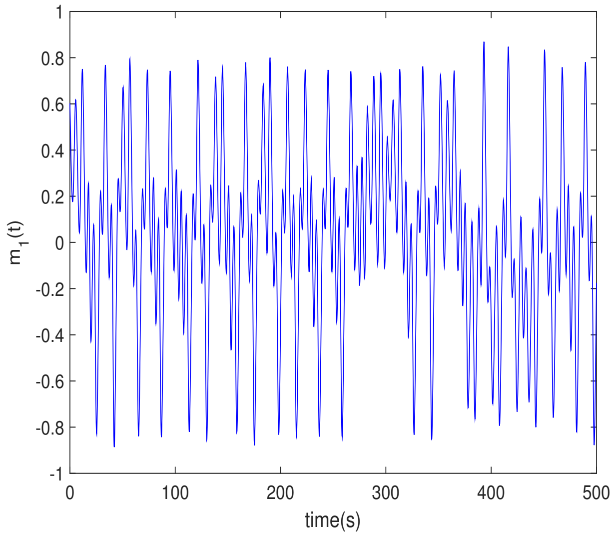

Figure 2. State trajectory $m_1(t)$ of NNs (23).

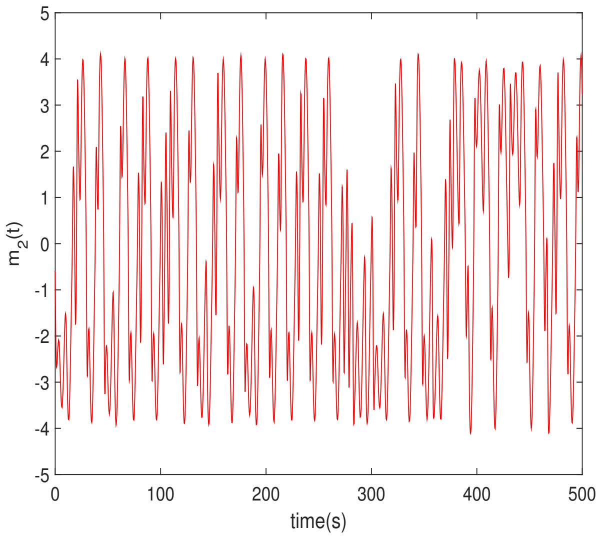

Figure 3. State trajectory $m_2(t)$ of NNs (23).

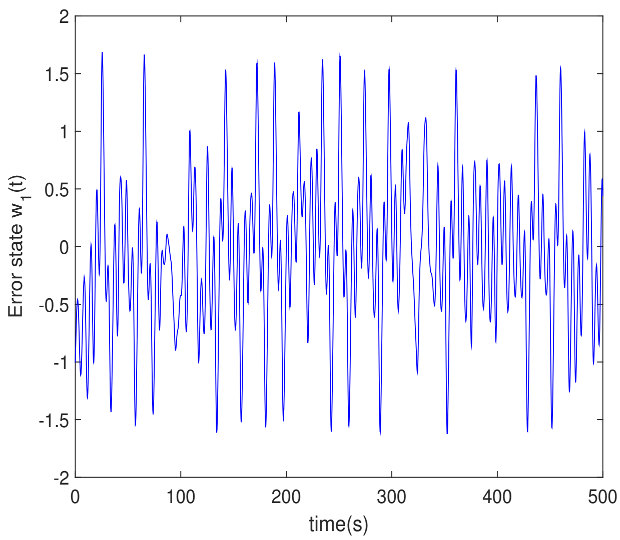

Figure 4. Error state trajectory $w_1(t)$ without control (15).

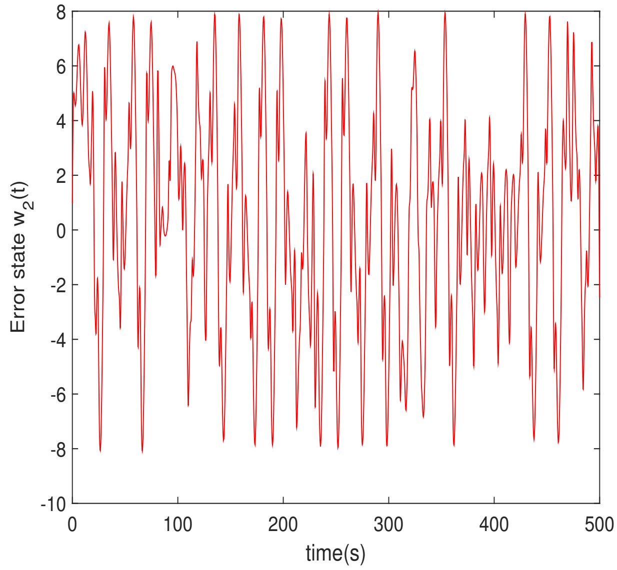

Figure 5. Error state trajectory $w_2(t)$ without control (15).

