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Robust Decentralized Dissipative Control Design for Uncertain Fractional-Order Interconnected Systems via Non-Fragile State Feedback

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Abstract

This study examines the problem of decentralized non-fragile dissipative control for a category of fractional-order linear uncertain large-scale systems. We assume that the subsystems uncertainty is norm-bounded and time-varying. Furthermore, we assume that the state-feedback gains for subsystems of the fractional-order large-scale system have norm-bounded controller gain variations. To achieve our goal, we introduce the concept of $(S, Q, R)-$dissipativity for fractional-order interconnected systems. By using this definition and mathematical transformations with fractional calculus, a decentralized non-fragile state-feedback controller is designed so that the closed-loop interconnected systems become asymptotically stable and satisfy a dissipative performance index. By utilizing the fractional-order Lyapunov method, sufficient LMI-based conditions are obtained to guarantee the existence of the desired controllers. Additionally, we provide a parameterized characterization of the robust non-fragile dissipative controller in terms of feasible solutions to certain LMIs. Finally, a numerical example and simulation results are provided to demonstrate the effectiveness of the proposed control design approach.

Keywords: decentralized non-fragile dissipative control, Fractional-order interconnected systems, fractional-order lyapunov method, linear matrix inequalities.

1 Introduction

In recent decades, considerable attention has been devoted to analyzing the stability and designing controls for large-scale systems. This motivation arises from the fact that many real-world engineering systems, including process control systems and communication networks, exhibit interconnected large-scale structures, transportation systems, and economic systems, that are made up of a large number of interconnected dynamical systems [1]. In many modern engineering applications, including smart grids, multi-agent robotic systems, large-scale industrial plants, and distributed sensor networks, decentralized control architectures are particularly

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desirable due to structural complexity, communication constraints, scalability requirements, and reliability considerations. Therefore, developing decentralized control strategies that guarantee both stability and performance for interconnected systems is of significant practical importance. Many interesting results have been achieved for integer-order large-scale interconnected systems [2–7, 9–11]. For instance, Thanh and Phat [5] addressed the decentralized H_∞ control problem for nonlinear time-delay systems within large-scale interconnected frameworks, utilizing a combination of Lyapunov-Krasovskii functional methodology and techniques based on linear matrix inequalities. Researchers in [6] explored decentralized stability issues in large-scale nonlinear systems with switching dynamics and time-varying delays, employing an effective integration of Lyapunov functional techniques integrated with the Newton–Leibniz formula and free-weighting matrix methods. Various studies in the literature have approached guaranteed cost control challenges for diverse integer-order large-scale interconnected systems, including those involving large-scale singular systems with interconnected delays [2], and uncertain systems with time delays in subsystem interconnections [4, 9, 11]. Furthermore, reference [7] addressed the design of finite-time decentralized non-fragile dissipative controllers for large-scale systems in the presence of actuator faults and saturation. Reference [3] delved into the design of distributed event-triggered state and unknown disturbances observers for a particular class of nonlinear interconnected systems, characterized by local state vectors affected by unknown delays and subject to the Lipschitz condition on the nonlinear function.

Fractional-order systems provide precise descriptions for various physical phenomena, including quantum mechanics [8], fluid mechanics [12], electromagnetics [25], and fractional-order neural networks [23]. Compared with integer-order models, fractional-order dynamics are capable of capturing memory and hereditary effects, which makes them more suitable for modeling viscoelastic materials, anomalous diffusion processes, and complex biological and networked systems. As a result, stability and control problems for interconnected large-scale systems governed by fractional-order models have attracted increasing attention. Recently, some researchers have directed their focus towards fractional-order interconnected systems. In [20], the authors employed definitions of positive and negative definite matrices alongside stability theory for fractional-order systems to establish several sufficient conditions ensuring the asymptotic stability of perturbed fractional-order linear interconnected systems in closed-loop configurations. Reference [24] adopted a transfer-function framework to specify the H_∞ performance from disturbance to output, formulating a decentralized H_∞ controller synthesis problem cast as a convex optimization problem under linear matrix inequality constraints. Although these works have significantly advanced decentralized stability and H_∞ control for fractional-order interconnected systems, they mainly focus on stability analysis or specific performance indices. The more general dissipativity framework, which unifies H_∞ performance, passivity, and energy-based supply rate characterizations, has not been systematically explored in the decentralized fractional-order interconnected setting.

Reference [14] investigated the functional observer design technique of a large-scale fractional-order interconnected systems, presenting a method for solving observer gain matrices using LMIs. Reference [26] focused on fully semi-decentralized and development of decentralized sliding mode control strategies for fractional-order nonlinear interconnected systems. The problem of stochastic stability and decentralized control for interconnected fractional-order stochastic systems with input saturation was addressed in [32], where the authors developed a decentralized observer-based partial variable control technique to solve the problem. In another study, the design of distributed functional observers for fractional-order interconnected systems with time-varying parameters and delays was investigated. In particular, a set of N observers was constructed to estimate the states of the corresponding N subsystems. Subsequently, conditions for the existence of such functional observers were established, along with effective algorithms for computing unknown observer matrices. Reference [16] delved into stabilization for fractional-order interconnected systems using an event-trigger control approach, establishing a new condition for the existence of state feedback controllers ensuring asymptotic stability through linear matrix inequalities.

Recently, more advanced control and estimation schemes have been proposed for Markovian and semi-Markovian jumping neural networks. For instance, an event-triggered resilient asynchronous estimation framework for stochastic Markovian jumping networks with missing measurements was developed in [31], where a co-design control approach was formulated to achieve improved disturbance attenuation despite communication limitations. In addition, stabilization of semi-Markovian jumping uncertain networks with time-varying delays was investigated via sliding-mode control in [22], where the memoryless restriction of classical Markov processes was relaxed. Moreover, a non-fragile asynchronous H_∞ estimation scheme for piecewise-homogeneous Markovian jumping neural networks with partly available transition rates was proposed in [21], employing a dynamic event-triggered mechanism to reduce communication burden while maintaining robust performance.

Notably, in these studies [14, 16, 20, 24, 26, 32], interconnected subsystems were consolidated into single systems with block matrices, enabling stability analyses and control design based on existing results for single systems. However, a systematic decentralized control framework that explicitly integrates fractional-order dynamics with general energy-based performance specifications and non-fragile design under uncertainties remains insufficiently developed. An intriguing question arises regarding how to design decentralized control utilizing information from interconnected subsystems to address the control design problem, which remains a challenging yet important area for study.

Recently, recent studies have shown significant interest in the dissipativity problem due to its relevance in various physical systems such as cooperative multi-agent systems, hybrid dynamic processes, nonlinear delayed systems, and cyber–physical control systems [30]. Passivity and its broader concept of dissipativity play a significant role in measuring the energy consumption of a system and have applications in fields including electrical, mechanical, chemical, and communication systems. The analysis and control design for dissipative systems have been extensively explored, particularly in fractional-order systems [15, 17, 28, 29, 33]. Although recent works such as [21, 22, 31] have made substantial progress in asynchronous estimation, sliding-mode stabilization, and non-fragile H_∞ control for Markovian or semi-Markovian jumping neural networks, these studies mainly concern neural network structures rather than general large-scale interconnected fractional-order systems under a unified (S, Q, R)-dissipativity framework.

From the perspective of current research, the problem of dissipative control for fractional-order interconnected systems with uncertainties has not been comprehensively addressed in the literature. This inspired us to study the problem of decentralized non-fragile dissipative control for interconnected fractional-order systems with uncertainties. Compared with the above-mentioned asynchronous estimation and robust control schemes developed for neural and Markovian jumping systems [21, 22, 31], the present work further extends the analysis to fractional-order large-scale interconnected systems and establishes decentralized non-fragile dissipative control conditions under parametric uncertainties. The necessity of this study lies in bridging the gap between decentralized fractional-order control and general dissipativity-based performance analysis while explicitly incorporating non-fragile controller design to enhance robustness in practical implementations.

