RESEARCH ARTICLE

On Mathematical Study of Juvenile Delinquency with Precautionary Measure, Public Education and Intervention Programs as Control Strategies

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Abstract

In this paper, we develop a mathematical model for juvenile delinquency transmission dynamics by incorporating key control strategies, namely precautionary measures, public education, and intervention programs. The model aims to identify effective prevention and control measures for curbing the spread of delinquent behavior among youths, with particular emphasis on evaluating the efficacy of public education. Adopting an epidemiological modelling framework, we derive a system of nonlinear differential equations governing the dynamics of juvenile delinquency over time. Stability analysis of the model is conducted, and the basic reproduction number along with the equilibrium points for both delinquency-free and endemic scenarios are established. Numerical simulations reveal that controlling the entry rate of juveniles into the population, reducing the transition rate from susceptible to delinquent status, and minimizing the rate at which individuals return to delinquency from correctional centers are critical for mitigating the spread of delinquent behavior. Moreover, while public education shows limited impact among susceptible individuals, it proves highly effective among the exposed, delinquent, and those in correctional facilities. Enhanced public education on the consequences of delinquency also contributes to reducing both arrest rates and juvenile homicides. This work offers valuable insights for researchers in applied mathematics, behavioral science, and healthcare management, while providing evidence-based guidance for policymakers seeking to manage and control juvenile delinquency.

Keywords: mathematical model, juvenile delinquency, nonlinear dynamics, basic JD-reproduction number, JD equilibrium points, precautionary measure, public education program, intervention program.

1 Introduction

Juvenile delinquency, or simply juvenile offending, which is a social and psychological menace and the act of engaging in unlawful behavior or violation of a criminal statue by a minor (that is, children under the age of 17 or 18) is becoming a major concern by all stakeholders in the world today. Most countries

Submitted: 12 December 2025
Accepted: 01 March 2026
Published: 04 March 2026

Vol. 2, **No.** 1, 2026.
10.62762/JNDA.2025.195550

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Citation
Nkeki, C. I., & Mbarie, I. A. (2026). On Mathematical Study of Juvenile Delinquency with Precautionary Measure, Public Education and Intervention Programs as Control Strategies. *Journal of Nonlinear Dynamics and Applications*, 2(1), 20–38.

specify persons under the age of 18 years of a age as a juvenile delinquent, or young offender while some countries have set the maximum age that is slightly different from 18 years, see Nuno *et al.* [1]. Most of the juvenile delinquent acts can be attributed to social and environmental, and perhaps hereditary factors such as family behavior, peer influence or social media influence. Some of the crimes that are common among juveniles are vandalism, burglary and theft, shoplifting, substance abuse and drug law violations, robbery, truancy, cybercrimes, physical assault, arson, joyriding, Graffiti, rape, homicide and trespassing. These acts would be seen as crimes if the individuals who committed them are adults, see [2]. According to Browning *et al.* [3], the word 'delinquent' is refers to juvenile delinquency, and is also generalized to mean a young individual who behaves an unacceptable manner. The stakeholders, especially teachers, parents, the government, the general public, and the world at large, are seeking for ways of preventing this menace among minors in our societies. In handling juvenile crime, there are laid down procedures. It is imperative to note that the procedures of handling juvenile crime differ from those for adult offenders. While adults are prosecuted and sentence to jail for criminal offences, juveniles who are delinquent, are sent to juvenile correction center. Juvenile delinquency cases are usually heard in family court or municipal court or juvenile court. In some states, if the crime committed is a serious one, such minor will be treated as adult and if found guilty, will go to jail. The government and private agencies have put modalities in place to prevent juvenile delinquency. Such preventive measures include substance abuse treatment, school counseling's, family counseling, and individual counseling, parenting education, family planning services and public enlightenment on the danger of delinquency. In United States, the Office of Juvenile Justice and Delinquency Prevention help in providing the necessary prevention and rehabilitation programs for juveniles.

The common structures of the juvenile legal system are juvenile arrest and detention centers, juvenile courts and monitoring. This juvenile courts are designs to treat juvenile offenses as civil rather than criminal cases, and it varies from states to states in the United States. It is imperative to note that some juvenile who are under the stipulated age as juvenile can be charged and treated as adults but it depends on the nature and gravity of the crime committed. According to [4], some scholars have found an increase in arrests for juvenile and have concluded that this may reflect more aggressive criminal justice and zero-tolerance policies rather than changes in juvenile behavior towards crimes. In some states, there are more correction centers than correction institutions. In this paper, we consider, in addition to arrest and aggressive criminal justice, the need to exploit a precaution to delinquent, public education, enlightenment and intervention programs in the control of juvenile delinquency among youths.

There have been recent studies on juvenile delinquency and other social misbehaviours in our society. Nuno *et al.* [1] considered a model that deal with the interactions of three social species: business owners, criminals, and security personnel. They found that the amount of crimes committed at a time determines the level of recruitment of security personnel, and the more resources available to fight crime, the greater the community of criminals. Lee *et al.* [5] stipulated that peer pressure play a role in juvenile misbehaviour. They stated that the number of arrests and sanctions required to stop the growth of youth gangs can be calculated using a simulation study on the threshold condition. Barrett *et al.* [6] examined influences on delinquency and recidivism using structural equation model. The sample considered in their work comprised 199,204 individuals: 99,602 youth whose cases had been processed by the South Carolina Department of Juvenile Justice and a matched control group of 99,602 youth without juvenile records. The structural equation model considered for the outcome of delinquent as against non-delinquent status shows a direct paths of two latent constructs, developmental exceptionalities and early parenting problems, to delinquent outcome, with developmental problems also mediating the relationship between adverse parenting and delinquency. Also, separate analysis of the delinquent group showed gender differences in the predictors of more serious offending, with adverse parenting a significant influence for girls only. Sooknanan *et al.* [7] stipulated that delinquent behaviour may be treated as a socially infectious disease spread by peer influence, and they adopted a compartmental model from mathematical epidemiology in the analysis of juvenile behavior. They used public health approach in designing their model. Mebratie *et al.* [8] considers mathematical model for the analysis of crime dynamics incorporating media coverage and police force. They put into account the impact of media coverage, police force and moral or religious activity on crime. Sooknanan *et al.* [9]

considers mathematical models for criminal behaviour used to support traditional crime prevention methods. Their paper focuses on recent efforts to model criminal behaviour associated with gangs, delinquency, terrorism and corruption by modifying infectious disease compartmental models. They carried out models that are based on the premise behaviours that are contagious, spread through association or contact with delinquent peers. Ibrahim [10] presented a mathematical model of juvenile delinquency in the New York State. They develop a juvenile delinquency system of non-linear differential equations using the mathematical epidemiology framework by assuming that juvenile delinquency can be studied as a socially infectious disease. Crokidakis [11] analyzed a mathematical model to understand the dynamics of bullying in schools. They considered a population that is divided into four groups: susceptible individuals, bullies, individuals exposed to bullying, and violent individuals. They stipulated that the transitions between these states occur at rates designed to capture the complex interactions among students, influenced by factors such as romantic rejection, conflicts with peers and teachers, and other school-related challenges, which can escalate into bullying and violent behavior. Such trends underscore the evolving nature of juvenile justice responses. For instance, the number of juveniles incarcerated in U.S. adult jails and prisons declined sharply from 2002 to 2021, as documented by Zeng *et al.* [17], which informs the parameterization of arrest, detention, and correctional flows in the proposed model.

The highlights of the paper

- JD is a highly contagious menace and can be analyzed using a compartmental model of seven classes of juveniles.
- The model is studied through nonlinear differential equations using mass action of contact rate.
- Public education, precautionary measure and intervention program can be used as control strategies of managing and preventing the spread of delinquency.
- The efficacy of public education can serve as a measure to determine the effectiveness of public education.
- The simulation results may be adopted by stakeholders and policy holders to develop inform policies on how to manage and prevent juvenile delinquency among youths.

The contributions of the paper

Our contribution in this paper is of sixfold,

- first, this paper considers mathematical model for juvenile delinquency involving seven classes of juveniles,
- second, three distinct control measures were considered to determine how juvenile delinquency can be controlled and prevented among youths,
- third, the basic JD reproduction number for JD model is determined,
- fourth, stability analysis of JD model is studied,
- fifth, sensitivity analysis of the JD-basic reproduction number is carried out in this paper,
- finally, numerical simulations of our JD model using US juveniles as a case study are carried out, in this paper.

The rest of the paper is structured as follows: In section 2, we presents the mathematical dynamics for juvenile delinquency. The stability analysis of our JD epidemic model is presented in section 3. Section 4 presents the numerical simulations of our model. Finally, section 5 present the concluding part of the paper and recommendations.

2 The mathematical dynamics for juvenile delinquency

Here, we consider the mathematical modelling for the dynamics of juvenile delinquency among young people in our society. First, we give the definition of the following concepts, as they relate to this paper:

Definition 1. *Precautionary measure: This is an action taken in advance by juveniles who are exposed to exposed or delinquent juveniles, to prevent himself or herself from being influence into the risks of becoming delinquent.*

Definition 2. *Public education program on juvenile delinquency: This is an enlightenment program aimed at providing the general public with information about the evils associated with juvenile delinquency so as to help them to make informed decision about the risks relating to delinquency.*

Definition 3. *Basic JD-reproduction number* ($\mathcal{R}_o^D$)*: This is an expected number of secondary delinquent cases*

