RESEARCH ARTICLE

Bifurcation and Stability Analysis for a Class of Discrete Singular Predator-Prey System

Jing Han [1,*] **and Wei Liu** [2]

[1] School of Information Engineering, Wuhan Business University, Wuhan 430000, China
[2] School of Information Management and Mathematics, Jiangxi University of Finance and Economics, Nanchang 330032, China

Abstract

A kind of discrete-time singular predator-prey system with time-varying harvesting term is investigated. By using theory of singular systems, bifurcation and center manifold theory, the stability and Neimark-Sacker bifurcation of such system is studied, and some conditions are used to judge local stability of its fixed points and ensure existence of the Neimark-Sacker bifurcation for the proposed discrete-time singular system are derived. Finally, numerical simulations are given to show the obtained results. The results of the paper complements some previous works, and we believe that the method of this paper can be used to study bifurcation for other discrete-time complex singular systems.

Keywords: stability, neimark-Sacker bifurcation, predator-prey, discrete-time, singular system.

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***Corresponding author:**
Jing Han
hjhust2014@163.com

1 Introduction

In recent years, predator-prey models and their dynamic behaviors have been investigated by many scholars [1–4]. In 2011, He *et al.* [3] studied bifurcation for a predator-prey model in discrete-time. In 2020, Li *et al.* [4] investigated flip bifurcation of a kind of discrete Holling-type III predator-prey model. In 2021, Mortuja *et al.* [5] systematically studied the dynamic behaviors of predator-prey system with fear, migration and switching. In 2023, Al Khabyah *et al.* [6] discussed bifurcation for a kind of discrete predator-prey system. In 2026, Huang *et al.* [7] studied bifurcation of a predator-prey model with discrete-time.

As we know, during the bio-economics's management, human intervention and harvesting are universal, so, it is more practical to research predator-prey models with harvesting. In 2017, Hu *et al.* [8] investigated a Michaelis-Menten harvesting in the predator-prey system, and some results about stability and bifurcation of such system were constructed. In 2023, Wang *et al.* [9] showed bifurcations of the linear harvesting in a diffusive predator-prey model. The purpose of human harvesting is usually for economic value and profit, so, discussing predator-prey system both with harvesting as well as economic profit has important theoretical and practical value.

In 2009, Liu *et al.* [10] studied a singular prey-predator system, which considered both harvesting and economic profit. Then, differential-algebraic

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prey-predator model has been attracted some scholars' attention and some interesting works have been reported [11–13]. In 2010, Zhang *et al.* [11] showed bifurcation and stability for a singular prey-predator system. In 2017, by introducing nonlinear prey harvesting, Li *et al.* [12] considered a singular predator-prey system. In 2026, Liu [13] studied a singular Holling-type IV predator-prey system.

However, most of the above singular predator-prey systems are continuous-time, and for discrete-time singular predator-prey system, there are few works. In order to enrich the relevant researches of this field, here, we will discuss a kind of singular predator-prey model in discrete-time, and some conditions are used to judge the local stability and ensure existence of the Neimark-Sacker bifurcation for such singular system in discrete-time will be derived. Based on [1] and considering prey harvesting in the following predator-prey system

$$\begin{cases} \frac{\mathrm{d}x}{\mathrm{d}t} = x[r(1-\frac{x}{k}) - \alpha y - qE], \\ \frac{\mathrm{d}y}{\mathrm{d}t} = y(-d + \frac{\beta x}{1+bx}), \end{cases} \tag{1}$$

and from the previous works [10–13], here, by applying forward Euler scheme to system (1), we discuss the following singular system in discrete-time

$$\begin{cases} x \to x + \delta x[r(1-\frac{x}{k}) - \alpha y - qE], \\ y \to y + \delta y(-d + \frac{\beta x}{1+bx}), \\ 0 = pE^2 x - cEx - m. \end{cases} \tag{2}$$

where the parameters $r, k, q, \beta, \alpha, b, d, p, c, m > 0$, δ is the step size, $pE^2(t)x(t)$ is harvesting reward, cEx is harvesting cost, and $m > 0$ is economic profit.

Here, let

$$\begin{aligned} f(X) &= \begin{pmatrix} f_1(x,y,E) \\ f_2(x,y,E) \end{pmatrix} \\ &= \begin{pmatrix} x + \delta x[r(1-\frac{x}{k}) - \alpha y - qE] \\ y + \delta y(-d + \frac{\beta x}{1+bx}) \end{pmatrix}, \\ g(X) &= pE^2 x - cEx - m. \end{aligned} \tag{3}$$

in which, $X = (x, y, E)^T$.

Remark 1. The previous works [10–13] discussed stability and bifurcation of continuous-time singular predator-prey system, different from these works, the discrete-time singular predator-prey system is considered here, so, the results of this paper enriches some previous relevant works.

