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RESEARCH ARTICLE

Asynchronous Intermittent Event-Triggered Control for a Class of Coupled Stochastic Strict-Feedback Nonlinear Systems

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Abstract

This paper investigates a class of coupled stochastic strict-feedback nonlinear systems under asynchronous intermittent event-triggered control (AIETC). Initially, stochastic analysis technique, Lyapunov method and backstepping design method are employed to design the virtual and actual controllers. AIETC is achieved by an auxiliary timer that grants each subsystem its own control and rest time. In the meantime the control input is applied only at the last node of each subsystem. Then, a global Lyapunov function is constructed. By utilizing graph theory, the global exponential ultimate boundedness in mean square of the systems can be obtained and Zeno behavior is eliminated successfully. Finally, a simulation example is provided to demonstrate the effectiveness of our results.

Keywords: stochastic nonlinear systems, intermittent control, event-triggered, backstepping.

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1 Introduction

Stochastic strict-feedback systems (SSFSs), which preserve the classical strict-feedback structure [1–4] while accounting for stochastic disturbances, have attracted sustained attention in recent years. In general, SSFSs typically employ backstepping techniques for controller design to achieve system stabilization. Backstepping designs for SSFSs were first proposed by [5, 6] and further developed by the work of [7–13]. However, the aforementioned results are confined to a single SSFS. Motivated by the ubiquitous interconnections among real-world systems, multiple stochastic strict-feedback networks are considered in a coupled configuration in [14–16]. Across these studies, Guo et al. [14] demonstrated that, even when the stochastic strict-feedback networks are not strongly connected and subject to time-varying delays, pinning control together with a graph theory can still guarantee exponential stabilisation or synchronization in mean square.

In many practical control systems, control inputs are not continuously available due to actuator faults, communication constraints, network interruptions, or energy-saving requirements. These limitations have motivated the development of intermittent control strategies. Intermittent control allows the controller to be inactive over certain time intervals, thereby reducing energy consumption and communication

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burden. Compared with continuous control, intermittent control exhibits greater practicality and flexibility and has been widely adopted in areas such as networked control systems [17–21]. For instance, Liu et al. [17] established synchronization criteria for complex networks under aperiodically intermittent pinning control. Li et al. [18] investigated the exponential synchronization of stochastic hybrid multi-weight coupled systems with mixed delays using aperiodically adaptive intermittent control, and derived less conservative conditions by introducing novel Halanay-type differential inequalities. Nevertheless, most existing intermittent control schemes rely on time-triggered mechanisms, in which control actions are executed according to predetermined schedules. Although such strategies are straightforward to implement, they fail to exploit real-time system state information, which may result in unnecessary control updates and inefficient use of resources. To overcome these drawbacks, event-triggered control (ETC) has been proposed and has received increasing attention in recent years [22–24]. In event-triggered control systems, control signals are updated only when predefined triggering conditions, determined by the system state, are satisfied. Recently, the ETC of stochastic strict-feedback nonlinear systems (SSFNSs) has attracted considerable research interest, with representative results reported in [25, 26]. Specifically, an adaptive fuzzy event-triggered tracking controller was developed in [25] to guarantee semi-global uniform ultimate boundedness of all closed-loop signals. Furthermore, Lu et al. [26] proposed an adaptive event-triggered tracking control scheme for SSFNSs with full-state constraints by integrating a tan-type stochastic barrier Lyapunov function with radial basis function neural networks.

To harness the flexibility of intermittent control and the resource efficiency of ETC, the intermittent event-triggered control strategy has been proposed and validated in [27–29]. Note that the aforementioned intermittent schemes require all nodes to switch simultaneously. In [30], an asynchronously intermittent decentralized framework was introduced that allows each subsystem to possess its own working and resting intervals, thereby accommodating practical demands such as staggered micro-grid duty cycles. Subsequently, input-to-state practically exponential stability was investigated by designing asynchronously intermittent event-triggered control in [31]. To the best of the authors' knowledge, there are few results on SSFNSs under intermittent event-triggered control, let alone asynchronous intermittent event-triggered control (AIETC) for coupled stochastic strict-feedback nonlinear systems (CSSFNSs).