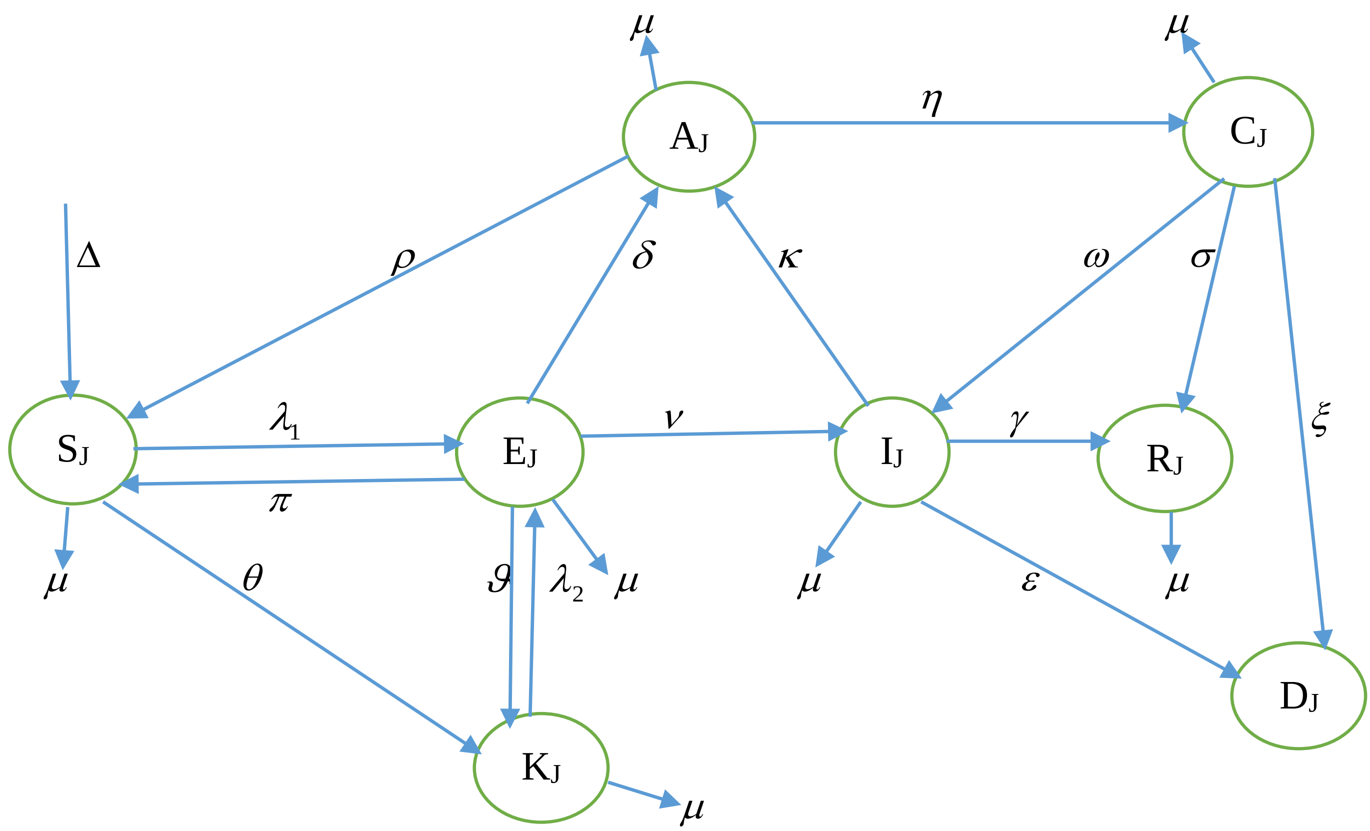

Figure 1. The schematic diagram for juvenile delinquency.

produced, in a completely JD-free population, by a typical delinquent juvenile. In other words, it is the average number of secondary delinquents produced when one delinquent is introduced into a juvenile population where all the juvenile population are free from delinquency.

Here, we consider the mathematical formulation of the spread dynamics of juvenile delinquency (JD). JD is a negatively influence behaviour which spread very fast among juvenile peer groups in our societies today. In this paper, we model this delinquent behaviour by following the approach of compartmental model for infectious disease. This JD model comprises of eight main compartments, $S_JK_JE_JA_JI_JC_JD_JR_J$, where S_J stands for juveniles who have never been involved in delinquency, K_J stands for juveniles who have received public education on consequences of JD, E_J stands for juveniles who are exposed to delinquents, A_J stands for juveniles arrested for delinquency, I_J stands for juveniles who are delinquent, C_J stands for juveniles who are found guilty of delinquency and are to remanded in juvenile correction centers, D_J stands for juveniles who dies in the cause of his or her involvement in delinquency, R_J stands for juveniles who are reformed through intervention program. The total the numbers of juveniles in the population is given by $N_J(t)$, at time t, and the force of delinquency of our model at time t is

$$\beta(\alpha_1 E_J + \alpha_2 A_J + \alpha_3 C_J + I_J),$$

where $\beta\alpha_1$ is the delinquent influence rates for compartment E_J, $0 \leq \alpha_1 < 1$, $\beta\alpha_2$ is the delinquent influence rates for compartment A_J, $0 \leq \alpha_2 < 1$, $\beta\alpha_3$ is the delinquent influence rates for compartment C_J, $0 \leq \alpha_3 < 1$.

And the incidence of JD are

$$\lambda_1 = \beta S_J(\alpha_1 E_J + \alpha_2 A_J + \alpha_3 C_J + I_J)$$

with respect to juveniles who are free from delinquency, and

$$\lambda_2 = \beta(1-\varphi)K_J(\alpha_1 E_J + \alpha_2 A_J + \alpha_3 C_J + I_J)$$

with respect to juveniles with public education on delinquency. The baseline values of the parameters for JD model are summarized in Table 1.

Figure 1 represents the schematic diagram for JD model from which we derived the following nonlinear

Table 1. The baseline values of the parameters for JD model.

Parameter	Description
μ_R	Juveniles' entry rate.
β	The contact rate with delinquents.
ρ	The rate at which juveniles are acquitted.
π	The rate at which juveniles are exposed delinquency take precautionary measure of being influenced.
θ	The rate at which juveniles who are free from delinquency receive public education on the evils of delinquency.
δ	The rate at which exposed juveniles are arrested and charge for delinquency.
ν	The rate at which exposed juveniles become delinquents.
μ	The exit rate of juveniles either through natural death or emigration.
κ	The rate at which delinquents are arrested and charged.
η	The rate at which delinquents are move to juvenile correction center.
ω	The rate juveniles move from correction center to become delinquent again.
γ	The rate at which juveniles who are delinquent are reformed by intervention program.
σ	The rate at which delinquents from correction center are reformed by intervention program.
ξ	The rate at which juveniles from correction center dies of delinquency.
ϵ	The delinquents induced death.
α_1	The proportion of exposed juvenile that are delinquents.
α_2	The proportion of arrested juvenile that are delinquents.
α_3	The proportion of juveniles in the correction center that are delinquents.
φ	The efficacy of public education on the evils of JD.
ϑ	The rate exposed juveniles receive public education on the evils of JD.
$\Lambda = \mu_R N_J$	Entry number of Juveniles, where N_J is the population of juveniles.

system of differential equations:

$$\frac{dS_J}{dt} = \Lambda + \rho A_J + \pi E_J - (\theta + \mu)S_J - \lambda_1, \quad t \geq 0, \tag{1}$$

$$\frac{dK_J}{dt} = \theta S_J + \vartheta E_J - \mu K_J - \lambda_2, \quad t \geq 0, \tag{2}$$

$$\frac{dE_J}{dt} = \lambda_1 + \lambda_2 - m_1 E_J, \quad t \geq 0, \tag{3}$$

$$\frac{dA_J}{dt} = \delta E_J + \kappa I_J - m_2 A_J, \quad t \geq 0, \tag{4}$$

$$\frac{dI_J}{dt} = \nu E_J + \omega C_J - m_3 I_J, \quad t \geq 0, \tag{5}$$

$$\frac{dC_J}{dt} = \eta A_J - m_4 C_J, \quad t \geq 0, \tag{6}$$

$$\frac{dR_J}{dt} = \sigma C_J + \gamma I_J - \mu R_J, \quad t \geq 0, \tag{7}$$

subject to the initial values $S_J(0) \geq 0$, $K_J(0) \geq 0$, $E_J(0) \geq 0$, $A_J(0) \geq 0$, $I_J(0 \geq 0$, $C_J(0) \geq 0$, $R_J(0) \geq 0$, where $m_1 = \pi + \vartheta + \nu + \delta + \mu$, $m_2 = \rho + \eta + \mu$, $m_3 = \kappa + \gamma + \epsilon + \mu$, $m_4 = \omega + \sigma + \xi + \mu$.

3 Stability analysis of JD model

In this section, we consider the boundedness and stability analysis of our JD epidemic model. The result of the boundedness of the system will tells us the maximum number of individual juveniles that the population can attain. In any system, stability play a vital role in determining the effectiveness of the performance of such system, hence, the need to carry out the stability analysis of our JD model. The results of the analysis will guide and give us the understanding on whether the juvenile population will be free from the spread of delinquency or not, and to provide a suitable way of controlling and preventing the spread of the social and psychological menace among our youths in the society.

3.1 The boundedness of juvenile population

In this subsection, we consider the boundedness of the system by first, determining the dynamics of the total JD-population at time t as follows:

$$N_J(t) = S_J(t) + K_J(t) + E_J(t) + A_J(t) + I_J(t) + R_J(t). \quad (8)$$

The derivative of $N_J(t)$ can be obtained as

$$\dot{N}_J(t) = \Lambda - \mu(S_J(t) + K_J(t) + E_J(t) + A_J(t) + I_J(t) + R_J(t)) - \epsilon I_J - \xi C_J. \quad (9)$$

Simplifying, we have

$$\dot{N}_J(t) = \Lambda - \mu N_J(t) - \epsilon I_J - \xi C_J. \quad (10)$$

Now, for $\epsilon I_J + \xi C_J \geq 0$, we have

$$\dot{N}_J(t) \leq \Lambda - \mu N_J(t). \quad (11)$$

From (11), we have that $\lim_{t\to\infty} \sup N_J(t) \leq \frac{\Lambda}{\mu}$ and when $N_J(t) > \frac{\Lambda}{\mu}$, $\dot{N}_J(t) < 0$, so the feasible region D of (1)-(7) is

$$D = \left\{ \begin{array}{l} (S_J, K_J, E_J, A_J, I_J, C_J, R_J) \in \mathbb{R}^7 | 0 < S_J(t) + K_J(t) \\ + E_J(t) + A_J(t) + C_J(t) + R_J(t) \leq N_J(t) \leq \frac{\Lambda}{\mu}, \\ S_J \geq 0, K_J \geq 0, E_J \geq 0, A_J \geq 0, I_J \geq 0, C_J, R_J \geq 0 \end{array} \right\}. \quad (12)$$

Since all solutions of (1)-(7) will remain or allow to tends to the field of D, it is easy to see that the feasible region D is the positive invariant set for the system of differential equations (1) - (7).

At time t, the new delinquent in compartment E_J arise by contacts between juvenile in compartments S_J and K_J, and delinquent in compartments E_J, A_J, C_J and I_J, at a rate λ_1 and λ_2, respectively. The quantities, λ_1 and λ_2 measures the incidence of JD at time t.

3.2 The rate of spread of juvenile delinquency: A unique delinquent-free equilibrium

Here, we determine the rate at which juvenile delinquency spread among young people in a population, by considering a unique delinquent-free equilibrium. Hence, the system has a unique delinquent-free equilibrium, with $S_J^o = \frac{\Lambda}{\theta+\mu}$ and $K_J^o = \frac{\theta\Lambda}{\mu(\theta+\mu)}$. Note that S_J^o and K_J^o measure the state of the juvenile population being free from delinquency. That is, when the population is free from delinquency, the susceptible population of juvenile is obtained as $S_J^o = \frac{\Lambda}{\theta+\mu}$ and juveniles with delinquency education is obtained as $K_J^o = \frac{\theta\Lambda}{\mu(\theta+\mu)}$, while other compartments are zeros.