2 Fixed point and local stability

For model (2), we consider in R^3_+, which fixed points satisfy

$$\begin{cases} x = x + \delta x[r(1-\frac{x}{k}) - \alpha y - qE], \\ y = y + \delta y(-d + \frac{\beta x}{1+bx}), \\ 0 = pE^2 x - cEx - m. \end{cases} \tag{4}$$

Lemma 2.1. Let $\beta > bd, r[1 - \frac{d}{k(\beta-bd)}] > [qc + q\sqrt{c^2 + 4pm(\beta - bd)/d}]/2p$, then the system (2) has only a positive fixed point $X_0 = (x_0, y_0, E_0) = (\frac{d}{\beta-bd}, [r - \frac{rx_0}{k} - qE_0]/\alpha, [c + \sqrt{c^2 + 4pm(\beta - bd)/d}]/2p)$.

From (3), one has $D_X g(x_0, y_0, E_0) = (\frac{m}{x_0}, 0, B)$, since $B = pE_0 x_0 + \frac{m}{E_0} > 0$. Based on [11–14], we define

$$[x, y, E]^T = \psi(P) = X_0^T + U_0 P + V_0 h(P), g(\psi(P)) = 0,$$

where $U_0 = \begin{bmatrix} I_2 \\ 0 \end{bmatrix}, I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, V_0 = (0,0,1)^T, P = (y_1, y_2)^T, h : R^2 \to R$ is a smooth mapping. From [14], one has

$$\begin{aligned} D\psi(P) =& (D_{y_1}\psi(P), D_{y_2}\psi(P)) \\ =& \begin{pmatrix} Dg(X) \\ U_0^T \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ I_2 \end{pmatrix}. \end{aligned} \tag{5}$$

for $\forall\, (x, y, E) \in B_1(x_0, y_0, E_0)$, and the system (2) is reduced to

$$P \to f(\psi(P)), P \in M \subset R^2, \tag{6}$$

where $M = \psi^{-1}(D_1(x_0, y_0, E_0))$, $D_1(x_0, y_0, E_0)$ is the neighborhood of X_0. Then, Jacobian matrix of $P_0 = 0$ is

$$\begin{aligned} D_P f(\psi(0)) =& Df(X_0) D\psi(0) \\ =& \begin{pmatrix} 1 - \frac{\delta r x_0}{k} + \frac{\delta q m}{B} & -\alpha\delta x_0 \\ \frac{\beta\delta y_0}{(1+bx_0)^2} & 1 \end{pmatrix}. \end{aligned} \tag{7}$$

From (7), one gets

$$\lambda^2 - (2 + Q_1\delta)\lambda + (1 + Q_1\delta + Q_2\delta^2) = 0, \tag{8}$$

where $Q_1 = -\frac{rx_0}{k} + \frac{qm}{B}, Q_2 = \frac{\alpha\beta x_0 y_0}{(1+bx_0)^2}$.

Let $J(\lambda) = \lambda^2 - (2 + Q_1\delta)\lambda + (1 + Q_1\delta + Q_2\delta^2)$, then we have

$$J(1) = \frac{\alpha\beta x_0 y_0 \delta^2}{(1 + bx_0)^2} > 0, J(-1) = 4 + 2Q_1\delta + Q_2\delta^2. \tag{9}$$

Now, by (9) and based on [3], one obtains the following Theorem 2.1.

Theorem 2.1. Model (2) has the positive fixed point X_0,
(a) X_0 is a sink if one of following inequality holds:

(a1)$-2\sqrt{Q_2} \leq Q_1 < 0$ and $0 < \delta < -\frac{Q_1}{Q_2}$;

(a2)$Q_1 < -2\sqrt{Q_2}$ and $0 < \delta < \frac{-Q_1-\sqrt{Q_1^2-4Q_2}}{Q_2}$;
(b) X_0 is a source if one of following inequality holds:

(b1)$-2\sqrt{Q_2} \leq Q_1 < 0$ and $\delta > -\frac{Q_1}{Q_2}$;

(b2)$Q_1 < -2\sqrt{Q_2}$ and $\delta > \frac{-Q_1+\sqrt{Q_1^2-4Q_2}}{Q_2}$; or $Q_1 \geq 0$;
(c) If $Q_1 < -2\sqrt{Q_2}$ and $\frac{-Q_1-\sqrt{L_1^2-4Q_2}}{Q_2} < \delta < \frac{-Q_1+\sqrt{Q_1^2-4Q_2}}{Q_2}$, then, X_0 is a saddle.
(d) X_0 is non-hyperbolic if one of following inequality holds:

(d1)$Q_1 < -2\sqrt{Q_2}, \delta = \frac{-Q_1\pm\sqrt{Q_1^2-4Q_2}}{Q_2}$ and $\delta \neq -\frac{2}{Q_1}, -\frac{4}{Q_1}$;

(d2)$-2\sqrt{Q_2} < Q_1 < 0$ and $\delta = -\frac{Q_1}{Q_2}$;

And now, Let

$$\Omega_B = \{(r,b,\alpha,\beta,d,p,q,c,m,\delta) : \delta = -\frac{Q_1}{Q_2}, \\ -2\sqrt{Q_2} < Q_1 < 0, r,b,\alpha,\beta,d,p,q,c,m, \\ \delta > 0\}.$$

Neimark-Sacker bifurcation will occur if parameter δ vary in small neighborhood of Ω_B at fixed point X_0.

3 Stability and Neimark-Sacker bifurcation

Now, the existence and local stability of Neimark-Sacker bifurcation in the small neighborhood of Ω_B will be discussed.

Because $(r,b,\alpha,\beta,d,p,q,c,m,\delta_1) \in \Omega_B, \delta_1 = -\frac{Q_1}{Q_2}$. Here, let $\delta^*(|\delta^*| \ll 1)$ is a parameter of the bifurcation, and consider

$$\begin{cases} x \to x + (\delta_1+\delta^*)x[r(1-\frac{x}{k}) - \alpha y - qE], \\ y \to y + (\delta_1+\delta^*)y(-d + \frac{\beta x}{1+bx}), \\ 0 = pE^2x - cEx - m. \end{cases} \tag{10}$$

Now, based on [15], (3) and (6), one has

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \to \begin{pmatrix} L_1 \\ L_2 \end{pmatrix}. \tag{11}$$

where

$$\begin{aligned} L_1 =& f_1^{(1)}y_1 + f_2^{(1)}y_2 + f_{11}^{(1)}y_1^2 + f_{12}^{(1)}y_1y_2 \\ &+ o(|y_1|+|y_2|)^4, \\ L_2 =& f_1^{(2)}y_1 + f_2^{(2)}y_2 + f_{11}^{(2)}y_1^2 + f_{12}^{(2)}y_1y_2 + f_{111}^{(2)}y_1^3 \\ &+ f_{112}^{(2)}y_1^2y_2 + o(|y_1|+|y_2|)^4, \end{aligned}$$

in which $f_1^{(1)}, f_2^{(1)}, ..., f_{112}^{(2)}$ can be same got as [16], which are omitted here.

$$\begin{aligned} f_1^{(1)} =& 1 - \frac{\delta_1 r x_0}{k} + \frac{\delta_1 qm}{B}, f_2^{(1)} = -\delta_1\alpha x_0, \\ f_1^{(2)} =& \frac{\delta_1\beta y_0}{(1+bx_0)^2}, f_2^{(2)} = 1, f_{11}^{(1)} = \frac{\delta_1 qm}{x_0 B} - \frac{\delta_1 r}{k}, \\ f_{12}^{(1)} =& -\delta_1\alpha, f_{11}^{(2)} = -\frac{\delta_1 b\beta y_0}{(1+bx_0)^3}, \\ f_{12}^{(2)} =& \frac{\delta_1\beta}{(1+bx_0)^2}, f_{111}^{(2)} = \frac{\delta_1 b^2\beta y_0}{(1+bx_0)^4}, \\ f_{112}^{(2)} =& -\frac{\delta_1 b\beta}{(1+bx_0)^3}, f_{22}^{(1)} = f_{22}^{(2)} = \cdots = f_{222}^{(2)} = 0. \end{aligned} \tag{12}$$

At $Y_0 = (0,0)$, the characteristic equation of the linearization part for (11) is

$$\lambda^2 - [2 + Q_1(\delta_1+\delta^*)]\lambda + [1 + Q_1(\delta_1+\delta^*) \\ + Q_2(\delta_1+\delta^*)^2] = 0, \tag{13}$$

Following the approach similar to [16] for continuous systems, one can derive for our discrete system that Theorem 2.1 (d2) holds and

$$l = \frac{d|\lambda|}{d\delta^*}|_{\delta^*=0} = -\frac{Q_1}{2} > 0, \\ |\lambda| = \sqrt{[1 + Q_1(\delta_1+\delta^*) + Q_2(\delta_1+\delta^*)^2]}. \tag{14}$$

In addition, when $\delta^* = 0$, we need $\lambda^i, \bar{\lambda}^i \neq 1 (i = 1,2,3,4)$ which is equivalent to $-(2+Q_1\delta_1) \neq -2,0,1,2$. In fact, $-(2+Q_1\delta_1) \neq 0,1$, is only need to hold, so, $\frac{Q_1^2}{Q_2} \neq 2,3$. Therefore, when $\delta^* = 0$, eigenvalues $\lambda, \bar{\lambda}$ of Y_0 do not lie on the intersection with unit circle.