The major findings and contributions of this paper are outlined as follows:

1. We introduced the concept of $(S, Q, R)-$dissipativity for fractional-order interconnected systems and solved the problem of decentralized non-fragile dissipative control for the considered fractional-order large-scale systems with parametric uncertainties in the system matrices. This extends existing decentralized stability and H_∞ results to a more general energy-based performance framework.
2. Decentralized dissipative controller design of fractional-order large-scale systems has been a challenge due to the lack of effective dissipativity analysis methods for fractional-order interconnected systems. We developed decentralized non-fragile control to ensure the closed-loop fractional-order interconnected system is asymptotically stable for all admissible uncertainties and satisfies a dissipative performance index. The proposed design explicitly incorporates controller gain perturbations, thereby improving robustness against implementation inaccuracies.
3. We presented the obtained conditions in the form of tractable strict linear LMIs, and their solutions provide a parameterized form of the controller. These LMI conditions can be handled effectively through available convex optimization methods. Therefore, the proposed framework is computationally efficient and applicable to practical large-scale fractional-order

interconnected systems.

Notation: In this work, $\mathbb{R}^n$ refers to the n-dimensional Euclidean space, while $\mathbb{R}^{m\times n}$ denotes the collection of real matrices of size $m \times n$. Moreover, A^T represents the transposed matrix of A, and $\mathrm{diag}\{\dots\}$ represents a block-diagonal matrix. Given a matrix $P \in \mathbb{R}^{n\times n}$, the inequality $P > 0$ ($P < 0$) implies that P is symmetric positive definite (symmetric negative definite). The notations $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ represent the maximum and minimum eigenvalues of P, respectively. Moreover, $C([a,b],\mathbb{R}^n)$ represents the Banach space of $\mathbb{R}^n$-valued continuous functions defined on $[a,b]$. The notation $AC([0,+\infty),\mathbb{R})$ refers to the class of real-valued functions that are absolutely continuous on $[0,+\infty)$. For a positive integer n, $AC^n([0,+\infty),\mathbb{R})$ designates the set of real-valued functions f whose derivatives up to order $n-1$ are continuous on $[0,+\infty)$ and whose $(n-1)$-th derivative belongs to $AC([0,+\infty),\mathbb{R})$. In addition, The symbol $*$ represents the symmetric terms in a block matrix.

2 Problem formulation and preliminaries

To begin with, we present several fundamental concepts and essential properties related to the Riemann–Liouville fractional integral and the Caputo fractional derivative. These concepts are fundamental and will be extensively explored throughout the paper.

Definition 1 [18] *Let $\alpha > 0$. The fractional integral of order α, defined in the Riemann–Liouville framework for the function $h(\cdot)$, takes the form:*

$$ {}_{t_0}\mathtt{I}_t^{\alpha} h(t) = \int_{t_0}^{t} \frac{1}{\Gamma(\alpha)} (t-s)^{\alpha-1} h(s) ds, $$

$t > t_0, \alpha > 0$, where $\Gamma(s) = \int_0^\infty \tau^{s-1} e^{-\tau} d\tau$ denotes the gamma function. When $\alpha = n \in \mathbb{N}$, the above definition coincides with the n-th integrals of the form

$$ {}_{t_0}\mathtt{I}_t^{n} h(t) = \int_{t_0}^{t} \frac{1}{(n-1)!} (t-s)^{n-1} h(s) ds. $$

Definition 2 [18] *For $\alpha > 0$, the Riemann–Liouville fractional derivative of the function $h(t)$ is defined by the following equation:*

$$ {}_{t_0}\mathtt{D}_t^{\alpha} h(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \left(\int_a^t (t-s)^{n-\alpha-1} h(s)\, ds \right), $$

$\forall t > t_0$, where $n = [\alpha] + 1$, and $[\alpha]$ represents the integer part of α. In the case where $0 < \alpha < 1$, we have:

$$ {}_{t_0}\mathtt{D}_t^{\alpha} h(t) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \left(\int_a^t (t-s)^{-\alpha} h(s)\, ds \right), \ t > t_0. $$

Definition 3 [18] *The Caputo derivative of order $\alpha > 0$ for the function $h(\cdot)$ can be defined as:*

$$ {}_{t_0}^{C}\mathtt{D}_t^{\alpha} h(t) = {}_{t_0}\mathtt{D}_t^{\alpha}\Big(h(t) - \sum_{k=0}^{n-1} \frac{h^{(k)}(t_0)}{k!} (t-t_0)^k \Big), $$

where $n = [\alpha] + 1$ if $\alpha \notin \mathbb{N}$, and $n = \alpha$ otherwise. In particular, when $0 < \alpha < 1$, then

$$ {}_{t_0}^{C}\mathtt{D}_t^{\alpha} h(t) = {}_{t_0}\mathtt{D}_t^{\alpha}\big(h(t) - h(t_0) \big). $$

Several fundamental properties of fractional-order calculus, which will be needed in what follows, are stated below.

Property 1 [18]: If $h(t) \in AC([0,+\infty),\mathbb{R})$ and $0 < \alpha \le 1$, then

$$ {}_0\mathtt{I}_t^{\alpha} \left({}_0^{C}\mathtt{D}_t^{\alpha} h(t) \right) = h(t) - h(0), \qquad t > 0. $$

Property 2 [18]: If $h(t) \in AC^n([0,+\infty),\mathbb{R})$, then

$$ {}_0\mathtt{I}_t^{\alpha_1} \left({}_0\mathtt{I}_t^{\alpha_2} h(t) \right) = {}_0\mathtt{I}_t^{\alpha_2} \left({}_0\mathtt{I}_t^{\alpha_1} h(t) \right) = {}_0\mathtt{I}_t^{\alpha_1+\alpha_2} (h(t)), $$

$\forall t \ge 0,\ \alpha_1, \alpha_2 > 0.$

A class of uncertain fractional-order interconnected systems comprising N coupled subsystems $\mathcal{S}_i$ ($i = 1, 2, \dots, N$) is considered, and its dynamics are given by

$$ \mathcal{S}_i : \begin{cases} {}_0^{C}\mathtt{D}_t^{\alpha} x_i(t) = [A_i + \Delta A_i(t)] x_i(t) \\ \qquad + \sum\limits_{j=1, j\neq i}^{N} [D_{ij} + \Delta D_{ij}(t)] x_j(t) \\ \qquad + [W_i + \Delta W_i(t)] \omega_i(t) + B_i u_i(t), t \ge 0, \\ z_i(t) = C_i x_i(t) + E_i \omega_i(t), \\ x_i(0) = x_i^0, \end{cases} \tag{1} $$

in which, the parameter $\alpha \in (0,1)$ denotes the fractional commensurate order. x_i^0 is the initial condition, $x_i(t) \in \mathbb{R}^{n_i}$ represent the state vector, $u_i(t) \in \mathbb{R}^{m_i}$ represent the control input, $\omega_i(t) \in \mathbb{R}^{p_i}$ and $z_i(t) \in \mathbb{R}^{q_i}$ represent the disturbance input, and performance output of the ith subsystems. The real matrices $A_i \in \mathbb{R}^{n_i\times n_i}, D_{ij} \in \mathbb{R}^{n_i\times n_i}, W_i \in \mathbb{R}^{n_i\times p_i}, B_i \in \mathbb{R}^{n_i\times m_i}, C_i \in \mathbb{R}^{q_i\times n_i}$ and $E_i \in \mathbb{R}^{q_i\times p_i}$ are given. The matrices D_{ij} and $\Delta D_{ij}(t)$ represent the interconnections from subsystem j to subsystem i. The matrices $\Delta A_i(t), \Delta W_i(t)$ and $\Delta D_{ij}(t)$ represent time-invariant uncertainties in the state, disturbance input, and interconnection matrices, respectively. Assuming that the uncertain matrices $\Delta A_i(t), \Delta W_i(t)$ and $\Delta D_{ij}(t)$ follow a certain form, the matrices can be represented in the following way:

$$ \Delta A_i(t) = G_i^a F_i^a(t) H_i^a,\ \Delta D_{ij}(t) = G_{ij}^d F_{ij}^d(t) H_{ij}^d, \tag{2} $$

$$\Delta W_i(t) = G_i^w F_i^w(t) H_i^w,$$

where $F_i^a(t), F_{ij}^d(t)$, and $F_i^w(t)$ are real uncertain matrices whose dimensions are compatible, and which fulfill the following conditions:

$$(F_i^a(t))^T F_i^a(t) \leq I, (F_{ij}^d(t))^T F_{ij}^d(t) \leq I, \tag{3}$$

$$(F_i^w(t))^T F_i^w(t) \leq I,$$

$G_i^a, G_{ij}^d, G_i^w, H_i^a, H_{ij}^d$, and H_i^w denote predefined real constant matrices with suitable dimensions, characterizing the structural embedding of the uncertainties in $F_i^a(t)$, $F_{ij}^d(t)$, and $F_i^w(t)$ modify the entries of the nominal system matrices A_i, D_{ij}, and W_i in the system.