The basic JD-reproduction number for our JD model, which is used to determine whether or not the delinquency will spread out among juveniles during the early phase of the delinquency, and it is obtained using the next generation matrix, and is obtained as follows:

$$\begin{aligned} \mathcal{R}_o^D = {} & \frac{\alpha_1\beta(S_J^o + K_J^o)}{m_1} + \frac{\alpha_2\beta m_4(\delta m_3 + \kappa\nu)(S_J^o + K_J^o)}{m_1(m_2 m_3 m_4 - \eta\kappa\omega)} \\ & + \frac{\beta(m_2 m_4 \nu + \delta\eta\omega)(S_J^o + K_J^o)}{m_1(m_2 m_3 m_4 - \eta\kappa\omega)} \\ & + \frac{\alpha_3\beta(\delta\eta m_3 + \eta\kappa\nu)(S_J^o + K_J^o)}{m_1(m_2 m_3 m_4 - \eta\kappa\omega)}. \end{aligned} \quad (13)$$

The obtained four terms of $\mathcal{R}_o^D$ are the number of secondary delinquents produced by an index case of a delinquent initially in compartment E_J. It is imperative to note that

- $\frac{1}{\beta}$ is the average time between delinquency contacts by a delinquent, and $\frac{1}{m_1}$ is the average time of being exposed to JD, then $\frac{\alpha_1\beta(S_J^o + K_J^o)}{m_1}$ is the average number delinquents that have contact with other juveniles before being arrested for the crime.
- the fraction $\frac{m_4(\delta m_3 + \kappa\nu)}{m_1(m_2 m_3 m_4 - \eta\kappa\omega)}$ is the average time a delinquent will spend in detention before being acquitted or send to juvenile correction center, it follows that $\frac{\alpha_2\beta m_4(\delta m_3 + \kappa\nu)(S_J^o + K_J^o)}{m_1(m_2 m_3 m_4 - \eta\kappa\omega)}$ is the average number of delinquents that have contact with other juveniles before being acquitted or send to juvenile correction center.
- the fraction $\frac{m_2 m_4 \nu + \delta\eta\omega}{m_1(m_2 m_3 m_4 - \eta\kappa\omega)}$ is the average time spent being delinquent before being arrested or reformed or dies due to delinquency, it follows that $\frac{\beta(m_2 m_4 \nu + \delta\eta\omega)(S_J^o + K_J^o)}{m_1(m_2 m_3 m_4 - \eta\kappa\omega)}$ is the average number of delinquents that will have contact with other juveniles before being arrested for the crime.
- the fraction $\frac{\delta\eta m_3 + \eta\kappa\nu}{m_1(m_2 m_3 m_4 - \eta\kappa\omega)}$ is the average time spent in correction center before being reformed or going back to delinquency or dies due to delinquency, then $\frac{\alpha_3\beta(\delta\eta m_3 + \eta\kappa\nu)(S_J^o + K_J^o)}{m_1(m_2 m_3 m_4 - \eta\kappa\omega)}$

is the average number of delinquents that have contact with other juveniles before being taken to correction center or become reformed or dies due to juvenile crimes.

3.3 The JD equilibrium points

In this subsection, we consider the JD equilibrium points. Thus, we determine the JD equilibrium points $P_0(S_J^o, K_J^o, 0, 0, 0, 0, 0)$ for JD-free case (that is, when the juvenile population is free from the social and psychological menace, called delinquency) and $P^*(S_J^*, K_J^*, E_J^*, A_J^*, I_J^*, C_J^*, R_J^*)$ for JD endemic case (that is, when the delinquency remain endemic in the population but stationary), where we obtain the following:

$$S_J^* = \frac{m_1 \Lambda \mathcal{R}_o^D}{(\theta+\mu)m_1\mathcal{R}_o^D + L_3\beta\Lambda(\mathcal{R}_o^D - 1)},$$

$$K_J^* = \frac{m_1\theta\Lambda\mathcal{R}_o^D}{\mu[(\theta+\mu)m_1\mathcal{R}_o^D + L_3\beta\Lambda(\mathcal{R}_o^D-1)]} + \frac{\Lambda(\vartheta-(1-\varphi)\beta L_3)(\mathcal{R}_o^D-1)}{\mu m_1\mathcal{R}_o^D},$$

$$E_J^* = \frac{\Lambda(\mathcal{R}_o^D-1)}{\mathcal{R}_o^D m_1},$$

$$I_J^* = \frac{m_2m_4}{m_2m_4-\mu\omega\kappa}\left(\frac{\nu}{m_3}+\frac{\mu\omega\delta}{m_2m_4}\right)E^*,$$

$$A_J^* = \left(\frac{\delta}{m_2}+\frac{m_2m_4\kappa}{m_2(m_2m_4-\mu\omega\kappa)}\left(\frac{\nu}{m_3}+\frac{\mu\omega\delta}{m_2m_4}\right)\right)E^*,$$

$$C_J^* = \frac{\mu}{m_4}\left(\frac{\delta}{m_2}+\frac{m_2m_4\kappa}{m_2(m_2m_4-\mu\omega\kappa)}\left(\frac{\nu}{m_3}+\frac{\mu\omega\delta}{m_2m_4}\right)\right)E^*,$$

$$R_J^* = \left(\frac{\delta\sigma}{m_2m_4}+\frac{m_2m_4(\mu\kappa\delta+m_2m_4\gamma)}{\mu m_2m_4(m_2m_4-\mu\omega\kappa)}\left(\frac{\nu}{m_3}+\frac{\mu\omega\delta}{m_2m_4}\right)\right)E^*,$$

where

$$L_1 = \frac{m_2m_4}{m_2m_4-\mu\omega\kappa}\left(\frac{\nu}{m_3}+\frac{\mu\omega\delta}{m_2m_4}\right),$$

$$L_2 = \left(\frac{\delta}{m_2}+\frac{\kappa L_1}{m_2}\right),$$

$$L_3 = \alpha_1 + L_1 + [\alpha_2+\alpha_3\frac{\mu}{m_4}]L_2.$$

In view of that, we determine that if $\mathcal{R}_o^D \leq 1$, there exists a unique JD equilibrium point P_0 i.e., the JD-free equilibrium point for the system of differential equations (1) - (7). In other words, if $\mathcal{R}_o^D \leq 1$, it means that the juvenile population will be free from delinquency; if $\mathcal{R}_o^D > 1$, there exists a unique endemic equilibrium $P^* \in D_s$, where D_s is a positive invariant subset of D. This simply tells us JD will persist among the young people who are below the adult age.

3.4 Stability analysis of JD equilibrium points

We now discuss the stability of equilibriums P_0 and P^* for the system of differential equations (1) - (7). The main results are discussed in the following Theorem 1 and Theorem 2 as follows:

Theorem 1. *If $\mathcal{R}_o^D \leq 1$, then JD-free equilibrium $P_0(S_J^o, K_J^o, 0, 0, 0, 0, 0)$ is globally asymptotically stable in D, and if $\mathcal{R}_o^D > 1$, P_0 will be unstable in D.*

Proof. See Appendix I. □

From Theorem 1, we conclude that $P_0(S_J^o, K_J^o, 0, 0, 0, 0, 0)$, is locally asymptotically stable in D for our system of differential equations (1) when $\mathcal{R}_o^D < 1$ and P_0 is unstable in D when $\mathcal{R}_o^D > 1$. It simply tells that if $\mathcal{R}_o^D < 1$, the society will be free from delinquency, and if $\mathcal{R}_o^D > 1$, the social menace called delinquency will continue to persist among our youths in the society.

Theorem 2. *If $\mathcal{R}_o^D > 1$, then the JD unique endemic equilibrium point,*

$P^*(S_J^*, K_J^*, E_J^*, A_J^*, I_J^*, C_J^*, R_J^*)$ is locally asymptotically stable in D for our system of differential equations (1) - (7).

Proof. See Appendix II. □

From Theorem 2, we have that the conditions of Routh–Hurwitz stability criterion are satisfied by our characteristic polynomial. We therefore conclude that $P^*(S_J^*, K_J^*, E_J^*, A_J^*, I_J^*, C_J^*, R_J^*)$, is locally asymptotically stable in D for our system of differential equations (1) - (7) when $\mathcal{R}_o^D > 1$. This tells us that when the delinquency is endemic among our youths, equilibrium point will be attained if the rate of spread of the social menace is greater than one.

4 Numerical simulations

In this section, we considers empirical results of JD model with precautionary measure, public education program and its efficacy, and intervention program, as control strategies using US juveniles as a case study. The aim of the control strategies is to determine the possible control and prevention of the spread of delinquency among juvenile-population. JD-model is developed to study the transmission dynamics of the spread of delinquency among youths in US population. According to [12], in 2021, juvenile courts in the US considered 437,300 delinquency cases that involved juvenile charged with criminal law violation. In the same year, there are 113,000 detention admissions of

Table 2. The baseline values of the parameters.

Parameter	Baseline value	References
β	0.000000356670046 year^{-1}	Ibrahim (2023)[10]
ρ	0.58 year^{-1}	Hockenberry and Puzzanchera (2024)[12]
δ	0.0006296694 year^{-1}	Estimated from Jackson (2024)[13]
κ	0.13 year^{-1}	Hockenberry and Puzzanchera (2024)[12]
μ	0.01228501228 year^{-1}	[4]
μ_R	0.00047 year^{-1}	Estimated
ν	0.012444224377361 year^{-1}	Estimated from Jackson (2024)[13]
η	0.34 year^{-1}	Hockenberry and Puzzanchera (2024)[12]
ω	0.09 year^{-1}	Hockenberry and Puzzanchera (2024)[12]
γ	0.90 year^{-1}	Hockenberry and Puzzanchera (2024)[12]
σ	0.07 year^{-1}	Hockenberry and Puzzanchera (2024)[12]
ξ	0.0685 year^{-1}	Estimated from Tapp *et.al* (2024)[14]
ϵ	0.0685 year^{-1}	Estimated from Tapp *et.al* (2024)[14]
α_1	0.01 year^{-1}	Assumed
α_2	0.005 year^{-1}	Assumed
α_3	0.0001 year^{-1}	Assumed

Table 3. Initial values for model's state variables.

Variable	Initial values	References
Λ	$\mu_R N_J$	Estimated
N_J	73,210,000	[15]
S_J	28,457,500	Estimated from Hockenberry and Puzzanchera (2024)[12]
E_J	0.48 N_J	Hockenberry and Puzzanchera (2024)[12]
A_J	244,100	Hockenberry and Puzzanchera (2024)[12]
K_J	0.12 N_J	Zeng and Carson (2023)[16]
I_J	437,300	Hockenberry and Puzzanchera (2024)[12]
C_J	113,000	Hockenberry and Puzzanchera (2024)[12]
R_J	0.07 C_J+0.05 I_J	Hockenberry and Puzzanchera (2024)[12]
D_J	0.0145 C_J+0.054 I_J	Tapp *et.al* (2024)[14]

juvenile on delinquency charges in the United States. The population of youths less than 18 years of age in the United States is approximately 73.21 million, according to the latest U.S. Census Bureau's American Community Survey estimates. In 2022, there are 21,590 homicides in the US, which was almost the same with that in 2021. Again, approximately 423,077 delinquency cases are adjudicated and disposed in juvenile courts annually. In US, about 39% of the population are susceptible youths, 48% of juvenile in US are exposed to delinquency, 12% of the population received public education, 244,100 are arrested for delinquency, 437,300 are delinquent, 113,000 juveniles are jailed of delinquency, 2,035 dies due to delinquency.

The baseline values of our parameters and their sources are presented in Table 2. Table 3 present the initial values for model's state variables for the JD-model.