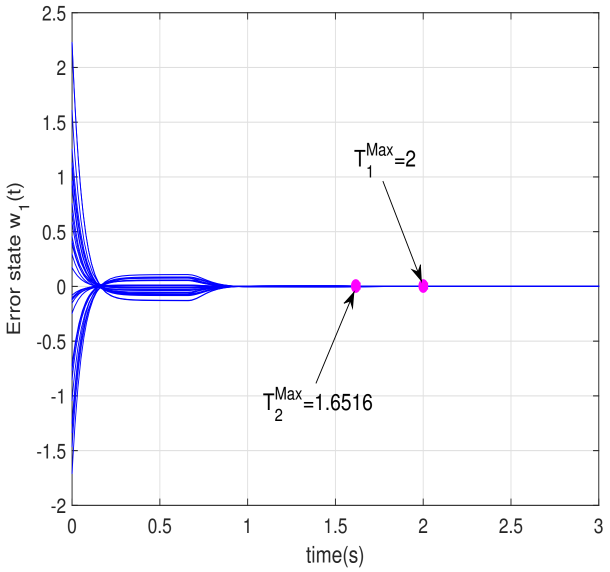

Figure 6. Error state trajectory $w_1(t)$ with control (15).

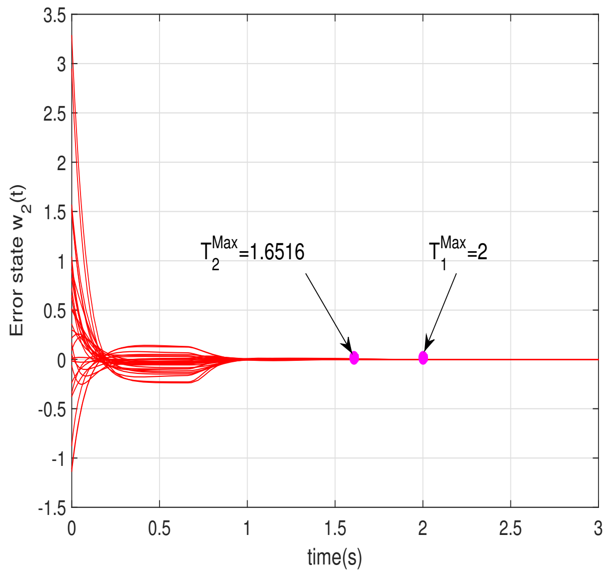

Figure 7. Error state trajectory $w_2(t)$ with control (15).

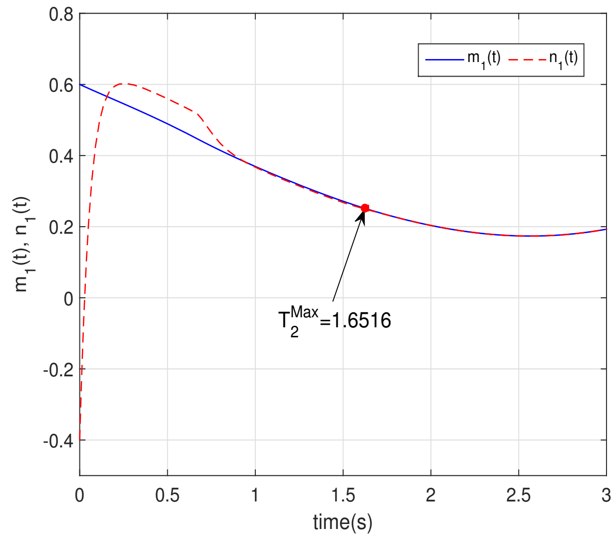

Figure 8. Synchronization curves between $m_1(t)$ and $n_1(t)$ with control (15).