Without a control input vector, system (1) transforms into

$$\mathcal{S}_i : \begin{cases} {}_0^C\mathrm{D}_t^\alpha x_i(t) = [A_i + \Delta A_i(t)]x_i(t) \\ \qquad + \sum\limits_{j=1, j\neq i}^{N} [D_{ij} + \Delta D_{ij}(t)]x_j(t) \\ \qquad +[W_i + \Delta W_i(t)]\omega_i(t), t \geq 0, \\ z_i(t) = C_i x_i(t) + E_i \omega_i(t), \\ x_i(0) = x_i^0. \end{cases} \tag{4}$$

Definition 4 *System (4) is said to be strictly $(Q_i, S_i, R_i)-$dissipative if the following two requirements are fulfilled:*
(i) When the disturbance input and performance output vectors are removed, system (4) is asymptotically stable.
(ii) With zero initial conditions, for every $t > 0$ and $\omega_i(t) \in L_2([0,\infty), \mathbb{R}^{p_i})$, there exists $\gamma > 0$ ensuring that the inequality below is fulfilled:

$$\sum_{i=1}^{N} \int_0^t \big(z_i^T(s) Q_i z_i(s) + 2 z_i^T(s) S_i \omega_i(s) + \omega_i^T(s) R_i \omega_i(s)\big) ds$$

$$> \gamma \sum_{i=1}^{N} \int_0^t \omega_i^T(s)\omega_i(s) ds,$$

where $S_i, Q_i, R_i, (i = 1, 2, \ldots, N)$ represent real matrices with $Q_i, R_i, (i = 1, 2, \ldots, N)$, are symmetric, $Q_i \leq 0, (i = 1, 2, \ldots, N)$.

We assume that each subsystem's controller $u_i(t)$ solely utilizes its local state $x_i(t)$. We employ a decentralized non-fragile state feedback controller represented by

$$u_i(t) = (K_i + \Delta K_i(t)) x_i(t), \tag{5}$$

in which $K_i \in \mathbb{R}^{m_i \times n_i}$ denotes the fixed gain, to be designed later, and $\Delta K_i(t)$ represents an a priori norm-bounded gain variation given by

$$\Delta K_i(t) = G_i^k F_i^k(t) H_i^k,$$

where G_i^k and H_i^k are known constant real matrices with appropriate dimensions. The matrix $F_i^k(t)$ is unknown and real, satisfying the condition $(F_i^k(t))^T F_i^k(t) \leq I$. Combining (1) and (5), we obtained the resulting fractional-order interconnected system in closed-loop form with uncertainties

$$\mathcal{S}_i : \begin{cases} {}_0^C\mathrm{D}_t^\alpha x_i(t) = [A_i + \Delta A_i(t) + B_i K_i \\ \qquad + B_i G_i^k F_i^k(t) H_i^k] x_i(t) \\ \qquad + \sum\limits_{j=1}^{N} [D_{ij} + \Delta D_{ij}(t)] x_j(t) \\ \qquad + [W_i + \Delta W_i(t)]\omega_i(t), t \geq 0, \\ z_i(t) = C_i x_i(t) + E_i \omega_i(t), \\ x_i(0) = x_i^0. \end{cases} \tag{6}$$

The objective of this paper is to construct a decentralized non-fragile state feedback controller, denoted by equation (5), such that (6), is strictly $(Q_i, S_i, R_i)-$ dissipative.

The lemmas presented below are essential for the analysis that follows.

Lemma 1 [13] *Given arbitrary vectors $x, y \in \mathbb{R}^n$ and a symmetric positive definite matrix $\mathcal{S} \in \mathbb{R}^{n\times n}$, one has:*

$$\pm 2x^T y \leqslant x^T \mathcal{S} x + y^T \mathcal{S}^{-1} y.$$

Lemma 2 [27] *Consider a symmetric positive definite matrix $\mathcal{S} \in \mathbb{R}^{n\times n}$ and a continuously differentiable vector function $x(t) \in \mathbb{R}^n$. Then, for all $\alpha \in (0,1)$ and $t \geq 0$, the following inequality is valid:*

$${}_0^C\mathrm{D}_t^\alpha (x^T(t) \mathcal{S} x(t)) \leq x^T(t) \mathcal{S}\, {}_0^C\mathrm{D}_t^\alpha x(t) + ({}_0^C\mathrm{D}_t^\alpha x(t))^T \mathcal{S} x(t).$$

Lemma 3 [19] *Consider a non-autonomous Caputo fractional-order system ${}_{t_0}^C\mathrm{D}_t^\alpha x(t) = g(t, x(t))$ and assume that the origin $x = 0$ is an equilibrium point. This equilibrium point is said to be asymptotically stable if one can determine positive constants such that $\beta_1, \beta_2, \beta_3, a, b$, and $V(t, x(t))$ be a continuously differentiable function satisfying the following conditions:*

$$\beta_1 \|x\|^a \leq V(t, x(t)) \leq \beta_2 \|x\|^{ab} \tag{7}$$

$${}_{t_0}^C D_t^\alpha V(t, x(t)) \leq -\beta_3 \|x\|^{ab}, \tag{8}$$

in which $\alpha \in (0,1)$, and $V(t, x(t))$ denotes a real-valued function defined on $[0,\infty) \times D$ is a function that satisfies a locally Lipschitz condition on x, where $D \subset \mathbb{R}^n$ is a domain including the origin.
If these conditions are fulfilled for all $x \in \mathbb{R}^n$, then $x = 0$ is globally asymptotically stable.

3 Decentralized non-fragile dissipative controller design

This section establishes sufficient criteria for the construction of the decentralized state-feedback controller (5) such that the closed-loop system (6) is strictly (Q_i, S_i, R_i)-dissipative. Before presenting the main result, we define the following notation for several matrix variables to simplify the discussion.