Table 4 shows the sensitivity analysis or change in the basic JD-reproduction number or simply, spread in the rate of juvenile delinquency, $\mathcal{R}_o^D$ with respect to the values of μ_R, ν, ω, and efficacy of public education, φ among US youths. From Table 4, we have that as the juvenile entry rate, μ_R increases from 0.0001 to 0.09, the spread of the social and moral misbehaviour among juveniles increases from 0.0825 to 74.2076, which is about 99.89% increase, which is a sharp increase in the spread of delinquency in the juvenile population. This simply tells us that, to reduce the spread of juvenile delinquency, the entry rate must be properly control and manage at the entry stage of juvenile into the population. In Table 4, as the rate at which juvenile become delinquent, ν increases, the spread of the social and moral misbehaviour among juveniles increases, that is when $\nu = 0.01$, $\mathcal{R}_o^D = 2.47552$, when $\nu = 0.10$, $\mathcal{R}_o^D = 9.06010$. This show that when ν is increased by 90%, $\mathcal{R}_o^D$ will increased by 72.68%. This tells us that the rate at which juvenile become delinquent, contributes

Table 4. Change in $\mathcal{R}_o^D$ for varying values of μ_R, ν, ω, and efficacy of public education of the spread of delinquency.

μ_R	$\mathcal{R}_o^D$	ν	$\mathcal{R}_o^D$	ω	$\mathcal{R}_o^D$	φ	$\mathcal{R}_o^D$
0.0001	0.0825	0.01	2.47552	0.01	2.45137	0.1	0.991285
0.0003	0.2474	0.02	3.48203	0.02	2.45558	0.2	0.991285
0.0005	0.4123	0.03	4.39825	0.03	2.45934	0.3	0.991285
0.0008	0.6596	0.04	5.23579	0.04	2.46272	0.4	0.991285
0.0010	0.8245	0.05	6.00437	0.05	2.46577	0.5	0.991285
0.0050	4.1226	0.06	6.71217	0.06	2.46854	0.6	0.991285
0.0090	7.4208	0.07	7.36613	0.07	2.47107	0.7	0.991285
0.0100	8.2453	0.08	7.97217	0.08	2.47339	0.8	0.991285
0.0500	41.2264	0.09	8.53536	0.09	2.47552	0.9	0.991285
0.0900	74.2076	0.10	9.06010	0.10	2.47748	1.0	0.991285

Table 5. Impact of the control strategies θ, ϑ, π and σ on $\mathcal{R}_o^D$.

θ	$\mathcal{R}_o^D$	ϑ	$\mathcal{R}_o^D$	π	$\mathcal{R}_o^D$	σ	$\mathcal{R}_o^D$
0.1	2.47552	0.02	3.51481	0.0	4.880920	0.01	1.002770
0.2	2.47552	0.04	3.08332	0.1	2.475520	0.02	1.001940
0.3	2.47552	0.06	2.74619	0.2	1.658280	0.03	1.001190
0.4	2.47552	0.08	2.47552	0.3	1.246710	0.04	1.000510
0.5	2.47552	0.10	2.25341	0.4	0.998814	0.05	0.999891
0.6	2.47552	0.12	2.06788	0.5	0.833150	0.06	0.999329
0.7	2.47552	0.14	1.91058	0.6	0.714622	0.07	0.998814
0.8	2.47552	0.16	1.77551	0.7	0.625619	0.08	0.998341
0.9	2.47552	0.18	1.65828	0.8	0.556330	0.09	0.997905
1.0	2.47552	0.20	1.55558	1.0	0.455446	0.10	0.997502

tremendously to the spread of delinquency among juveniles in the population. In the same table, it is observed that as the rate at which juveniles move from correction center to delinquent class, ω increases, the rate of spread of the social and moral misbehaviour among juveniles' $\mathcal{R}_o^D$ increases as well. It is observed that when $\omega = 0.01$, $\mathcal{R}_o^D = 2.45137$ and when $\omega = 0.10$, $\mathcal{R}_o^D = 2.47748$, which simply implies that when ω increases by 90%, $\mathcal{R}_o^D$ is increases by 1.0539%. This tells us that the rate at which juveniles move from correction center to delinquent class, ω contributes slightly to the spread of delinquency among youths in the population. Also in Table 4, it is observed that as the efficacy of public education, φ increases, the spread of juvenile delinquency will remain unchanged. This simply implies that efficacy of public education does not have any effect on the spread of juvenile delinquency in the juvenile population, but help to reduce the number of youths from being exposed to delinquency or of becoming delinquent.

Table 5 shows the impact of the control strategies, ϑ, θ, ϑ, π and σ on $\mathcal{R}_o^D$. From Table 5, we have that as the rate susceptible received public education, θ increases, the rate of spread of delinquency among US juveniles remain unchanged. This simply tells us that, if the aim of the stakeholders is to eradicates the spread of juvenile delinquency, public education for the susceptible juveniles will not be necessary.

In Table 5, as the rate at which exposed juveniles received public education, ϑ increases, the rate of spread of juvenile delinquency among US juveniles decreases. That is, when $\vartheta = 0.02$, $\mathcal{R}_o^D = 3.51481$ and when $\vartheta = 0.20$, $\mathcal{R}_o^D = 1.55558$. This simply tells us that if the rate at which exposed juveniles received public education, ϑ is increased by 90%, juvenile delinquency will decrease by 55.74%. Hence, stakeholders should channel their attentions and efforts on providing public education for juveniles who are exposed to delinquency, so as to reduce or possibly eradicate the spread of delinquency in the US population. Also in Table 5, it is observed that as the rate at which exposed juveniles continue to take precautionary measure π on being delinquent, the rate of spread of delinquency among US juveniles will decrease. That is, when $\pi = 0.0$, $\mathcal{R}_o^D = 4.880920$, when $\pi = 0.1$, $\mathcal{R}_o^D = 2.475520$ and when $\pi = 1.0$, $\mathcal{R}_o^D = 0.455446$. This simply tells us that if juvenile can take 100% precautions, the spread of delinquency will reduce by 90.69%, which is huge and highly recommended. In the same Table 5, it is observed that as intervention program σ continue to take place for US juveniles in the correction center, the rate of spread of juvenile delinquency will continue to reduce. That is, when $\sigma = 0.01$, $\mathcal{R}_o^D = 1.002770$, when $\sigma = 0.02$, $\mathcal{R}_o^D = 1.001940$ and when $\sigma = 0.10$, $\mathcal{R}_o^D = 0.997502$. It simply implies that as intervention program continue to take place for US juveniles in the correction center up to 90% , the spread of delinquency will reduce by 0.5253%. Hence, this shows that intervention program will go a long way in slightly reducing delinquency among US youths in the correction center.

Figure 2 shows the number of juveniles that will remain susceptible for a 20-year for a vary values

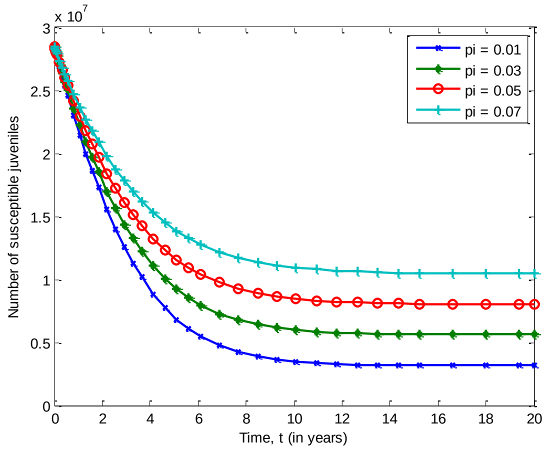

Figure 2. The graph showing the number of juveniles that will remain susceptible for a 20-year for a vary values of the precautionary measure.

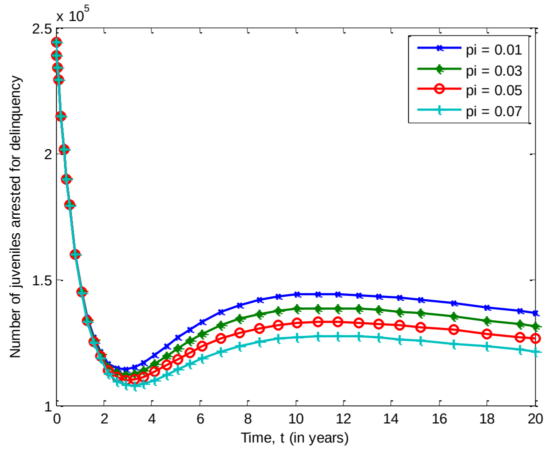

Figure 4. The graph showing the number of juveniles who are arrested for delinquent for a 20-year for a vary values of the precautionary measure.

of the precautionary measure, π. It is observed that as π increases, the number of juveniles who are susceptible increases. This tells us that juvenile taking precaution on becoming delinquent will go a long way in increasing the number of susceptible juveniles in a juvenile population.

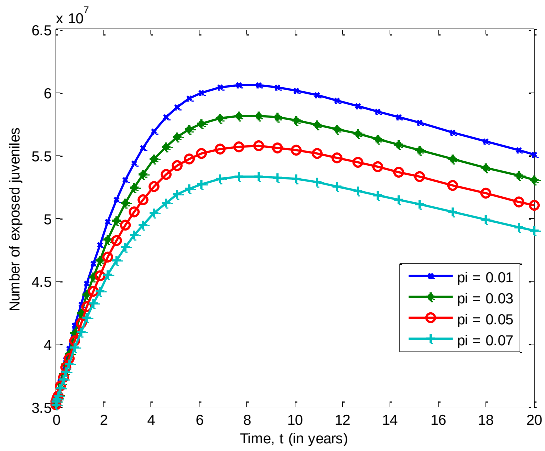

Figure 3. The graph showing the number of exposed juveniles for a 20-year for a vary values of the precautionary measure.

Figure 3 shows the number of juveniles who are exposed to delinquency for a 20-year for a vary values of the precautionary measure, π. We observed that as π increases, the number of juveniles who are exposed to delinquency decreases. This simply tells us that taking precautionary measure will reduce the number of delinquents in the juvenile population.

Figure 4 shows the number of juveniles who are arrested of delinquency for a 20-year for a vary values of the precautionary measure, π. It is observed that as π increases, the number of juveniles who are arrested for delinquency decreases. Hence, taking precaution on becoming delinquent will reduce the number of juveniles arrest and on detention.

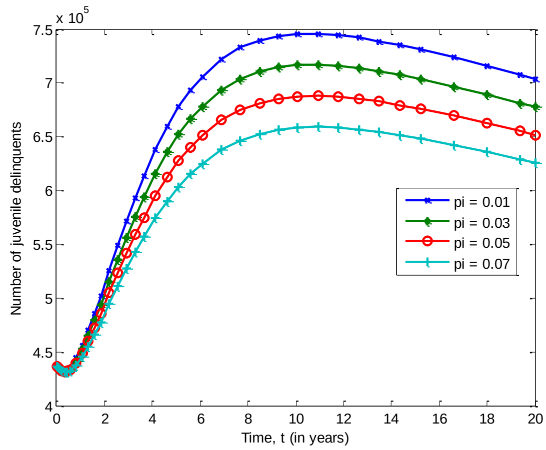

Figure 5. The graph showing the number of juveniles who are delinquent for a 20-year for a vary values of the precautionary measure.

Figure 5 shows the number of juveniles who are delinquent for a 20-year for a vary values of π. It is observed that as π increases, the number of juveniles who are delinquent decreases. This tells us that precautionary measure will tremendously reduce delinquency among youths.