Now, under $\delta^* = 0$, the normal form of map (11) is discussed. Choose $\delta^* = 0$, $\mu = 1 + \frac{Q_1\delta_1}{2}$, $\omega = \frac{\delta_1}{2}\sqrt{4Q_2 - Q_1^2}$, now by using the translation $\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} f_2^{(1)} & 0 \\ \mu - f_1^{(1)} & -\omega \end{pmatrix}\begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$, then, we can get the normal form of (11) as follows:

$$\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \to \begin{pmatrix} W_1 \\ W_2 \end{pmatrix}, \tag{15}$$

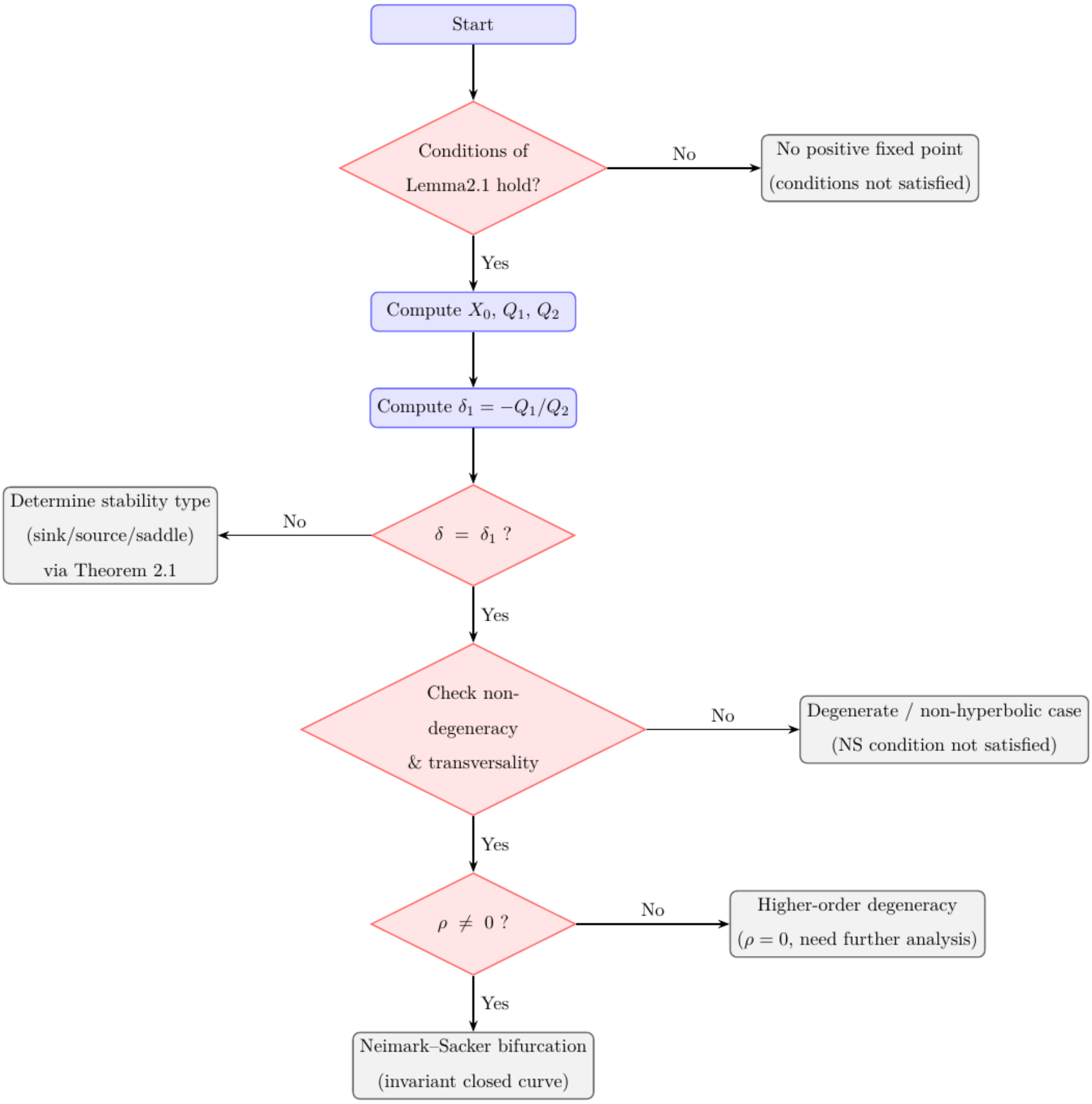

Figure 1. Flowchart for judging Neimark-Sacker bifurcation.