$$\mathcal{D}_i = \sum_{j\neq i, j=1}^{N} D_{ij}D_{ij}^T,\ \mathcal{G}_i = \sum_{j\neq i, j=1}^{N} G_{ij}^d(G_{ij}^t)^T,$$

$$\mathcal{H}_i = \sum_{j\neq i, j=1}^{N} (I + (H_{ij}^d)^T H_{ij}^d).$$

Theorem 1 *For given constant matrices Q_i, S_i, R_i with $Q_i \leq 0, Q_i = Q_i^T$ and $R_i = R_i^T$. The interconnected system (1) with the decentralized non-fragile state feedback controller (5) is strictly $(Q_i, S_i, R_i)-$dissipative if there exist symmetric positive definite matrices P_i, matrices Y_i, and positive scalars $\epsilon_i, \beta_i, i = 1, 2, \ldots, N$, for which the following linear matrix inequalities are feasible*

$$\Xi^i = \begin{bmatrix} \Xi_{11}^i & \Xi_{12}^i & P_i(H_i^k)^T & P_i(H_i^a)^T & P_i\mathcal{H}_i & P_iC_i^T \\ * & \Xi_{22}^i & 0 & 0 & 0 & 0 \\ * & * & -\epsilon_i I & 0 & 0 & 0 \\ * & * & * & -\beta_i I & 0 & 0 \\ * & * & * & * & -\mathcal{H}_i & 0 \\ * & * & * & * & * & -\hat{Q}_i \end{bmatrix} < 0, \tag{9}$$

$$i = 1, 2, \ldots, N,$$

where

$$\Xi_{11}^i = A_iP_i + P_iA_i^T + B_iY_i + Y_i^TB_i^T + \epsilon_i B_i G_i^k(G_i^k)^T B_i^T + \beta_i G_i^a\left(G_i^a\right)^T + \mathcal{D}_i + \mathcal{G}_i + G_i^w\left(G_i^w\right)^T,$$

$$\Xi_{12}^i = \left[W_i - P_iC_i^TS_i - PC_i^TQ_iE_i\right],$$

$$\Xi_{22}^i = \left(H_i^w\right)^T H_i^w - R_i + \gamma I - E_i^TS_i - S_i^TE_i - E_i^TQ_iE_i,$$

$$\hat{Q}_i = -Q_i.$$

Moreover, a decentralized non-fragile dissipative controller gain matrix is determined by

$$K_i = Y_iP_i^{-1}, i = 1, 2, \ldots, N.$$

Proof For the stability analysis of the closed-loop system, we consider the Lyapunov function:

$$V(t, x(t)) = \sum_{i=1}^{N} V_i(t, x_i(t)) = \sum_{i=1}^{N} x_i^T(t)P_i^{-1}x_i(t).$$

It follows directly that

$$\kappa_1\|x(t)\|^2 \leq V(t, x(t)) \leq \kappa_2\|x(t)\|^2, \tag{10}$$

where $\|x(t)\| = \sqrt{\sum_{i=1}^{N}\|x_i(t)\|^2}$,

$$\kappa_1 = \min_{i=1,\ldots,N}\{\lambda_{\min}(P_i^{-1})\}, \kappa_2 = \max_{i=1,\ldots,N}\{\lambda_{\max}(P_i^{-1})\}.$$

Evaluating the fractional-order derivative of $V(t, x(t))$ along the solution of (6) and using Lemma 2, we can obtain the following estimates

$$\begin{aligned}
{}_0^C\mathrm{D}_t^\alpha V(t, x(t)) &\leq \sum_{i=1}^{N} 2x_i^T(t)P_i^{-1}\,{}_0^C\mathrm{D}_t^\alpha x_i(t) \\
&= \sum_{i=1}^{N}\Big(x_i^T(t)[P_i^{-1}A_1 + P_i^{-1}B_iK_i \\
&\quad + A_i^TP_i^{-1} + K_i^TB_i^TP_i^{-1}]x_i(t) \\
&\quad + 2x_i^T(t)P_i^{-1}B_iG_i^kF_i^k(t)H_i^kx_i(t) \\
&\quad + 2x_i^T(t)P_i^{-1}G_i^aF_i^a(t)H_i^ax_i(t) \\
&\quad + 2x_i^T(t)P_i^{-1}\sum_{j=1, j\neq i}^{N}(D_{ij} + G_{ij}^dF_{ij}^d(t)H_{ij})x_j(t) \\
&\quad + 2x_i^T(t)P_i^{-1}W_i\omega_i(t) \\
&\quad + 2x_i^T(t)P_i^{-1}G_i^wF_i^w(t)H_i^w\omega_i(t)\Big).
\end{aligned} \tag{11}$$

By utilizing Lemma 1 along with the conditions (2) and (3), we can derive the following result:

$$\begin{aligned}
&\sum_{i=1}^{N} 2x_i^T(t)P_i^{-1}B_iG_i^kF_i^k(t)H_i^kx_i(t) \\
&\leq \sum_{i=1}^{N}\left[\epsilon_ix_i^T(t)P_i^{-1}B_iG_i^k\left(G_i^k\right)^TB_i^TP_i^{-1}x_i(t)\right. \\
&+ \left.\epsilon_i^{-1}x_i^T(t)(H_i^k)^T(F_i^k(t))^TF_i^k(t)H_i^kx_i(t)\right] \\
&\leq \sum_{i=1}^{N}\left[\epsilon_ix_i^T(t)P_i^{-1}B_iG_i^k\left(G_i^k\right)^TB_i^TP_i^{-1}x_i(t)\right. \\
&+ \left.\epsilon_i^{-1}x_i^T(t)(H_i^k)^TH_i^kx_i(t)\right] \\
&= \sum_{i=1}^{N} x_i^T(t)\left[\epsilon_iP_i^{-1}B_iG_i^k\left(G_i^k\right)^TB_i^TP_i^{-1} + \right. \\
&\left.\epsilon_i^{-1}(H_i^k)^TH_i^k\right]x_i(t),
\end{aligned} \tag{12}$$

$$\sum_{i=1}^{N} 2x_i^T(t)P_i^{-1}G_i^a F_i^a(t)H_i^a x_i(t)$$
$$\leq \sum_{i=1}^{N} [\beta_i x_i^T(t)P_i^{-1}G_i^a(G_i^a)^T P_i^{-1}x_i(t)$$
$$+ \beta_i^{-1}x_i^T(t)(H_i^a)^T H_i^a x_i(t)]$$
$$= \sum_{i=1}^{N} x_i^T(t)[\beta_i P_i^{-1}G_i^a(G_i^a)^T P_i^{-1} + \beta_i^{-1}(H_i^a)^T H_i^a]x_i(t), \tag{13}$$

$$\sum_{i=1}^{N} 2x_i^T(t)P_i^{-1} \sum_{j=1,j\neq i}^{N} D_{ij}x_j(t)$$
$$\leq \sum_{i=1}^{N} (x_i^T(t)P_i^{-1} \sum_{j=1,j\neq i}^{N} D_{ij}D_{ij}^T P_i^{-1}x_i(t)$$
$$+ \sum_{j=1,j\neq i}^{N} x_j^T(t)x_j(t))$$
$$= \sum_{i=1}^{N} [x_i^T(t)P_i^{-1}\mathcal{D}_i P_i^{-1}x_i(t) + \sum_{j=1,j\neq i}^{N} x_j^T(t)x_j(t)], \tag{14}$$

$$\sum_{i=1}^{N} 2x_i^T(t)P_i^{-1} \sum_{j=1,j\neq i}^{N} G_{ij}^d F_{ij}^d(t)H_{ij}x_j(t)$$
$$\leq \sum_{i=1}^{N} (x_i^T(t)P_i^{-1} \sum_{j=1,j\neq i}^{N} G_{ij}^d(G_{ij}^d)^T P_i^{-1}x_i(t)$$
$$+ \sum_{j=1,j\neq i}^{N} x_j^T(t)(H_{ij}^d)^T H_{ij}^d x_j(t)) \tag{15}$$
$$= \sum_{i=1}^{N} [x_i^T(t)P_i^{-1}\mathcal{G}_i P_i^{-1}x_i(t)$$
$$+ \sum_{j=1,j\neq i}^{N} x_j^T(t)(H_{ij}^d)^T H_{ij}^d x_j(t)],$$

$$\sum_{i=1}^{N} 2x_i^T(t)P_i^{-1}G_i^w F_i^w(t)H_i^w \omega_i(t)$$
$$\leq \sum_{i=1}^{N} (x_i^T(t)P_i^{-1}G_i^w(G_i^w)^T P_i^{-1}x_i(t) \tag{16}$$
$$+ \omega_i^T(t)(H_i^w)^T H_i^w \omega_i(t)).$$