Figure 6 shows the number of juveniles who are in the correction center for a 20-year for a vary values of π. It is observed that as π increases, the number of juveniles who are in the correction center decreases. Also, taking precautionary measure will reduce the number of juveniles in the correction center.

Figure 7 shows the number of juveniles who received public education for a 20-year for a vary values of θ. We found that as θ increases, the number of juveniles

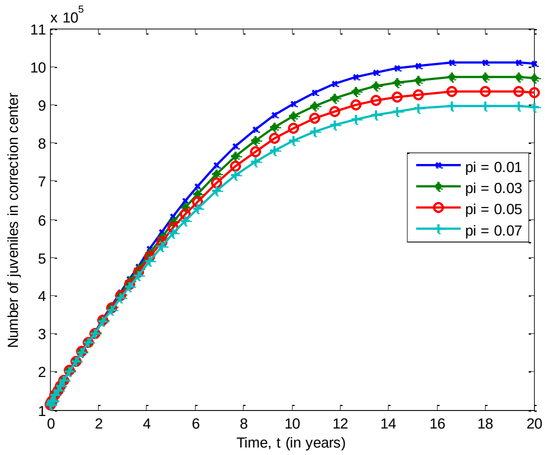

Figure 6. The graph showing the number of juveniles who are in correction center over delinquent offence for a 20-year for a vary values of the precautionary measure.

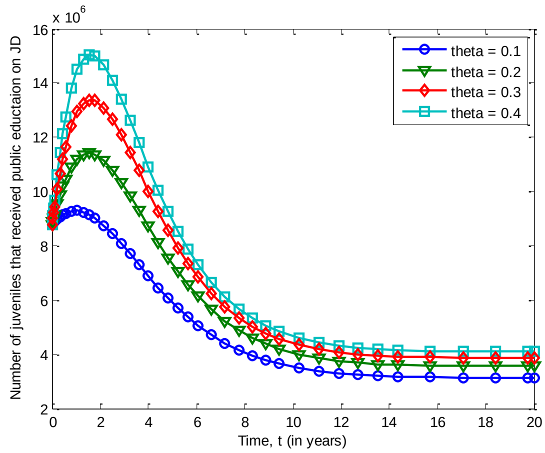

Figure 7. The graph showing the number of susceptible juveniles who received public education on JD for a 20-year for a vary values of θ.

who received public education on JD increases.

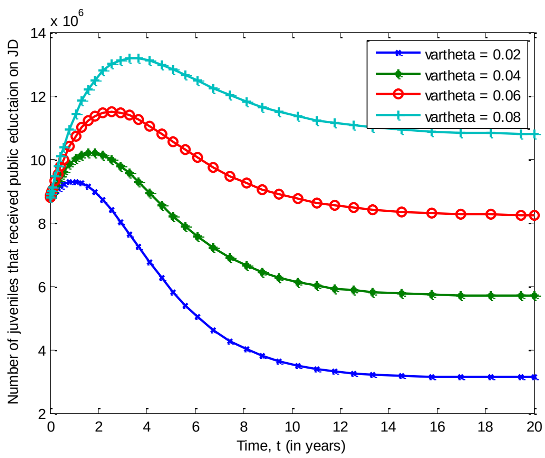

Figure 8. The graph showing the number of susceptible juveniles who received public education on JD for a 20-year for a vary values of ϑ.

Figure 8 shows the number of juveniles who received public education for a 20-year for a vary values of ϑ. It is observed that as ϑ increases, the number of juveniles who received public education on JD increases. This tells us that giving public education on the evils on juvenile delinquency will increase the number of juveniles that received public education on delinquency.

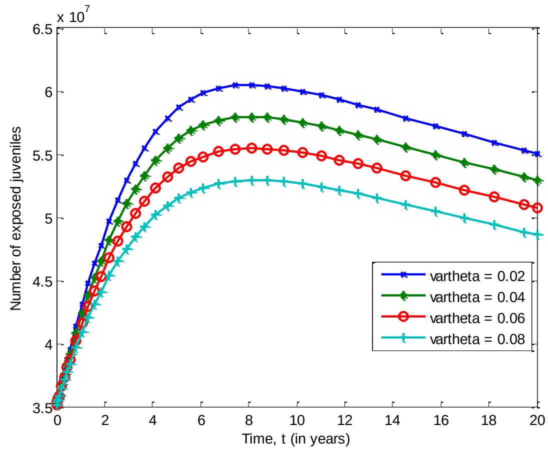

Figure 9. The graph showing the number of exposed juveniles who received public education on JD for a 20-year for a vary values of ϑ.

Figure 9 shows the number of juveniles who are exposed for a 20-year for a vary values of ϑ. It is found that as ϑ increases, the number of juveniles who are exposed to delinquency decreases. This tells us that giving public education on the evils on juvenile delinquency will reduce the number of delinquency among the youths.

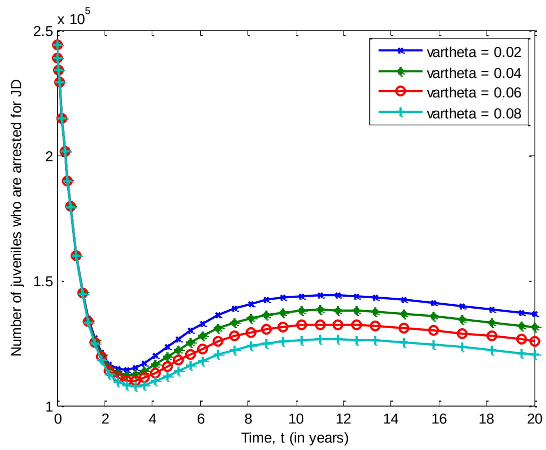

Figure 10. The graph showing the number of juveniles who are arrested over delinquent for a 20-year for a vary values of ϑ.

Figure 10 shows the number of juveniles who are arrested of JD for a 20-year for a vary values of ϑ. It is observed that as ϑ increases, the number of juveniles who are arrested decreases. This tells us that giving public education on the evils on juvenile delinquency will reduce the number of arrest of juveniles on delinquency.

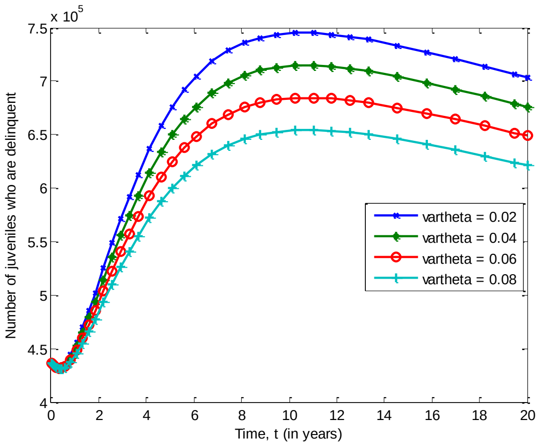

Figure 11. The graph showing the number of juveniles who are delinquent for a 20-year for a vary values of ϑ.

Figure 11 shows the number of juveniles who are delinquent for a 20-year for a vary values of ϑ. It is observed that as ϑ increases, the number of juveniles who are delinquent decreases. This simply tells us that giving public education on the evils on juvenile delinquency will greatly reduce the number of juveniles who are delinquent.

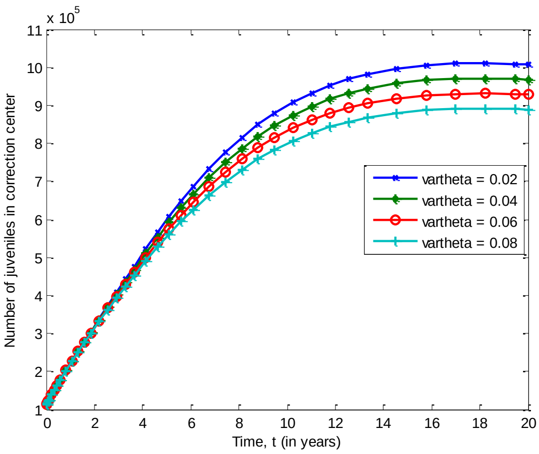

Figure 12. The graph showing the number of juveniles who are in correction center for a 20-year for a vary values of ϑ.

Figure 12 shows the number of juveniles who are in correction center for a 20-year for a vary values of ϑ. It is observed that as ϑ increases, the number of juveniles who are in correction center decreases. This also tells us that giving public education on the evils on juvenile delinquency will reduce the number of delinquency among the youths.

Figure 13 shows the number of juveniles who received public education on JD for a 20-year for a vary values

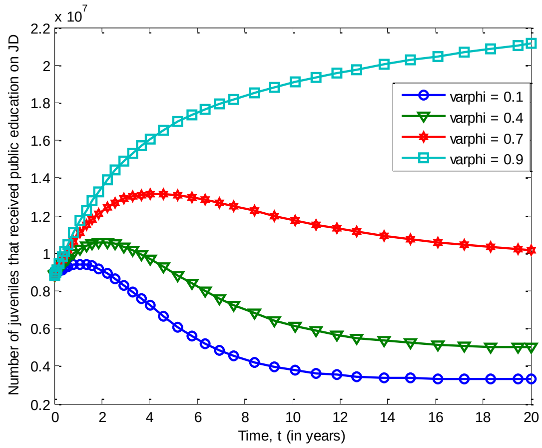

Figure 13. The graph showing the number of juvenile who received public education on JD for a 20-year for a vary values of the efficacy measure.

of φ. It is observed that as φ increases, the number of juveniles who received public education increases. This tells us that the efficacy of public education on the evils on juvenile delinquency will increase the number of juvenile who received public education on the menace of delinquency among the youths.

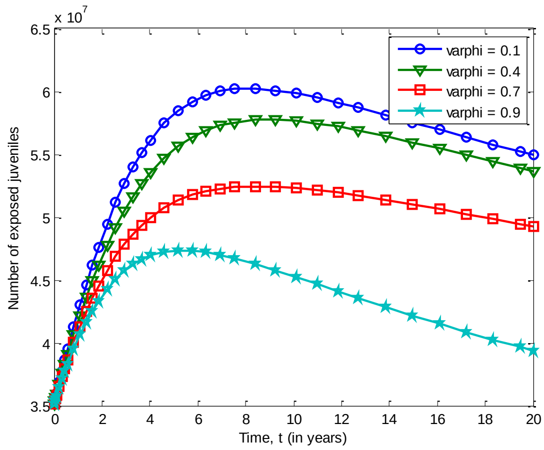

Figure 14. The graph showing the number of exposed juvenile for a 20-year for a vary values of the efficacy measure.

Figure 14 shows the number of juveniles who are exposed for a 20-year for a vary values of φ. It is observed that as φ increases, the number of juveniles who are exposed to JD decreases. This tells us that the efficacy of public education on the evils on juvenile delinquency will decrease the number of juvenile who are exposed to the menace of delinquency among the youths.