Motivated by the above discussions, we propose an asynchronous intermittent event-triggered control scheme is proposed for CSSFNSs. By designing an auxiliary timer and employing the backstepping technique as well as graph theory, AIETC is designed for each subsystem, which can ensure the global exponential ultimate boundedness in mean square of the systems. The main contributions of this article can be listed as follows:

(1) In contrast to the ETC for a single SSFNS reported in previous works [25, 26], an intermittent event-triggered control scheme for CSSFNSs is investigated in this paper, which simultaneously adopts intermittent control and explicitly accounts for the topological structure among multiple SSFNSs.

(2) We devise an AIETC strategy that assigns each subsystem its own working and resting intervals for CSSFNSs. Compare to [30, 31], the control input is applied only to the last state of each subsystem in this paper.

The rest of the paper is organized as follows. Section 2 presents the corresponding preliminary work and the model description. Section 3 details the design of the controllers and primary results together with proofs. In Section 4, the effectiveness of the our results is verified through an example. The paper is concluded in Section 5.

2 Preliminaries and Model Description

2.1 Preliminaries

Throughout this paper, the following notations will be used. Let $\mathbb{Z}^+$, $\mathbb{R}$ and $\mathbb{R}^+$ be the sets of positive integers, real numbers and non-negative real numbers, respectively. Define $\tilde{\mathbb{N}} = \{1, 2, \cdots, n-1\}$, $\mathbb{N} = \{1, 2, \cdots, n\}$, where $n \in \mathbb{Z}^+, n > 1$. Let $\mathcal{N}_0 = \{0, 1, 2, \cdots\}$ and $\mathbb{K} = \{1, 2, \cdots, k\}$, $k \in \mathbb{Z}^+$. $\mathbb{R}^n$ is the n-dimensional Euclidean space with norm $|\cdot|$. For a matrix or vector A, the symbol A^T stands for its transpose, and $\mathrm{Tr}(A)$ denotes its trace when A is square. $(\Omega, \mathcal{F}, \{\mathcal{F}\}_{t\geq 0}, \mathbb{P})$ represents a complete probability space with a filtration $\{\mathcal{F}\}_{t\geq 0}$ satisfying the usual conditions. Let $B(t)$ be a one-dimensional Brownian motion defined on the probability space. $\mathbb{E}(\cdot)$ denotes the expectation with respect to the probability measure

$\mathbb{P}$. Assign $C^{1,2}(\mathbb{R}^+ \times \mathbb{R}^n; \mathbb{R}^+)$ be the family of all nonnegative functions $W(t,x)$ on $\mathbb{R}^+ \times \mathbb{R}^n$ which are continuously once differentiable in t and twice in x.

Consider the following stochastic differential equation

$$\mathrm{d}z(t) = f(t, z(t))\mathrm{d}t + g(t, z(t))\mathrm{d}B(t),$$

where $z \in \mathbb{R}^n$. For nonnegative function $W(t,z) \in C^{1,2}(\mathbb{R}^+ \times \mathbb{R}^n; \mathbb{R}^+)$, the operators $\mathcal{L}$ and $\mathcal{H}$ are defined as

$$\begin{aligned}\mathcal{L}W =& W_t(t,z) + W_z(t,z)f(t,z) \\ &+ \frac{1}{2}\mathrm{Tr}(g^T(t,z)W_{zz}(t,z)g(t,z)), \\ \mathcal{H}W =& W_z(t,z(t))g(t,z),\end{aligned}$$

where $W_t(t,z) = \frac{\partial W(t,z)}{\partial t}$, $W_{zz}(t,z) = \left(\frac{\partial^2 W(t,z)}{\partial z_i \partial z_j}\right)_{n\times n}$, $W_z(t,z) = \left(\frac{\partial W(t,z)}{\partial z_1}, \cdots, \frac{\partial W(t,z)}{\partial z_n}\right)$.