Substituting conditions (12)–(16) into (13), we obtain

$$
\begin{aligned}
&{}_0^C\mathrm{D}_t^\alpha V(t, x(t)) \\
&\leq \sum_{i=1}^{N} \Big(x_i^T(t)\Big(P_i^{-1}A_i + A_i^T P_i^{-1} + P_i^{-1}B_i K_i \\
&+ K_i^T B_i^T P_i^{-1} + \epsilon_i^{-1}(H_i^k)^T H_i^k \\
&+ \epsilon_i P_i^{-1}B_i G_i^k(G_i^k)^T B_i^T P_i^{-1} + \beta_i P_i^{-1}G_i^a(G_i^a)^T P_i^{-1} \\
&+ \beta_i^{-1}(H_i^a)^T H_i^a + P_i^{-1}\mathcal{D}_i\mathcal{D}_i^T P_i^{-1} \\
&+ P_i^{-1}\mathcal{G}_i P_i^{-1} + P_i^{-1}G_i^w(G_i^w)^T P_i^{-1}\Big) x_i(t) \\
&+ \omega_i^T(t)(H_i^w)^T H_i^w \omega_i(t) + 2x_i^T(t)P_i^{-1}W_i\omega_i(t) \\
&+ \sum_{j=1,j\neq i}^{N} x_j^T(t)[I + (H_{ij}^d)^T H_{ij}^d]x_j(t)\Big).
\end{aligned} \tag{17}
$$

Note that

$$
\begin{aligned}
&\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} x_j^T(t)[I + (H_{ij}^d)^T H_{ij}^d]x_j(t) \\
&= \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} x_i^T(t)[I + (H_{ji}^d)^T H_{ji}^d]x_i(t) \\
&= \sum_{i=1}^{N} x_i^T(t)[\sum_{j=1,j\neq i}^{N} [I + (H_{ji}^d)^T H_{ji}^d]]x_i(t) \\
&= \sum_{i=1}^{N} x_i^T(t)\mathcal{H}_i x_i(t).
\end{aligned} \tag{18}
$$

Setting $J_i = 2z_i^T(t)S_i\omega_i(t) + z_i^T(t)Q_i z_i(t) + \omega_i^T(t)(R_i - \gamma I)\omega_i(t), i = 1, 2, \ldots, N$. By some simple direct calculations, we obtain

$$
\begin{aligned}
J_i &= 2[x_i^T(t)C_i^T + \omega_i^T(t)E_i^T]S_i\omega_i(t) \\
&\quad + [x_i^T(t)C_i^T + \omega_i^T(t)E_i^T]Q_i[C_i x_i(t) + E_i\omega_i(t)] \\
&\quad + \omega_i^T(t)(R_i - \gamma I)\omega_i(t) \\
&= x_i^T(t)C_i^T Q_i C_i x_i(t) + \omega_i^T(t)[R_i - \gamma I + E_i^T S_i \\
&\quad + S_i^T E_i + E_i^T Q_i E_i]\omega_i(t) \\
&\quad + 2x_i^T(t)C_i^T S_i\omega_i(t) + 2x_i^T(t)C_i^T Q_i E_i\omega_i(t).
\end{aligned} \tag{19}
$$

Combining conditions (17), (18), and (19), we obtain

$$
\begin{aligned}
&{}_0^C\mathrm{D}_t^\alpha V(t,x(t)) = \sum_{i=1}^N {}_0^C\mathrm{D}_t^\alpha V_i(t,x(t))\\
&\le \sum_{i=1}^N \Big(x_i^T(t)\Big) P_i^{-1}A_i + A_i^T P_i^{-1} + P_i^{-1}B_iK_i\\
&\quad + K_i^T B_i^T P_i^{-1} + \epsilon_i^{-1}(H_i^k)^T H_i^k\\
&\quad + \epsilon_i P_i^{-1}B_iG_i^k(G_i^k)^T B_i^T P_i^{-1} + \beta_i P_i^{-1}G_i^a(G_i^a)^T P_i^{-1}\\
&\quad + \beta_i^{-1}(H_i^a)^T H_i^a\\
&\quad + P_i^{-1}\mathcal{D}_iP_i^{-1} + P_i^{-1}\mathcal{G}_iP_i^{-1} + P_i^{-1}G_i^w(G_i^w)^T P_i^{-1}\\
&\quad + \mathcal{H}_i - C_i^TQ_iC_i\Big(x_i(t)\\
&\quad + \omega_i^T(t)\big[(H_i^w)^T H_i^w - R_i + \gamma I\\
&\quad - E_i^TS_i - S_i^TE_i - E_i^TQ_iE_i\big]\omega_i(t)\\
&\quad + 2x_i^T(t)\big[P_i^{-1}W_i - C_i^TS_i - C_i^TQ_iE_i\big]\omega_i(t) + J_i\Big)\\
&= \sum_{i=1}^N \Big(\xi_i^T(t)\Omega^i\xi_i(t) + J_i\Big),
\end{aligned}
\tag{20}
$$

where

$$
\begin{aligned}
&\xi_i^T(t) = \begin{bmatrix} x_i^T(t) & \omega_i^T(t)\end{bmatrix},\\
&\Omega^i = \begin{bmatrix}\Omega_{11}^i & \Omega_{12}^i\\ * & \Omega_{22}^i\end{bmatrix},\\
&\Omega_{11}^i = P_i^{-1}A_i + A_i^TP_i^{-1} + P_i^{-1}B_iK_i + K_i^TB_i^TP_i^{-1}\\
&\quad + \epsilon_i^{-1}(H_i^k)^TH_i^k + \epsilon_iP_i^{-1}B_iG_i^k(G_i^k)^TB_i^TP_i^{-1}\\
&\quad + \beta_iP_i^{-1}G_i^a(G_i^a)^TP_i^{-1} + \beta_i^{-1}(H_i^a)^TH_i^a\\
&\quad + P_i^{-1}\mathcal{D}_iP_i^{-1} + P_i^{-1}\mathcal{G}_iP_i^{-1} + P_i^{-1}G_i^w(G_i^w)^TP_i^{-1}\\
&\quad + \mathcal{H}_i - C_i^TQ_iC_i,\\
&\Omega_{22}^i = (H_i^w)^TH_i^w - R_i + \gamma I - E_i^TS_i - S_i^TE_i\\
&\quad - E_i^TQ_iE_i,\\
&\Omega_{12}^i = P_i^{-1}W_i - C_i^TS_i - C_i^TQ_iE_i.
\end{aligned}
$$

Hence

$$
\begin{aligned}
{}_0^C\mathrm{D}_t^\alpha V(t,x(t)) &= \sum_{i=1}^N \big({}_0^C\mathrm{D}_t^\alpha V_i(t,x(t))\big)\\
&\le \sum_{i=1}^N \Big(\xi_i^T(t)\Omega^i\xi_i(t) + J_i\Big).
\end{aligned}
\tag{21}
$$

We will present the proof in two steps.

Step 1. We will show that (6) with $\omega_i(t) = 0, z_i(t) = 0 (i = 1, 2, ..., N)$, is asymptotically stable. Since $\omega_i(t) = 0$ and $z_i(t) = 0$, we have $J_i = 0$. Using equation (21), we can conclude that

$$
{}_0^C\mathrm{D}_t^\alpha V(t,x(t)) = \sum_{i=1}^N \big({}_0^C\mathrm{D}_t^\alpha V_i(t,x(t))\big) \le \sum_{i=1}^N \xi_i^T(t)\Omega^i\xi_i(t).
\tag{22}
$$

Let's start by setting $K_i = Y_iP_i^{-1}, (i = 1, 2, \dots, N)$. By pre- and post-multiplying Ω^i by $\mathrm{diag}P_i, I$ and then applying the Schur complement lemma, condition $\Omega_i < 0$ is shown to be equivalent to (9). By using (9) and (22), we obtain

$$
{}_0^C\mathrm{D}_t^\alpha V(t,x(t)) < 0, \forall t \ge 0.
\tag{23}
$$

From conditions (10) and (23), we can deduce that the conditions stated in Lemma 3 have been met. Thus, we can conclude that the closed-loop system (6) with $\omega_i(t) = 0, z_i(t) = 0 (i = 1, 2, ..., N)$, is asymptotically stable.