where

$$\begin{aligned} W_1 =& \mu z_1 - \omega z_2 + \bar{f}^{(1)}_{11} z_1^2 + \bar{f}^{(1)}_{12} z_1 z_2 \\ &+ o(|z_1| + |z_2|)^4, \\ W_2 =& \omega z_1 + \mu z_2 + \bar{f}^{(2)}_{11} z_1^2 + \bar{f}^{(2)}_{12} z_1 z_2 \\ &+ \bar{f}^{(2)}_{111} z_1^3 + \bar{f}^{(2)}_{112} z_1^2 z_2 + o(|z_1| + |z_2|)^4, \end{aligned}$$

in which,

$$\begin{aligned} \bar{f}^{(1)}_{11} =& f^{(1)}_2 f^{(1)}_{11} + f^{(1)}_{12}(\mu - f^{(1)}_1), \bar{f}^{(1)}_{12} = -\omega f^{(1)}_{12}, \\ \bar{f}^{(2)}_{11} =& \frac{1}{\omega}[f^{(1)}_{12}(\mu - f^{(1)}_1)^2 - f^{(2)}_{11}(f^{(1)}_2)^2 \\ &+ f^{(1)}_{11} f^{(1)}_2 (\mu - f^{(1)}_1) - f^{(2)}_{12} f^{(1)}_2 (\mu - f^{(1)}_1)], \\ \bar{f}^{(2)}_{112} =& f^{(2)}_{112}(f^{(1)}_2)^2, \bar{f}^{(2)}_{12} = -f^{(1)}_{12}(\mu - f^{(1)}_1) + f^{(2)}_{12} f^{(1)}_2, \\ \bar{f}^{(2)}_{111} =& -\frac{1}{\omega}[f^{(2)}_{111}(f^{(1)}_2)^3 + f^{(2)}_{112}(f^{(1)}_2)^2(\mu - f^{(1)}_1)], \\ \bar{f}^{(1)}_{22} =& \cdots = \bar{f}^{(2)}_{222} = 0. \end{aligned} \quad (16)$$

If the following discriminatory quantity is not zero, then, by using [15, 16], the map (2) will occur Neimark-Sacker bifurcation.

$$\begin{aligned} \rho = \{ &- \mathrm{Re}[\frac{(1-2\lambda)\bar{\lambda}^2}{1-\lambda}\zeta_{11}\zeta_{20}] \\ &- \frac{1}{2}|\zeta_{11}|^2 - |\zeta_{02}|^2 + \mathrm{Re}(\bar{\lambda}\zeta_{21})\}|_{\delta^*=0}, \end{aligned} \quad (17)$$

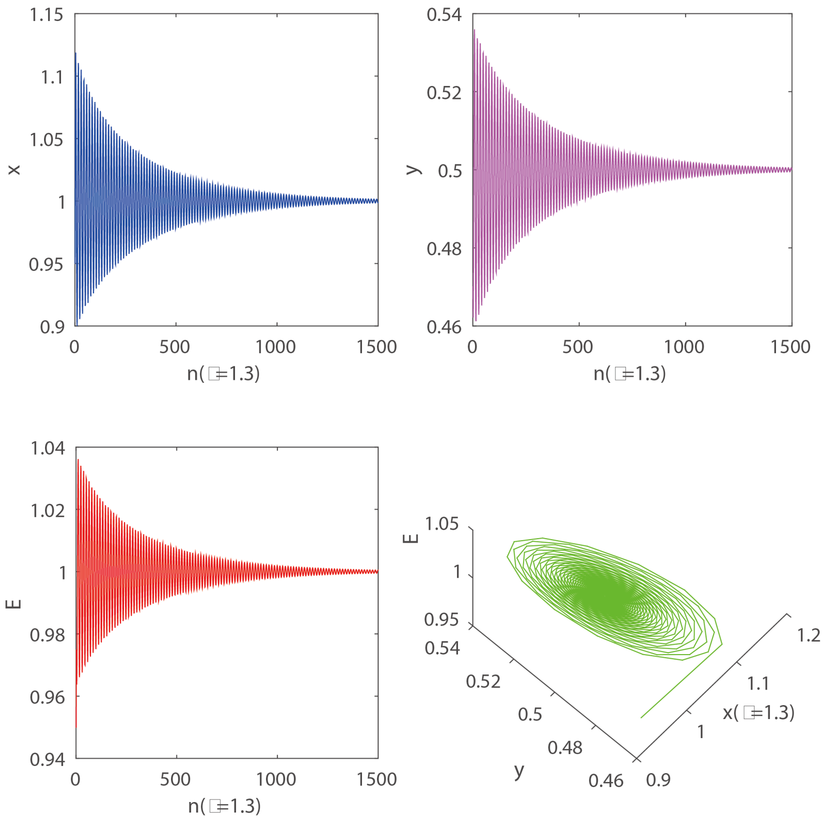

Figure 2. The positive fixed point $X_0 = (1, 0.5, 1)$ of (18) is asymptotically stable when $\delta = 1.3 < \delta_1 = \frac{4}{3}$.