Figure 15 shows the number of juveniles who are arrested for a 20-year for a vary values of φ. It is observed that as φ increases, the number of juveniles who are arrested of JD decreases. This tells us that the

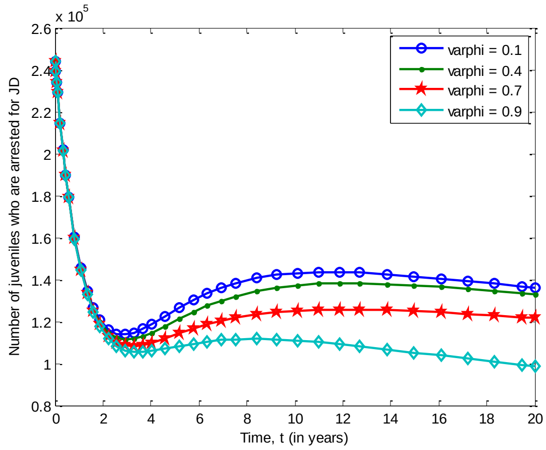

Figure 15. The graph showing the number of juvenile who are arrested for a 20-year for a vary values of the efficacy measure.

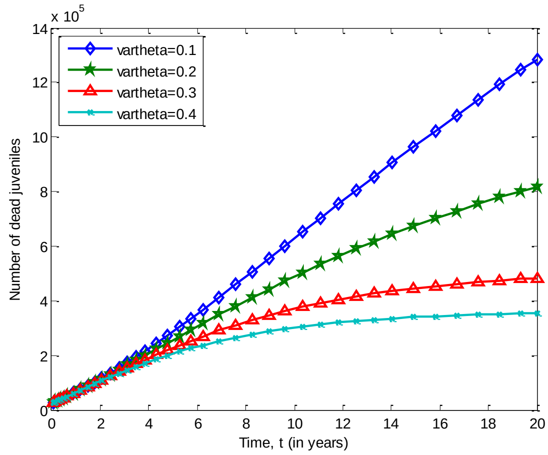

Figure 17. The graph showing the number of dead juveniles who received public education on JD for a period of 20 years.

efficacy of public education on the evils on juvenile delinquency will reduce the number of juveniles who are arrested of delinquency over time.

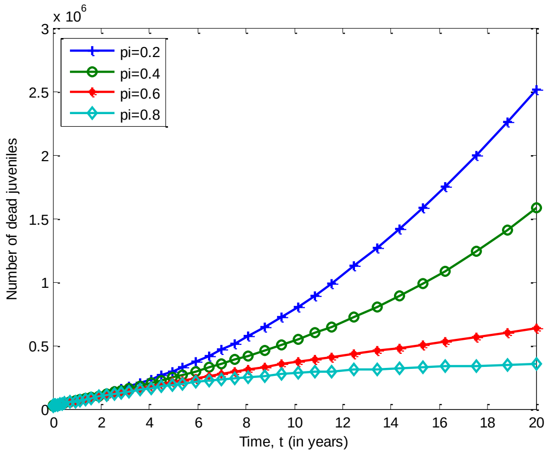

Figure 16. The graph showing the number of dead juveniles who take precautionary measure on JD for a period of 20 years.

Figure 16 shows the number of juveniles who are dead as a result of JD for a 20-year for a vary values of π. It is observed that as π increases, the number of juveniles who are dead of JD decreases. This simply tells us that precautionary measure on the evils on juvenile delinquency will reduce the number of homicide juveniles over time.

Figure 17 shows the number of juveniles who are dead as a result of JD for a 20-year for a vary values of ϑ. It is observed that as ϑ increases, the number of juveniles who are dead of JD decreases. This tells us that public education for the exposed juveniles on the evils of juvenile delinquency will reduce the number of homicide juveniles over time.

5 Conclusion

We examined in this paper, mathematical model for juvenile delinquency, so as to determine the transmission dynamics of the spread of delinquency among youths in a population. Precautionary measure, public education and intervention programs are used as control strategies for the control and prevention of delinquency among youths. The efficacy of public education on the control of juvenile delinquency was studied. Stability analysis for the juvenile delinquency model was established, in this paper. We obtained the basic JD-reproduction number and the JD equilibrium points for epidemic and endemic. We found from the numerical simulations that

(i) to reduce the spread of juvenile delinquency in US juvenile population, the juvenile entry rate in US must be properly control and managed at the entry stage into the population.

(ii) as the rate at which juvenile in US become delinquent increases, the spread of juvenile delinquency will increase.

(iii) as the rate at which juveniles in US move from correction center to delinquent class increases, the rate of spread of the delinquency among juveniles US will increase.

(iv) public education should be unnecessary among the susceptible juveniles, but highly necessary in the exposed, delinquent and correction center stages, so as to reduce delinquency among youths in US.

(v) the efficacy of public education on the evils on juvenile delinquency will go a long way in reducing the number of juveniles who are arrested of delinquency and the number of homicide juveniles in US.

(vi) as the rate at which exposed juveniles continue to take precautionary measure of being delinquent, the rate of spread of delinquency among juveniles will decrease, reduce the number of delinquents and arrest, increase the number of susceptible juveniles, and reduce the number of homicide juveniles in US.

(vii) as intervention program continue to take place for juveniles in the correction center, the rate of spread of juvenile delinquency will continue to decline.

Data Availability Statement

Data will be made available on request.

Funding

This work was supported without any funding.

Conflicts of Interest

The authors declare no conflicts of interest.

AI Use Statement

The authors declare that no generative AI was used in the preparation of this manuscript.

Ethical Approval and Consent to Participate

Not applicable.

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Appendix

A: Proof of Theorem 1

We now construct a Lyapunov function for our system of differential equations (1) - (7) as follows:

$$V(t) = (S_J(t) - S_J^* \ln S_J(t)) + (K_J(t) - K_J^* \ln K_J(t)) + E_J(t) + A_J(t) + I_J(t) + C_J(t) + R_J(t). \quad \text{(A1)}$$

Differentiating with respect to t, we have

$$\dot{V}(t) = \dot{S}_J(t)\left(1 - \frac{S_J^*(t)}{S_J(t)}\right) + \dot{K}_J(t)\left(1 - \frac{K_J^*(t)}{K_J(t)}\right) + \dot{E}_J(t) + \dot{A}_J(t) + \dot{I}_J(t) + \dot{C}_J(t) + \dot{R}(t). \quad \text{(A2)}$$

Adopting (1) - (7), we have

$$\begin{aligned}\dot{V}(t) = \Lambda\left(1 - \frac{S_J^*(t)}{S_J(t)}\right) + (\theta+\mu)S_J^*(t) + \frac{\lambda_1 S_J^*(t)}{S_J(t)} - \frac{\rho A_J^*(t) S_J^*(t)}{S_J(t)} - \frac{\pi E_J S_J^*(t)}{S_J(t)} \\ - \frac{\theta S_J(t) K_J^*(t)}{K_J(t)} - \frac{\vartheta E_J K_J^*(t)}{K_J(t)} + \frac{\lambda_2 K_J^*(t)}{K_J(t)} + \mu K_J^*(t) - \epsilon I_J(t) - \xi C_J(t) - \mu N_J(t).\end{aligned} \quad \text{(A3)}$$

In the set D, we have $N_J = \frac{\Lambda}{\mu}$ and $S_J^* = S_J$ and $K_J^* = K_J$. If $E_J = A_J = I_J = C_J = 0$, then (A3) becomes

$$\dot{V}(t) = \mu(S_J^*(t) + K_J^*(t)) - \Lambda. \quad \text{(A4)}$$

Using the fact that $S_J^* = \dfrac{m_1 \Lambda \mathcal{R}_o^D}{(\theta+\mu)m_1\mathcal{R}_o^D + L_3\beta\Lambda(\mathcal{R}_o^D - 1)}$ and

$K_J^* = \dfrac{m_1\theta\Lambda\mathcal{R}_o^D}{\mu[(\theta+\mu)m_1\mathcal{R}_o^D + L_3\beta\Lambda(\mathcal{R}_o^D - 1)]} + \dfrac{\Lambda(\vartheta - (1-\varphi)\beta L_3)(\mathcal{R}_o^D - 1)}{\mu m_1 \mathcal{R}_o^D}$,

we have

$$\dot{V}(t) = \frac{m_1(\theta+\mu)\Lambda\mathcal{R}_o^D}{(\theta+\mu)m_1\mathcal{R}_o^D + L_3\beta\Lambda(\mathcal{R}_o^D - 1)} + \frac{\Lambda(\vartheta - (1-\varphi)\beta L_3)(\mathcal{R}_o^D - 1)}{m_1\mathcal{R}_o^D} - \Lambda. \quad \text{(A5)}$$

Hence,

$$\dot{V}(t) = \frac{m_1(\theta+\mu)\Lambda\mathcal{R}_o^D}{(\theta+\mu)m_1\mathcal{R}_o^D + L_3\beta\Lambda(\mathcal{R}_o^D - 1)} + \frac{\Lambda(\vartheta - (1-\varphi)\beta L_3)(\mathcal{R}_o^D - 1)}{m_1\mathcal{R}_o^D} - \Lambda \le 0, \quad \text{(A6)}$$

if and only if $\mathcal{R}_o^D \le 1$ and $\frac{m_1(\theta+\mu)\mathcal{R}_o^D}{(\theta+\mu)m_1\mathcal{R}_o^D + L_3\beta\Lambda(\mathcal{R}_o^D-1)} + \frac{(\vartheta-(1-\varphi)\beta L_3)(\mathcal{R}_o^D-1)}{m_1\mathcal{R}_o^D} \le 1$. By adopting LaSalle's (1976) extension to Lyapunov's method, we see the limit set of each solution is contained in the largest invariant set D, we obtain that $P_0(S_J^o, K_J^o, 0, 0, 0, 0, 0)$ is globally asymptotically stable in D. If $\mathcal{R}_o^D > 1$, we have that for $E_J \ne 0, A_J \ne 0, I_J \ne 0, C_J, R_J \ne 0$, and $S_J \to \frac{\Lambda}{\theta+\mu}$ and $S_J > \frac{m_1\Lambda\mathcal{R}_o^D}{(\theta+\mu)m_1\mathcal{R}_o^D + L_3\beta\Lambda(\mathcal{R}_o^D-1)}$, and $K_J \to \frac{\theta\Lambda}{\mu(\theta+\mu)}$ and $K_J > \frac{m_1\theta\Lambda\mathcal{R}_o^D}{\mu[(\theta+\mu)m_1\mathcal{R}_o^D + L_3\beta\Lambda(\mathcal{R}_o^D-1)]} + \frac{\Lambda(\vartheta-(1-\varphi)\beta L_3)(\mathcal{R}_o^D-1)}{\mu m_1\mathcal{R}_o^D}$, then $\dot{V}(t) > 0$. We therefore conclude that any solutions of D, which are close to P_0 will be away come from P_0, hence, P_0 is unstable in D.