Step 2. We prove that (6) is strictly $(Q_i, S_i, R_i)-$dissipative under zero-initial condition. The inequality (21) and the condition (9) ensure that

$$
\sum_{i=1}^N \big({}_0^C\mathrm{D}_t^\alpha V_i(t,x_i(t)) - J_i\big) \le \sum_{i=1}^N \xi_i^T(t)\Omega^i\xi_i(t) < 0.
\tag{24}
$$

From (24), we have

$$
\sum_{i=1}^N \big({}_0^C\mathrm{D}_t^\alpha V_i(t,x_i(t)) - J_i\big) < 0.
\tag{25}
$$

Integrating both sides of (25) from 0 to t gives

$$
\begin{aligned}
&\sum_{i=1}^N {}_0\mathrm{I}_t^1\, {}_0^C\mathrm{D}_t^\alpha V_i(t,x_i(t)) - \sum_{i=1}^N \int_0^t \big[2z_i^T(s)S_i\omega_i(s)\\
&+ z_i^T(s)Q_iz_i(s) + \omega_i^T(s)(R_i - \gamma I)\omega_i(s)\big]ds < 0.
\end{aligned}
\tag{26}
$$

By utilizing Properties 2.1 and 2.2, we make the following estimates

$$
\begin{aligned}
&\sum_{i=1}^N {}_0\mathrm{I}_t^1\, {}_0^C\mathrm{D}_t^\alpha V_i(t,x(t)) = \sum_{i=1}^N {}_0\mathrm{I}_t^{1-\alpha}\, {}_0\mathrm{I}_t^\alpha\, {}_0^C\mathrm{D}_t^\alpha V_i(t,x(t))\\
&= \sum_{i=1}^N {}_0\mathrm{I}_t^{1-\alpha}\big({}_0\mathrm{I}_t^\alpha\, {}_0^C\mathrm{D}_t^\alpha V_i(t,x_i(t))\big)\\
&= \sum_{i=1}^N {}_0\mathrm{I}_t^{1-\alpha}\big(V_i(t,x_i(t)) - V_i(0,x_i(0))\big)\\
&= \sum_{i=1}^N {}_0\mathrm{I}_t^{1-\alpha}V_i(t,x_i(t)) - \sum_{i=1}^N {}_0\mathrm{I}_t^{1-\alpha}V_i(0,x_i(0)).
\end{aligned}
$$

We obtain the following result when the initial condition is set to zero:

$$\begin{aligned}&\sum_{i=1}^{N} {}_0\mathrm{I}_t^{1-\alpha} V_i(0, x_i(0))\\&= \sum_{i=1}^{N} \int_0^t \frac{(t-s)^{-\alpha}}{\Gamma(1-\alpha)} x_i^T(0) P_i^{-1} x_i(0) ds = 0.\end{aligned} \tag{27}$$

Using Definition 1, we have

$$\begin{aligned}&\sum_{i=1}^{N} {}_0\mathrm{I}_t^{1-\alpha} V_i(t, x_i(t))\\&= \sum_{i=1}^{N} \int_0^t \frac{(t-s)^{-\alpha}}{\Gamma(1-\alpha)} x_i^T(s) P_i^{-1} x_i(s) ds \geq 0.\end{aligned} \tag{28}$$

By combining conditions (26), (27), and (28), we derive the following estimates

$$\sum_{i=1}^{N} {}_0\mathrm{I}_t^1 \, {}_0^C\mathrm{D}_t^\alpha V_i(t, x_i(t)) \geq 0. \tag{29}$$

From (26) and (29), we get

$$\begin{aligned}&\sum_{i=1}^{N} \int_0^t \big[2z_i^T(s) S_i \omega_i(s) + z_i^T(s) Q_i z_i(s)\\&+\omega_i^T(s)\big(R_i - \gamma I\big)\omega_i(s)\big] ds > 0.\end{aligned}$$

Hence

$$\begin{aligned}&\sum_{i=1}^{N} \int_0^t \big(z_i^T(s) Q_i z_i(s) + 2z_i^T(s) S_i \omega_i(s) + \omega_i^T(s) R_i \omega_i(s)\big) ds\\&> \gamma \sum_{i=1}^{N} \int_0^t \omega_i^T(s) \omega_i(s) ds.\end{aligned}$$

Thus, the closed-loop fractional-order interconnected system (6) is strictly $(Q_i, S_i, R_i)-$dissipative.

Remark 1 Several papers have explored the design of decentralized control for fractional-order interconnected systems [14, 16, 20, 24, 26, 32]. However, decentralized non-fragile dissipative control has been neglected due to the lack of significant results regarding the dissipativity of fractional-order large-scale systems. In this work, the decentralized non-fragile dissipative control problem for fractional-order interconnected systems with uncertainties is studied.

Remark 2 If the fractional-order $\alpha = 1$, system (1) becomes an integer-order interconnected system. Hence, the results of Theorem 1 on decentralized non-fragile dissipative control design still apply to integer-order large-scale systems.

Remark 3 The dissipativity condition adopted in this paper admits a clear physical interpretation from an energy perspective. For the considered fractional-order interconnected large-scale system, dissipativity characterizes a generalized energy balance relation between the internal stored energy and the supplied energy. Due to the intrinsic memory property of fractional-order dynamics, the system exhibits hereditary energy accumulation behavior. Therefore, ensuring strict (Q_i, S_i, R_i)-dissipativity implies that, despite the presence of subsystem interconnections, plant uncertainties, and controller gain perturbations, the overall closed-loop system maintains a bounded energy-like storage function and satisfies a prescribed supply rate inequality. This guarantees not only asymptotic stability but also robust performance against disturbances and implementation imperfections.

4 An illustrative example

As a numerical example, we examine system (1) consisting of two coupled subsystems ($N = 2$) to verify the feasibility of the derived LMI conditions. The system is described as

$$\mathcal{S}_1 : \begin{cases} {}_0^C\mathrm{D}_t^\alpha x_1(t) = \big[A_1 + G_1^a F_1^a(t) H_1^a\big] x_1(t) \\ \quad +\big[D_{12} + G_{12}^d F_{12}^d(t) H_{12}^d\big] x_2(t) \\ \quad +\big[W_1 + G_1^w F_1^w(t) H_1^w\big]\omega_1(t) + B_1 u_1(t), t \geq 0, \\ z_1(t) = C_1 x_1(t) + E_1 \omega_1(t), \\ x_1(0) = x_1^0, \end{cases} \tag{30}$$

$$\mathcal{S}_2 : \begin{cases} {}_0^C\mathrm{D}_t^\alpha x_2(t) = \big[A_2 + G_2^a F_2^a(t) H_2^a\big] x_2(t) \\ \quad +\big[D_{21} + G_{21}^d F_{21}^d(t) H_{21}^d\big] x_1(t) \\ \quad +\big[W_2 + G_2^w F_2^w(t) H_2^w\big]\omega_2(t) + B_2 u_2(t), t \geq 0, \\ z_2(t) = C_2 x_2(t) + E_2 \omega_2(t), \\ x_2(0) = x_2^0, \end{cases} \tag{31}$$

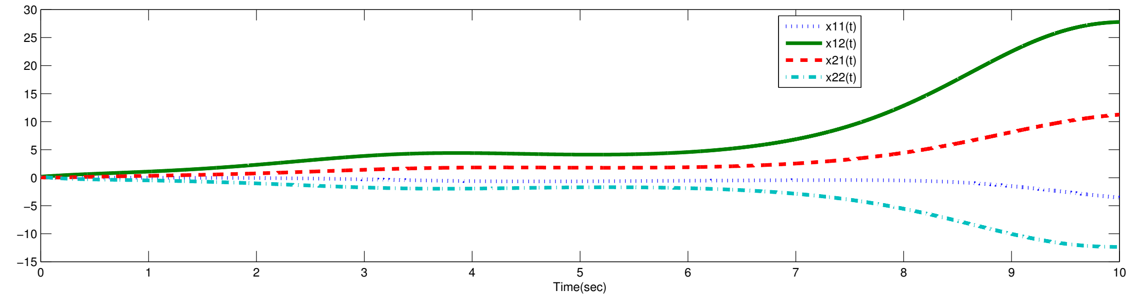

Figure 1. State responses of the open-loop system with $\alpha = 0.6$ in Example 1.