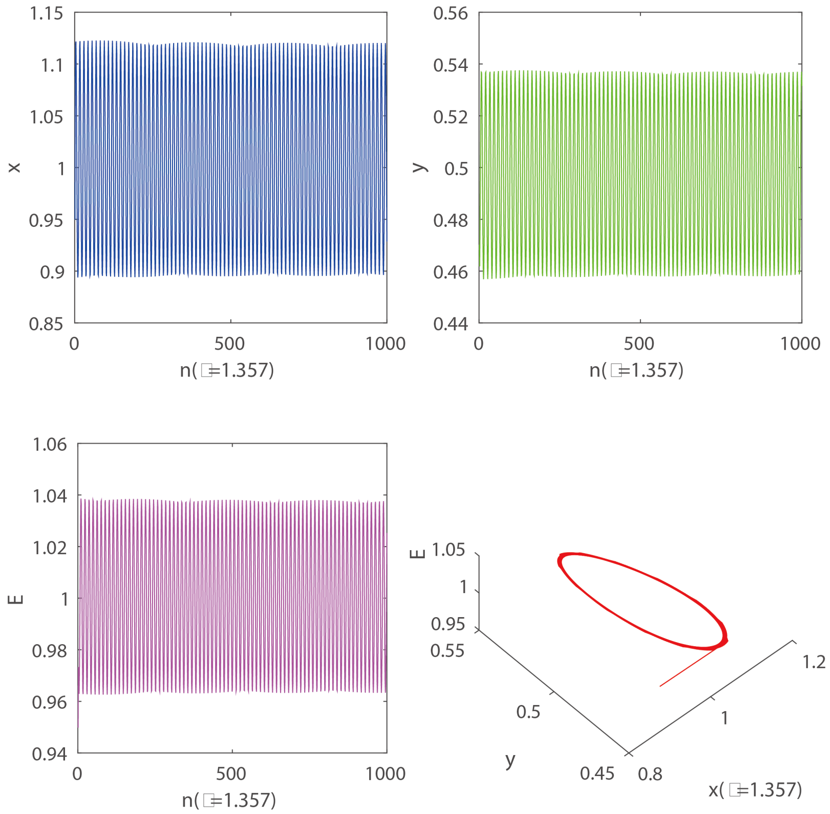

Figure 3. The Neimark-Sacker bifurcation occurs and bifurcating periodic orbit from positive fixed point $X_0 = (1, 0.5, 1)$ of (18) when $\delta = 1.357 > \delta_1 = \frac{4}{3}$.

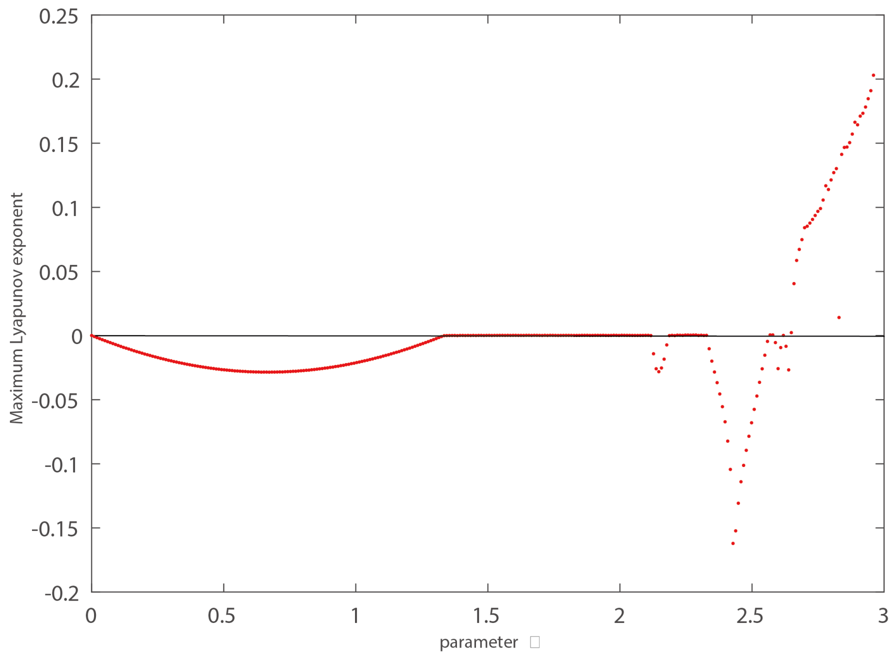

Figure 4. Maximum Lyapunov exponents corresponding of (18).