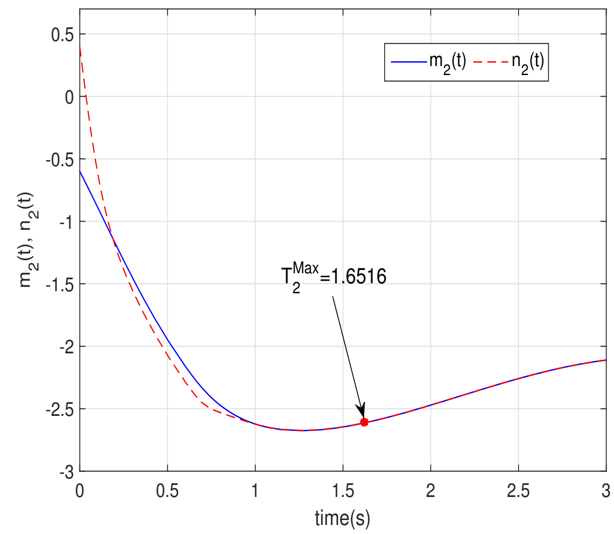

Figure 9. Synchronization curves between $m_2(t)$ and $n_2(t)$ with control (15).

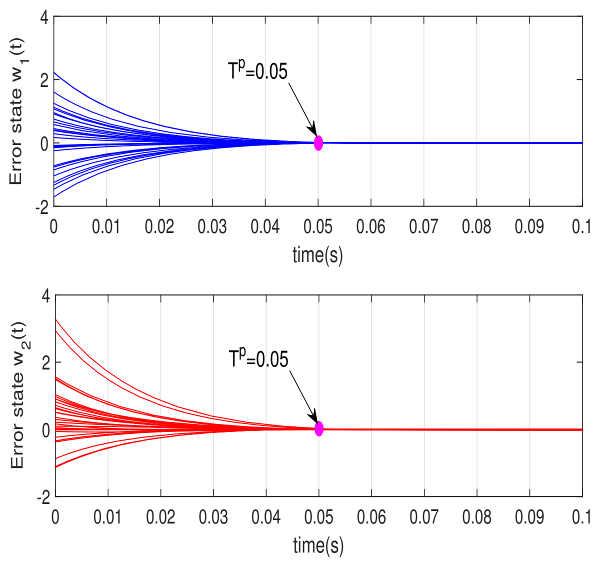

Figure 10. Error state trajectories $w_1(t), w_2(t)$ with control (21).

$0.5, \eta_1 = \eta_2 = 1$, then, all the conditions of Theorem 1 are hold, so, by using control (15), we know that drive-response systems (23) and (24) achieve FTS. By simple computing, one gets that $c = \sigma = 1$, so, the settling-time of the FTS is $\mathbf{T}_2^{\max} = 1.6516$. The error state trajectories with control (15) between drive-response systems (23) and (24) are, respectively, shown in Figures 6 and 7. We randomly select 30 initial values, and the curves of the FTS between $m_1(t)$ and $n_1(t)$, $m_2(t)$ and $n_2(t)$ are given in Figures 8 and 9, respectively.

Table 1. Comparisons of some pervious works with this article.

FTS	Settling-time
$\sigma = 0, b = e$ hold in [24, 25]	$\mathbf{T}_1^{\max} = \frac{1}{qc} = 2$
$\sigma \geqslant 0, b > 1$ hold in this paper	$\mathbf{T}_2^{\max} = 1.6516$

Now, we use the control input (21) to realize PRTS between drive-response systems (23) and (24). From the parameters given above, we get $\zeta_2 = 1.6, \zeta_2 = 2.2$. Let $\mathbf{T}^p = 0.05$, and Theorem 2 are all hold, and by using control (21), we get drive-response systems (23) and (24) achieve PRTS at settling-time $\mathbf{T}^p = 0.05$, which error state trajectories with control (21) between systems (23) and (24) are displayed in Figure 10.

Remark 7. Through the above simulations, we can find that the results of this paper are very effective. In addition, from Table 1, we know the if $\sigma = 0, b = e$, one can the results used in previous works [24, 25], which settling-time of FTS is $\mathbf{T}_1^{\max} = 2 > \mathbf{T}_2^{\max} = 1.6516$ of this paper. What' more, in this paper, the parameters σ, b satisfy $\sigma \geqslant 0, b > 1$, which are more general.

5 Conclusion

A kind of NNs with with time-varying delays has been discussed in this article. Based on new inequalities, novel Lemmas on FTS and PRTS were constructed. In order to eliminate the influence of time delays for the delayed NNs, two new feedback controllers (15) and (21) contain the term $-\gamma_k \text{sign}(w_k(t)), k \in \mathcal{N}$ were designed, then, some novel criteria ensure FTS and PRTS of the delayed NNs were established. At last, example simulations were also shown the effectiveness the given FTS and PRTS results. As we know, NNs with complex-valued has some critical applications in the field of engineering, so, FTS and PRTS of complex-valued NNs will be investigated in the future works.

Data Availability Statement

Data will be made available on request.

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Conflicts of Interest

The authors declare no conflicts of interest.

Ethical Approval and Consent to Participate

Not applicable.

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