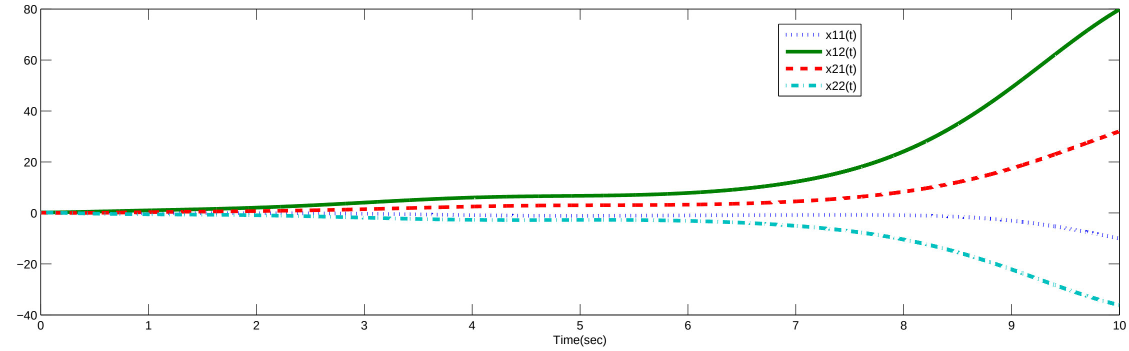

Figure 2. State responses of the open-loop system with $\alpha = 0.9$ in Example 1.

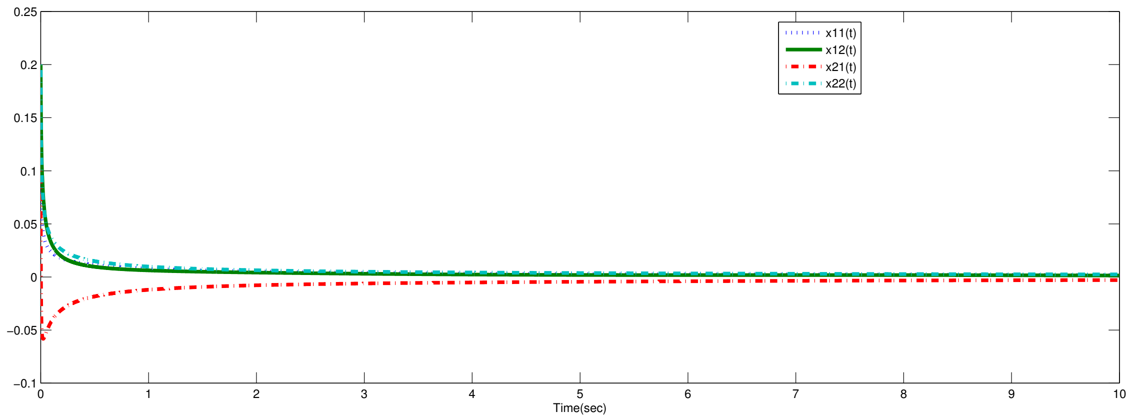

Figure 3. State responses of the closed-loop system with $\alpha = 0.6$ in Example 1.

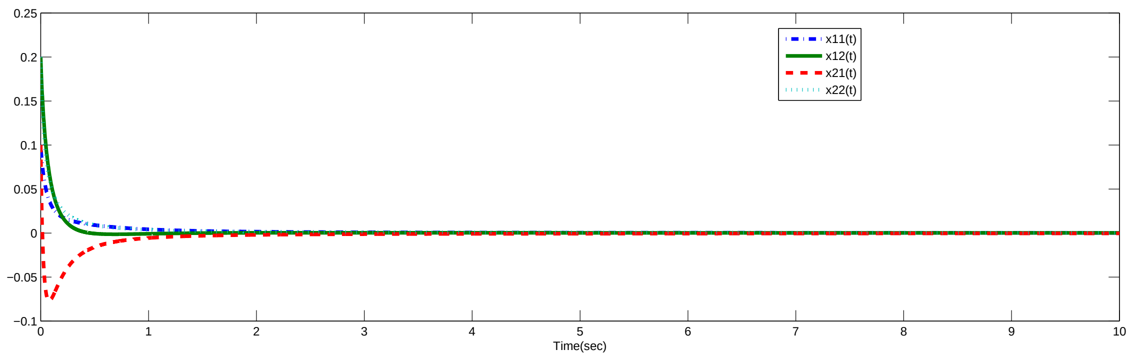

Figure 4. State responses of the closed-loop system with $\alpha = 0.9$ in Example 1.

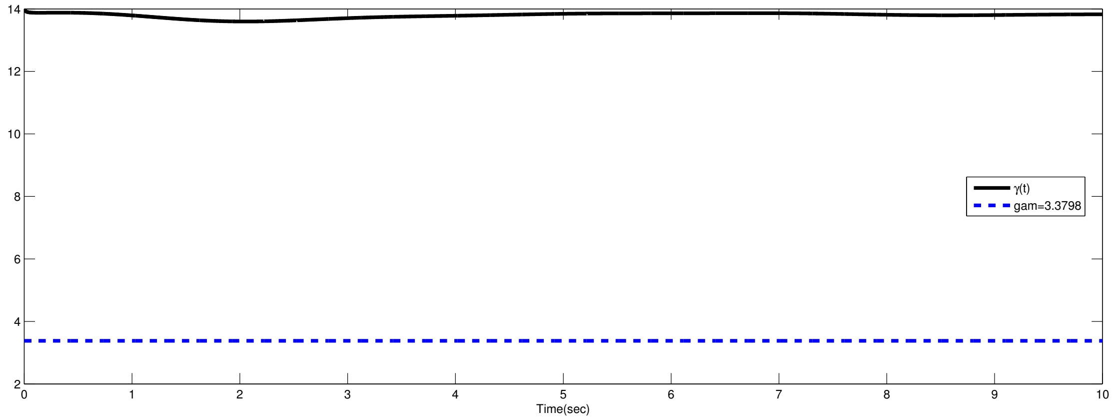

Figure 5. The response of $\gamma(t)$ with $\alpha = 0.6$ in Example 1.

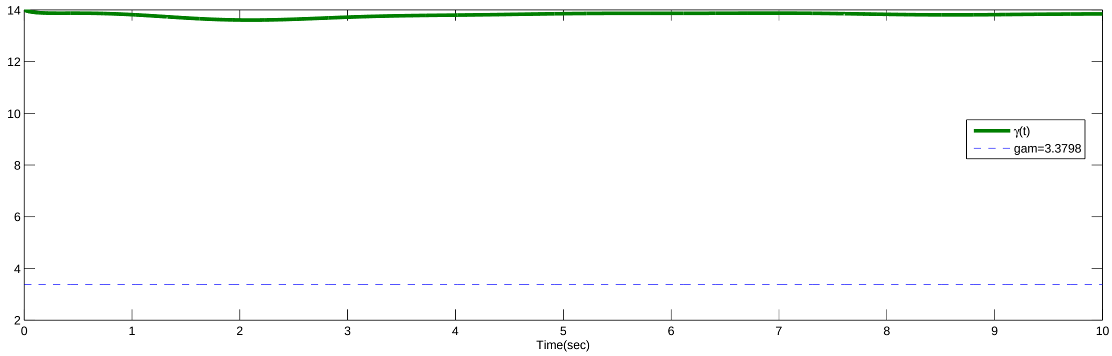

Figure 6. The response of $\gamma(t)$ with $\alpha = 0.9$ in Example 1.