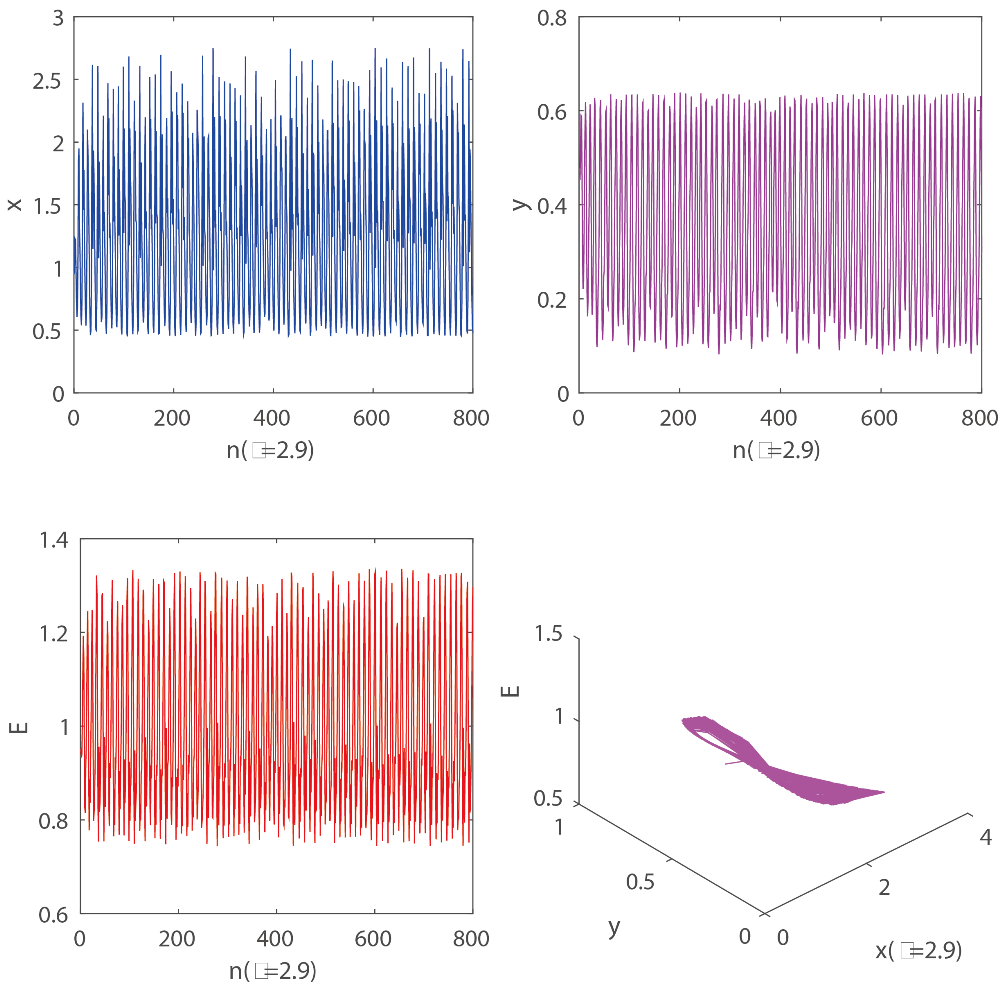

Figure 5. Chaotic behaviors of (18) with $\delta = 2.9$.

in which,

$$\begin{aligned}\zeta_{11} =& \frac{1}{4}[i(\bar{f}_{11}^{(2)} + \bar{f}_{22}^{(2)}) + \bar{f}_{11}^{(1)} + \bar{f}_{22}^{(1)}],\\ \zeta_{20} =& \frac{1}{8}[i(\bar{f}_{11}^{(2)} - \bar{f}_{22}^{(2)} - 2\bar{f}_{12}^{(1)}) + \bar{f}_{11}^{(1)} - \bar{f}_{22}^{(1)} + 2\bar{f}_{12}^{(2)}],\\ \zeta_{02} =& \frac{1}{8}[i(\bar{f}_{11}^{(2)} - \bar{f}_{22}^{(2)} + 2\bar{f}_{12}^{(1)}) + \bar{f}_{11}^{(1)} - \bar{f}_{22}^{(1)} - 2\bar{f}_{12}^{(2)}],\\ \zeta_{21} =& \frac{1}{16}[i(\bar{f}_{111}^{(2)} + \bar{f}_{122}^{(2)} - \bar{f}_{112}^{(1)} - \bar{f}_{222}^{(1)}) + \bar{f}_{111}^{(1)} + \bar{f}_{122}^{(1)}\\ &+ \bar{f}_{112}^{(2)} + \bar{f}_{222}^{(2)}].\end{aligned}$$

From [15], one gets the following Theorem 3.1.