Again using the Routh-Hurwitz stability criterion, we have the following:

The Jacobian matrix of the system (1) in the point P^* is obtained as

$$J|_{P_0}(S_J^o, K_J^o, 0, 0, 0, 0, 0) = \begin{pmatrix} -\theta-\mu & 0 & \pi - \beta\alpha_1 s & \rho - \beta\alpha_2 s & -\beta s & -\beta\alpha_3 s & 0 \\ \theta & -\mu & \vartheta - \beta\alpha_1 k & -\beta\alpha_2 k & -\beta k & -\beta\alpha_3 k & 0 \\ 0 & 0 & -m_1 + \beta\alpha_1(s+k) & \beta\alpha_2(s+k) & \beta(s+k) & \beta\alpha_3(s+k) & 0 \\ 0 & 0 & \delta & -m_2 & \kappa & 0 & 0 \\ 0 & 0 & \nu & 0 & -m_3 & \omega & 0 \\ 0 & 0 & 0 & \eta & 0 & -m_4 & 0 \\ 0 & 0 & 0 & 0 & \gamma & \sigma & -\mu \end{pmatrix}.$$

We now find the characteristics equation of $J|_{P^*}$ by using the relation $Det\left(\lambda \mathbf{I}_J - J|_{P^*}\right) = 0$, where $\mathbf{I}_J$ is an identity matrix which has the dimension with matrix $J|_{P^*}$ and $\lambda' s$ are the eigenvalues of the system. It then follows that

$$(\lambda+\mu)^2(\lambda+\theta+\mu)\left(a_4\lambda^4+a_3\lambda^3+a_2\lambda^2+a_1\lambda+a_0\right)=0, \tag{A7}$$

where

$$\begin{aligned}
a_4 =&1,\\
a_3 =&m_1+m_2+m_3+m_4-\alpha_1\beta(K_J^o+S_J^o),\\
a_2 =&m_1m_2+m_1m_3+m_2m_3+(m_1+m_2+m_3)m_4-\alpha_2\beta\delta(K_J^o+S_J^o)\\
&-\alpha_1\beta(m_2+m_3+m_4)(K_J^o+S_J^o)-\beta\nu(K_J^o+S_J^o),\\
a_1 =&m_1m_2m_3+m_2m_3m_4+m_1(m_2+m_3)m_4-\alpha_1\beta K_J^o(m_3m_4+m_2(m_3+m_4))\\
&-\beta K_J^o(m_2+m_4)\nu-\eta\kappa\omega-\beta(\alpha_1m_2m_3+\alpha_1(m_2+m_3)m_4+(m_2+m_4)\nu)S_J^o\\
&-\beta(\alpha_3\delta\eta+\alpha_2\delta(m_3+m_4)+\alpha_2\kappa\nu)(K_J^o+S_J^o),\\
a_0 =&-m_2m_4(\alpha_1\beta K_J^om_3-m_1m_3+\beta K_J^o\nu)-\eta(\beta K_J^o(\delta-\alpha_1\kappa)+\kappa m_1)\omega\\
&-\beta(m_2m_4(\alpha_1m_3+\nu)+\eta(\delta-\alpha_1\kappa)\omega)S_J^o-\beta(\alpha_3\eta+\alpha_2m_4)(\delta m_3+\kappa\nu)(K_J^o+S_J^o).
\end{aligned}$$

Clearly, $\lambda=-\mu$ and $\lambda=-(\theta+\mu)$ are two eigenvalues of $J|_{P_0}$. The other eigenvalues of $J|_{P_0}$ are determined by the following characteristic equation:

$$a_4\lambda^4+a_3\lambda^3+a_2\lambda^2+a_1\lambda+a_0=0. \tag{A8}$$

Obviously, $a_4>0$. We also see that $a_3>0$, if and only if $m_1+m_2+m_3+m_4>\alpha_1\beta(K_J^o+S_J^o)$. Again, $a_2>0$ if and only if $m_1m_2+m_1m_3+m_2m_3+(m_1+m_2+m_3)m_4>\alpha_2\beta\delta(K_J^o+S_J^o)+\alpha_1\beta(m_2+m_3+m_4)(K_J^o+S_J^o)+\beta\nu(K_J^o+S_J^o)$. In a similar manner, $a_1>0$ and $a_0>0$.

It follows from the Routh–Hurwitz criteria that all the eigenvalues associated to $J|_{P^*}$ have negative real parts if and only if $a_3>a_2a_0$, and $a_2a_3>a_0a_2^2+a_1a_4$.

Hence, the conditions of Routh–Hurwitz stability criterion are satisfied by our characteristic polynomial. We therefore conclude that $P_0(S_J^o,K_J^o,0,0,0,0,0)$, is locally asymptotically stable in D for our system of differential equations (1) when $\mathcal{R}_o^D<1$ and P_0 is unstable in D when $\mathcal{R}_o^D>1$.

B: Proof of Theorem 2

We commence by obtaining the Jacobian matrix of our system of differential equations (1) - (7) in $P^*(S_J^*,K_J^*,E_J^*,A_J^*,I_J^*,C_J^*,R_J^*)$ as follows: 0

$$J|_{P^*}(S_J^*,K_J^*,E_J^*,A_J^*,I_J^*,C_J^*,R_J^*)$$
$$=\begin{pmatrix}
-(\theta+\mu)-\beta Y^* & 0 & \pi-\beta\alpha_1S_J^* & \rho-\beta\alpha_2S_J^* & -\beta S_J^* & -\beta\alpha_3S_J^* & 0\\
\theta & \mu-\beta Y^* & \vartheta-\beta\alpha_1K_J^* & -\beta\alpha_2K_J^* & -\beta K_J^* & -\beta\alpha_3K_J^* & 0\\
\beta Y^* & \beta Y^* & -m_1+\beta\alpha_1(S_J^*+K_J^*) & \beta\alpha_2(S_J^*+K_J^*) & \beta(S_J^*+K_J^*) & \beta\alpha_3(S_J^*+K_J^*) & 0\\
0 & 0 & \delta & -m_2 & \kappa & 0 & 0\\
0 & 0 & \nu & 0 & -m_3 & \omega & 0\\
0 & 0 & 0 & \eta & 0 & -m_4 & 0\\
0 & 0 & 0 & 0 & \gamma & \sigma & -\mu
\end{pmatrix},$$

where $Y^*=\alpha_1E_J^*+\alpha_2A_J^*+I_J^*+\alpha_1C_J^*$. Setting $z_1=\beta Y^*$, $z_2=\beta S_J^*\alpha_1$, $z_3=\beta S_J^*\alpha_2$, $z_4=\beta S_J^*$, $z_5=\beta S_J^*\alpha_3$, $z_6=\beta K_J^*\alpha_1$, $z_7=\beta K_J^*\alpha_2$, $z_8=\beta K_J^*$, $z_9=\beta K_J^*\alpha_3$, $k_1=\beta(S_J^*+K_J^*)\alpha_1$, $k_2=\beta(S_J^*+K_J^*)\alpha_2$, $k_3=\beta(S_J^*+K_J^*)$, $k_4=\beta(S_J^*+K_J^*)\alpha_3$ and the characteristic equation is $Det(\lambda\mathbf{I}-J|_{P^*})=0$, where $\lambda' s$ are the eigenvalues of the system and $\mathbf{I}$ is an identity matrix of the same dimension with matrix $J|_{P^*}$

$$(\lambda+\mu)(b_6\lambda^6+b_5\lambda^5+b_4\lambda^4+b_3\lambda^3+b_2\lambda^2+b_1\lambda+b_0)=0 \tag{A9}$$

with

$$\begin{aligned}
b_6 =& 1, \\
b_5 =& -k_1 + m_1 + m_2 + m_3 + m_4 + \theta + 2z_1,
\end{aligned}$$

$$\begin{aligned}
b_4 =& -\delta k_2 + m_2m_3 + m_2m_4 + m_3m_4 - \mu^2 - k_3\nu + m_2\theta + m_3\theta + m_4\theta - \mu\theta \\
&+ 2m_2z_1 + 2m_3z_1 + 2m_4z_1 - \pi z_1 + \theta z_1 - \vartheta z_1 + z_1^2 - k_1(m_2 + m_3 + m_4 + \theta \\
&+ 2z_1) + m_1(m_2 + m_3 + m_4 + \theta + 2z_1) + z_1z_2 + z_1z_6,
\end{aligned}$$

$$\begin{aligned}
b_3 =& m_1m_2m_3 + m_1m_2m_4 + m_1m_3m_4 + m_2m_3m_4 - m_1\mu^2 - m_2\mu^2 - m_3\mu^2 - m_4\mu^2 \\
&- k_3m_2\nu - k_3m_4\nu - \kappa(k_2\nu + \eta\omega) + (m_2m_3 + m_2m_4 + m_3m_4 + m_1(m_2 + m_3 \\
&+ m_4 - \mu) - (m_2 + m_3 + m_4)\mu - k_3\nu)\theta - \delta(\eta k_4 + k_2(m_3 + m_4 + \theta)) - k_1(m_4\theta \\
&+ m_3(m_4 + \theta) + m_2(m_3 + m_4 + \theta) - \mu(\mu + \theta)) + z_1^2(-k_1 + m_1 + m_2 + m_3 + m_4 \\
&- \pi - \vartheta + z_2 + z_6) + z_1(2m_2m_3 + 2m_2m_4 + 2m_3m_4 - 2k_1(m_2 + m_3 + m_4) - m_2\pi \\
&- m_3\pi - m_4\pi + \mu\pi - k_1\theta + m_2\theta + m_3\theta + m_4\theta - \pi\theta + m_1(2(m_2 + m_3 + m_4) + \theta) \\
&- m_2\vartheta - m_3\vartheta - m_4\vartheta - \mu\vartheta - \theta\vartheta + m_2z_2 + m_3z_2 + m_4z_2 - \mu z_2 + \theta z_2 + (m_2 \\
&+ m_3 + m_4 + \mu + \theta)z_6 + \delta(-2k_2 - \rho + z_3 + z_7) + \nu(-2k_3 + z_4 + z_8)),
\end{aligned}$$