where $\alpha \in (0,1)$, and

$$A_1 = \begin{bmatrix} -2 & 1 \\ 4 & 0 \end{bmatrix}, \; G_1^a = \begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix}, \; H_1^a = \begin{bmatrix} 0.5 & 0.4 \end{bmatrix},$$

$$D_{12} = \begin{bmatrix} -1 & 2 \\ 0 & -2 \end{bmatrix}, \; G_{12}^d = \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix}, \; H_{12}^d = \begin{bmatrix} 0.6 & 0.5 \end{bmatrix},$$

$$F_1^a(t) = \sin t, \; F_{12}^d(t) = \cos t,$$

$$W_1 = \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix}, \; G_1^w = \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix}, \; H_1^w = \begin{bmatrix} 1 \end{bmatrix}, \; F_1^w(t) = \cos t,$$

$$B_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \; C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \; E_1 = \begin{bmatrix} 1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -2 & -2 \\ 3 & -1 \end{bmatrix}, \; G_2^a = \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix}, \; H_2^a = \begin{bmatrix} 0.3 & 0.2 \end{bmatrix},$$

$$D_{21} = \begin{bmatrix} -1 & 0 \\ -2 & -2 \end{bmatrix}, \; G_{21}^d = \begin{bmatrix} 0.1 \\ 0.3 \end{bmatrix}, \; H_{21}^d = \begin{bmatrix} 0.3 & 0.4 \end{bmatrix},$$

$$F_2^a(t) = \cos t, \; F_{21}^d(t) = \sin t,$$

$$W_2 = \begin{bmatrix} 0.1 \\ 0.3 \end{bmatrix}, \; G_2^w = \begin{bmatrix} 0.4 \\ 0.5 \end{bmatrix}, \; H_2^w = \begin{bmatrix} 1 \end{bmatrix}, \; F_2^w(t) = \sin t,$$

$$B_2 = \begin{bmatrix} 4 \\ 2 \end{bmatrix}, \; C_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}, \; E_2 = \begin{bmatrix} 1 \end{bmatrix}.$$

Synthesis of decentralized sliding mode controllers for fractional-order nonlinear interconnected systems

$$u_i(t) = (K_i + G_i^k F_i^k(t) H_i^k) x_i(t), (i = 1, 2),$$

where $G_1^k = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix}, H_1^k = \begin{bmatrix} 1 \end{bmatrix}, F_1^k(t) = \sin t, G_2^k = \begin{bmatrix} 0.2 & 0.3 \end{bmatrix}, H_2^k = \begin{bmatrix} 1 \end{bmatrix}, F_2^k(t) = \cos t$, of systems (30), (31) can be represented as follows:

$$\mathcal{S}_1 : \begin{cases} {}_0^C\mathrm{D}_t^\alpha x_1(t) = \big[A_1 + G_1^a F_1^a(t) H_1^a \\ \quad + B_1 K_1 + B_1 G_1^k F_1^k(t) H_1^k\big] x_1(t) \\ \quad + \big[D_{12} + G_{12}^d F_{12}^d(t) H_{12}^d\big] x_2(t) \\ \quad + \big[W_1 + G_1^w F_1^w(t) H_1^w\big] \omega_1(t), t \geq 0, \\ z_1(t) = C_1 x_1(t) + E_1 \omega_1(t), \\ x_1(0) = x_1^0, \end{cases}$$

$$\mathcal{S}_2 : \begin{cases} {}_0^C\mathrm{D}_t^\alpha x_2(t) = \big[A_2 + G_2^a F_2^a(t) H_2^a \\ \quad + B_2 K_2 + B_2 G_2^k F_2^k(t) H_2^k\big] x_2(t) \\ \quad + \big[D_{21} + G_{21}^d F_{21}^d(t) H_{21}^d\big] x_1(t) \\ \quad + \big[W_2 + G_2^w F_2^w(t) H_2^w\big] \omega_2(t), t \geq 0, \\ z_2(t) = C_2 x_2(t) + E_2 \omega_2(t), \\ x_2(0) = x_2^0. \end{cases}$$

Let us choose $Q_1 = Q_2 = \begin{bmatrix} -5 \end{bmatrix}, R_1 = R_2 = \begin{bmatrix} 15 \end{bmatrix}, S_1 = S_2 = \begin{bmatrix} 2 \end{bmatrix}$. We study the problem of decentralized non-fragile dissipative control for the considered systems. By employing the MATLAB LMI Control Toolbox, the conditions stated in Theorem 1 are verified to be feasible with $\gamma = 3.3798, \epsilon_1 = 15.4675, \epsilon_2 = 15.3962, \beta_1 = 14.6104, \beta_2 = 14.6709$, and

$$P_1 = \begin{bmatrix} 2.3062 & -1.2767 \\ -1.2767 & 3.2477 \end{bmatrix}, Y_1 = \begin{bmatrix} -3.9384 & -3.4250 \end{bmatrix},$$
$$P_2 = \begin{bmatrix} 3.8363 & -1.2630 \\ -1.2630 & 1.8436 \end{bmatrix}, Y_2 = \begin{bmatrix} -5.8176 & -5.1227 \end{bmatrix}.$$

Based on Theorem 1, the system under consideration is strictly (Q_i, S_i, R_i)-dissipative $(i = 1, 2)$ with dissipativity level $\gamma = 3.3798$. Moreover, stabilizing state-feedback gain matrices are given by

$$K_1 = \begin{bmatrix} -2.9291 & -2.2061 \end{bmatrix}, K_2 = \begin{bmatrix} -3.1392 & -4.9291 \end{bmatrix}.$$

The evolution of the system states for both open-loop and closed-loop cases under the prescribed initial conditions $x_1(0) = (0.1, 0.2)^T$ and $x_2(0) = (0.1, 0.2)^T \in \mathbb{R}^2$ are illustrated in Figures 1, 2, 3, 4 for $\alpha = 0.6$ and $\alpha = 0.9$, respectively. To check the (Q_i, S_i, R_i)-dissipative $(i = 1, 2)$ with the dissipativity level $\gamma = 3.3798$, define the dissipativity performance function as

$$\gamma(t) := \frac{\sum_{i=1}^{2} \int_0^t \begin{pmatrix} z_i^T(s) Q_i z_i(s) + 2 z_i^T(s) S_i \omega_i(s) \\ + \omega_i^T(s) R_i \omega_i(s) \end{pmatrix} ds}{\sum_{i=1}^{2} \int_0^t \omega_i^T(s) \omega_i(s)\, ds}.$$

Figures 5 and 6 show the response of the function $\gamma(t)$ subject to the initial conditions $x_1(0) = (0, 0)^T$ and $x_2(0) = (0, 0)^T \in \mathbb{R}^2$, for $\alpha = 0.6$ and $\alpha = 0.9$, respectively. It is evident that the function $\gamma(t)$ remains greater than $\gamma = 3.3798$, indicating that the condition (ii) in Definition 4 holds.

5 Conclusions

This work considers the problem of decentralized non-fragile dissipative control for fractional-order interconnected systems with uncertainties. The approach used leverages the fractional-order Lyapunov method along with the properties of fractional calculus to derive novel sufficient conditions for designing a decentralized non-fragile controller. The controller ensures both asymptotic stability and dissipative performance index for the obtained closed-loop large-scale system with fractional order. The conditions are formulated as linear matrix inequalities, which can be easily solved with the aid of numerical software such as MATLAB's LMI Control Toolbox. To validate the practicality and effectiveness of the obtained results, an illustrative example is presented. Beyond the current setting, fractional-order interconnected systems often exhibit additional complexities such as time-varying delays, switching dynamics, and communication constraints. Therefore, future research will focus on extending the proposed decentralized non-fragile dissipative control framework to fractional-order systems with time delays, event-triggered control mechanisms, and distributed optimization strategies for improved scalability. Investigating output-feedback designs and adaptive non-fragile schemes also constitutes a promising direction.

Data Availability Statement

Data will be made available on request.

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Conflicts of Interest

The authors declare no conflicts of interest.

AI Use Statement

The authors declare that no generative AI was used in the preparation of this manuscript.

Ethical Approval and Consent to Participate

Not applicable.

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