Theorem 3.1. Let $\beta > bd, r[1 - \frac{d}{k(\beta - bd)}] > [qc + q\sqrt{c^2 + 4pm(\beta - bd)/d}]/2p$, if $l = \frac{d|\lambda|}{d\delta^*}|_{\delta^*=0} = -\frac{Q_1}{2} > 0$, (d2), $\frac{Q_1^2}{Q_2} \neq 2, 3$ hold and $\rho \neq 0$, then, at the fixed point X_0 and δ varies in a small-neighborhood of δ_1, the discrete-time singular predator-prey system (2) occurs Neimark-Sacker bifurcation. And if $\rho < 0$ $(resp., \rho > 0)$, then an invariant closed curve which is attracting $(\rho < 0)$ or repelling $(\rho > 0)$ bifurcates from fixed point for $\delta > \delta_1$ or $\delta < \delta_1$, respectively.

Remark 2. For convenience, we give the flowchart of the procedure for determining the occurrence of Neimark-Sacker bifurcation for the discrete-time singular predator-prey system (2) , which is displayed in the Figure 1.

4 Simulation results

Now, for discrete-time singular predator-prey system (2), let $r = p = 2, k = 4, \alpha_1 = \beta = b = q = c = m = 1, d = \frac{1}{2}$, that is

$$\begin{cases} x \to x + \delta x[2(1 - \frac{x}{4}) - y - E],\\ y \to y + \delta y(-\frac{1}{2} + \frac{x}{1+x}),\\ 0 = 2E^2x - Ex - 1. \end{cases} \quad (18)$$

From above discussion, we show the stability of the positive fixed point and Neimark-Sacker bifurcation. The only positive fixed point of system (18) is $X_0 = (1, 0.5, 1)$ and the bifurcation value $\delta_1 = \frac{4}{3}$, $\beta = 1 > bd = \frac{1}{2}, r[1 - \frac{d}{k(\beta - bd)}] = \frac{3}{2} > [qc + q\sqrt{c^2 + 4pm(\beta - bd)/d}]/2p = 1$, $Q_1 = -\frac{1}{6}, Q_2 = \frac{1}{8}, \frac{Q_1^2}{Q_2} = \frac{2}{9}$ and $\rho = -0.3328 < 0$. So, from the Theorem 3.1, when $\delta = 1.3 < \delta_1 = \frac{4}{3}$, one knows that X_0 of (18) is local asymptotical stable, and when $\delta = 1.357 > \delta_1 = \frac{4}{3}$, an attracting invariant closed curve occurs that is , respectively, displayed in Figures 2 and 3. With increasing the step δ, system (18) will show that the complicated dynamical behaviors. The maximum Lyapunov exponents of (18) is showed in Figure 4, which confirms the existence of the chaos. Based on Figure 4, if we choose $\delta = 2.9$, we get that the system (18) becomes a chaotic one that is illustrated by Figure 5. Here, we choose $x(0) = 0.95, y(0) = 0.47, E(0) = 0.95$ are the initial values.

5 Conclusions

Here, a kind of discrete-time singular predator-prey system was investigated. Let the step size δ be the bifurcation parameter, some results on stability and Neimark-Sacker bifurcation were derived. The analysis process of this paper provides us with a theoretical basis for studying more reasonable discrete-time singular predator-prey system, such as with non-linear harvesting [12, 13], which will be further discussed in our future works.

Data Availability Statement

Data will be made available on request.

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Conflicts of Interest

The authors declare no conflicts of interest.

AI Use Statement

The authors declare that no generative AI was used in the preparation of this manuscript.

Ethical Approval and Consent to Participate

Not applicable.

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Jing Han received her Ph.D. degree in systems science from the College of Science, Wuhan University of Science and Technology, in June 2023. In September 2023, she joined Wuhan Business University. She is currently a Lecturer with the School of Information Engineering, Wuhan Business University, China. Her current research interests include stability analysis of dynamic systems, synchronization, and robust control for neural networks (Email: hjhust2014@163.com)

Wei Liu received his Ph.D. degree in mathematics from the School of Mathematics and Statistics, Xi'an Jiaotong University in 2019. He is currently a master supervisor in the School of Information Management and Mathematics, Jiangxi University of Finance and Economics, Nanchang, China. He has published over 20 international journal papers, he received the 2012 Outstanding Master's Degree Thesis Award of Hubei Province. His current research interests include nonlinear dynamics and mathematical biology. (Email: weiliu@jxufe.edu.cn)