$$\begin{aligned}
b_2 =& -k_1m_2m_3m_4 + m_1m_2m_3m_4 + k_1m_2\mu^2 - m_1m_2\mu^2 + k_1m_3\mu^2 - m_1m_3\mu^2 - m_2m_3\mu^2 \\
&+ k_1m_4\mu^2 - m_1m_4\mu^2 - m_2m_4\mu^2 - m_3m_4\mu^2 - \eta k_4\kappa\nu - k_2\kappa m_4\nu - k_3m_2m_4\nu + k_3\mu^2\nu \\
&+ \eta k_1\kappa\omega - \eta\kappa m_1\omega - (-m_2m_3m_4 - m_1(m_2m_3 + (m_2 + m_3)m_4) + m_2m_3\mu + m_2m_4\mu \\
&+ m_3m_4\mu + m_1(m_2 + m_3 + m_4)\mu + k_1(m_2m_3 + m_2m_4 + m_3m_4 - (m_2 + m_3 + m_4)\mu) \\
&+ k_3(m_2 + m_4 - \mu)\nu + \kappa(k_2\nu + \eta\omega))\theta - \delta(\eta k_3\omega + k_2m_4\theta + \eta k_4(m_3 + \theta) + k_2m_3(m_4 + \theta) \\
&- k_2\mu(\mu + \theta)) + z_1^2(m_2m_3 + m_2m_4 + m_3m_4 - k_1(m_2 + m_3 + m_4) + m_1(m_2 + m_3 + m_4) \\
&- m_2\pi - m_3\pi - m_4\pi - m_2\vartheta - m_3\vartheta - m_4\vartheta + m_2z_2 + m_3z_2 + m_4z_2 + (m_2 + m_3 + m_4)z_6 \\
&+ \delta(-k_2 - \rho + z_3 + z_7) + \nu(-k_3 + z_4 + z_8)) + 2m_1m_2(m_3 + m_4) + 2m_1m_3m_4 + 2m_2m_3m_4 \\
&- 2k_1(m_3m_4 + m_2(m_3 + m_4)) - 2k_2\kappa\nu - 2k_3m_2\nu - 2k_3m_4\nu - 2\eta\kappa\omega - (m_2m_3 + m_2m_4 \\
&+ m_3m_4 - m_2\mu - m_3\mu - m_4\mu)\pi - \kappa\nu\rho + (m_1m_2 + m_1m_3 + m_2m_3 + m_1m_4 + m_2m_4 + m_3m_4 \\
&- k_1(m_2 + m_3 + m_4) - k_3\nu - m_2\pi - m_3\pi - m_4\pi)\theta - \delta(2\eta k_4 + k_2(2(m_3 + m_4) + \theta) \\
&+ \rho(m_3 + m_4 - \mu + \theta)) - (m_4(\mu + \theta) + m_3(m_4 + \mu + \theta) + m_2(m_3 + m_4 + \mu + \theta))\vartheta \\
&+ m_4(-\mu + \theta)z_2 + m_3(m_4 - \mu + \theta)z_2 + m_2(m_3 + m_4 - \mu + \theta)z_2 + \kappa\nu z_3 + \delta(m_3 + m_4 - \mu \\
&+ \theta)z_3 + \nu(m_2 + m_4 - \mu + \theta)z_4 + \delta\eta z_5 + m_4(\mu + \theta)z_6 + m_3(m_4 + \mu + \theta)z_6 + m_2(m_3 + m_4 \\
&+ \mu + \theta)z_6 + \kappa\nu z_7 + \delta(m_3 + m_4 + \mu + \theta)z_7 + \nu(m_2 + m_4 + \mu + \theta)z_8 + \delta\eta z_9,
\end{aligned}$$

$$
\begin{aligned}
b_1 =&\mu^2(\delta\eta k_4 + k_1 m_2 m_3 - m_1 m_2 m_3 - m_1 m_2 m_4 - m_1 m_3 m_4 - m_2 m_3 m_4 + k_1(m_2 + m_3)m_4 \\
&+ \delta k_2(m_3 + m_4) + k_3(m_2 + m_4)\nu + \kappa(k_2\nu + \eta\omega)) + (m_1 m_2 m_3 m_4 - m_2 m_3 m_4 \mu \\
&- m_1(m_3 m_4 + m_2(m_3 + m_4))\mu + k_1(-m_2 m_3 m_4 + m_3 m_4 \mu + m_2(m_3 + m_4)\mu) + (-\eta k_4 \kappa \\
&- (k_2\kappa + k_3 m_2)m_4 + k_2\kappa\mu + k_3(m_2 + m_4)\mu)\nu + \eta\kappa(k_1 - m_1 + \mu)\omega + \delta(-k_2 m_3 m_4 \\
&+ k_2(m_3 + m_4)\mu - \eta(k_4 m_3 - k_4\mu + k_3\omega)))\theta + z_1^2(m_1 m_2 m_3 + m_1 m_2 m_4 + m_1 m_3 m_4 \\
&+ m_2 m_3 m_4 - k_1(m_2 m_3 + (m_2 + m_3)m_4) - k_3 m_2\nu - k_3 m_4\nu - \kappa(k_2\nu + \eta\omega) - (m_2 m_3 \\
&+ m_2 m_4 + m_3 m_4)\pi - \kappa\nu\rho - m_2 m_3\vartheta - m_2 m_4\vartheta - m_3 m_4\vartheta + (m_2 m_3 + m_2 m_4 + m_3 m_4)z_2 \\
&+ \kappa\nu z_3 + m_2\nu z_4 + m_4\nu z_4 + m_2 m_3 z_6 + m_2 m_4 z_6 + m_3 m_4 z_6 + \kappa\nu z_7 + (m_2 + m_4)\nu z_8 \\
&+ \delta(-(m_3 + m_4)(k_2 + \rho - z_3 - z_7) + \eta(-k_4 + z_5 + z_9))) + z_1(-2\eta k_4\kappa\nu - 2k_2\kappa m_4\nu \\
&- 2k_3 m_2 m_4\nu - (m_2 m_3 m_4 - m_2 m_3\mu - m_2 m_4\mu - m_3 m_4\mu - \eta\kappa\omega)\pi - \kappa m_4\nu\rho + \kappa\mu\nu\rho \\
&- (k_2\kappa\nu + k_3 m_4\nu + \eta\kappa\omega + m_3 m_4\pi + m_2(-m_3 m_4 + k_3\nu + (m_3 + m_4)\pi) + \kappa\nu\rho)\theta \\
&+ m_1(2m_2 m_3 m_4 - 2\eta\kappa\omega + m_3 m_4\theta + m_2(m_3 + m_4)\theta) + -k_1(2m_2 m_3 m_4 - 2\eta\kappa\omega + m_3 m_4\theta \\
&+ m_2(m_3 + m_4)\theta) - \delta(2k_2 m_3 m_4 + 2\eta k_3\omega + m_3 m_4\rho - m_3\mu\rho - m_4\mu\rho + (m_3 + m_4)(k_2 + \rho)\theta \\
&+ \eta k_4(2m_3 + \theta)) - (-\eta\kappa\omega + m_2 m_4(\mu + \theta) + m_3 m_4(\mu + \theta) + m_2 m_3(m_4 + \mu + \theta))\vartheta \\
&+ (-\eta\kappa\omega + m_2 m_4(-\mu + \theta) + m_3 m_4(-\mu + \theta) + m_2 m_3(m_4 - \mu + \theta))z_2 + (\delta m_4(-\mu + \theta) \\
&+ \delta m_3(m_4 - \mu + \theta) + \kappa\nu(m_4 - \mu + \theta))z_3 + (\delta\eta\omega + m_4\nu(-\mu + \theta) + m_2\nu(m_4 - \mu + \theta))z_4 \\
&+ \eta(\kappa\nu + \delta(m_3 - \mu + \theta))z_5 + (-\eta\kappa\omega + m_2 m_4(\mu + \theta) + m_3 m_4(\mu + \theta) + m_2 m_3(m_4 + \mu + \theta))z_6 \\
&+ (\delta m_4(\mu + \theta) + \delta m_3(m_4 + \mu + \theta) + \kappa\nu(m_4 + \mu + \theta))z_7 + (\delta\eta\omega + m_4\nu(\mu + \theta) + m_2\nu(m_4 \\
&+ \mu + \theta))z_8 + \eta(\kappa\nu + \delta(m_3 + \mu + \theta))z_9),
\end{aligned}
$$

$$
\begin{aligned}
b_0 =&\mu(\eta k_4\kappa\nu + m_4(k_1 m_2 m_3 - m_1 m_2 m_3 + k_2\kappa\nu + k_3 m_2\nu) + \eta\kappa(-k_1 + m_1)\omega)(\mu + \theta) - (\eta k_4\kappa\nu \\
&+ k_1(m_2 m_3 m_4 - \eta\kappa\omega) + m_1(-m_2 m_3 m_4 + \eta\kappa\omega) + m_4\nu(k_3 m_2 + \kappa(k_2 + \rho)))\theta z_1 - (\eta k_4\kappa\nu \\
&+ m_4(k_1 m_2 m_3 - m_1 m_2 m_3 + k_2\kappa\nu + k_3 m_2\nu) + \eta\kappa(-k_1 + m_1)\omega)z_1^2 + \delta(\eta(k_4 m_3 + k_3\omega)(\mu \\
&- z_1)(\mu + \theta + z_1) + m_3 m_4(-\rho\theta z_1 + k_2(\mu - z_1)(\mu + \theta + z_1))) + \eta(\delta m_3 + \kappa\nu)z_1(\mu + \theta + z_1)z_9 \\
&+ z_1(\mu + \theta + z_1)(\delta m_3 m_4 z_7 + \kappa m_4\nu z_7 + m_2 m_4\nu z_8 + \delta\eta\omega z_8) + z_1(m_4(\delta m_3 + \kappa\nu)(-\rho z_1 \\
&+ \mu(\rho - z_3) + (\theta + z_1)z_3) + m_2 m_4(\nu(-\mu + \theta + z_1)z_4 - m_3(\theta + z_1)(\pi + \vartheta - z_2 - z_6) \\
&+ m_3\mu(\pi - \vartheta - z_2 + z_6)) + \eta(-\delta(\mu - \theta - z_1)(\omega z_4 + m_3 z_5) + \kappa(\theta + z_1)(\nu z_5 + \omega(\pi + \vartheta \\
&- z_2 - z_6)) - \kappa\mu(\nu z_5 + \omega(\pi - \vartheta - z_2 + z_6)))).
\end{aligned}
$$

Clearly, $\lambda = -\mu$ is an eigenvalue of $J|_{P^*}$. The other eigenvalues of $J|_{P^*}$ are determined by the following characteristic equation:

$$\lambda^6 + b_5\lambda^5 + b_4\lambda^4 + b_3\lambda^3 + b_2\lambda^2 + b_1\lambda + b_0 = 0. \tag{A10}$$

By a direct calculation, we have that $b_j > 0$ for $j = 0, 1, 2, 3, 4, 5$. It therefore follows from the Routh–Hurwitz criteria that all the eigenvalues associated with $J|_{P^*}$ have negative real parts if and only if $b_j > 0$ for $j = 0, 1, 2, 3, 4, 5$, and

$$
\begin{gathered}
b_5 > b_4 b_0, \\
b_0 b_2 + b_4 b_5 > b_3 + b_0 b_4^2, \\
2b_0 b_2 b_3 + b_1 b_5 + b_0 b_2 b_4 b_5 + b_3 b_4 b_5 > b_0^2 b_2^2 + b_3^2 + b_0 b_1 b_4 + b_0 b_3 b_4^2 + b_2 b_5^2,
\end{gathered}
$$

$$
\begin{aligned}
&2b_0 b_2^2 b_3 + b_1 b_3 b_4 + b_0 b_1 b_4^2 + b_2(2b_1 + (b_0 b_2 + b_3)b_4)b_5 > b_1^2 + b_1 b_4(3b_0 b_2 + b_4 b_5) \\
&+ b_2(b_0^2 b_2^2 + b_3^2 + b_0 b_3 b_4^2 + b_2 b_5^2),
\end{aligned}
$$

$$
\begin{aligned}
&2b_0 b_2^2 b_3 + b_1 b_3 b_4 + b_0 b_1 b_4^3 + b_2(2b_1 + (b_0 b_2 + b_3)b_4)b_5 > b_1^2 + b_1 b_4(3b_0 b_2 + b_4 b_5) \\
&+ b_2(b_0^2 b_2^2 + b_3^2 + b_0 b_3 b_4^2 + b_2 b_5^2),
\end{aligned}
$$

Hence, the conditions of Routh–Hurwitz stability criterion are satisfied by our characteristic polynomial. We therefore conclude that $P^*(S_J^*, K_J^*, E_J^*, A_J^*, I_J^*, C_J^*, R_J^*)$, is locally asymptotically stable in D for our system of differential equations (1) - (7) when $\mathcal{R}_o^D > 1$.

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