2.2 Model Description

Consider the following CSSFNSs constructed on a strongly connected digraph $(\mathcal{G}^j, A^j)$ with $A^j = (a^j_{im})_{k\times k}$, $j \in \mathbb{N}$,

$$\begin{aligned}\mathrm{d}x_i^j(t) =& (\sigma_i^{j+1}x_i^{j+1} + f_i^j(t, x_i^{[j]}) + \sum_{m=1}^{k} a^j_{im} \\ &\cdot \Gamma^j_{im}(x_i^j, x_m^j))\mathrm{d}t + g_i^j(t, x_i^{[j]})\mathrm{d}B(t), j \in \tilde{\mathbb{N}},\end{aligned} \tag{1}$$

$$\begin{aligned}\mathrm{d}x_i^n(t) =& (u_i + f_i^n(t, x_i^{[n]}) + \sum_{m=1}^{k} a^n_{im}\Gamma^n_{im}(x_i^n, x_m^n))\mathrm{d}t \\ &+ g_i^n(t, x_i^{[n]})\mathrm{d}B(t), i \in \mathbb{K},\end{aligned} \tag{2}$$

where $x_i^j \in \mathbb{R}$ is the j-th node state of the i-th SSFNS, $x_i^{[j]} = (x_i^1, \cdots, x_i^j)^T$ and $x_i^{[n]} = (x_i^1, \cdots, x_i^n)^T$; $u_i \in \mathbb{R}$ is the control input; $f_i^j : \mathbb{R}^+ \times \mathbb{R}^j \to \mathbb{R}$, $g_i^j : \mathbb{R}^+ \times \mathbb{R}^j \to \mathbb{R}$ are all nonlinear functions; $\Gamma^j_{im} : \mathbb{R}\times\mathbb{R} \to \mathbb{R}$ is coupling function and a^j_{im} represents coupling strength; σ_i^j is a positive constant and let $\sigma_i^1 = 1$.

Denote $x = ((x_1^{[n]})^T, \cdots, (x_k^{[n]})^T)^T \in \mathbb{R}^{kn}$. Assume that f_i^j, g_i^j and Γ^j_{im} satisfy the necessary conditions to ensure system (1)–(2) has a unique solution for any initial value $x(0) = x_0 \in \mathbb{R}^{kn}$. Suppose that $f_i^j(t,0) = g_i^j(t,0) = \Gamma^j_{im}(0,0) = 0$, then system (1)–(2) has a trivial solution $x(t) \equiv 0$.

To this end, we introduce the following assumptions and definitions.

Assumption 1. For $i \in \mathbb{K}, j \in \mathbb{N}$, there exist positive constants $c_i^{j,1}$ and $c_i^{j,2}$ such that

$$\begin{aligned}|f_i^j(t, x_i^{[j]})| \le c_i^{j,1}(|x_i^1| + \cdots + |x_i^{j-1}| + |x_i^j|), \\ |g_i^j(t, x_i^{[j]})| \le c_i^{j,2}(|x_i^1| + \cdots + |x_i^{j-1}| + |x_i^j|).\end{aligned}$$

Assumption 2. For any $i, m \in \mathbb{K}, j \in \mathbb{N}$, there exists a nonnegative constant h^j_{im} such that

$$|\Gamma^j_{im}(x_i^j, x_m^j)| \le h^j_{im}(|x_i^j| + |x_m^j|).$$

Remark 1. The two assumptions are crucial for the boundedness analysis, and their rationality is justified as follows. Assumption 1 imposes linear growth constraints on $f_i^j(\cdot)$ and $g_i^j(\cdot)$ with bounded constants $c_i^{j,1}$, $c_i^{j,2}$ which is a standard condition in SSFNSs [13, 14] to ensure controller design tractability. It accommodates practical nonlinearities like $a\sin t$ and $b\cos t$ by choosing appropriate constants a and b. Assumption 2 characterizes the coupling functions are restricted to linear functions, which is general in coupled nonlinear system analysis, as seen in [18, 31].

Definition 1. (see [32]) The intermittent control of i-th subsystem is said to have an average control ratio $\pi_i \in (0,1)$, if there exists positive constant $\breve{N}_{\pi_i}$ such that the following inequality holds:

$$\breve{N}_i(t_2, t_1) \ge \pi_i(t_2 - t_1) - \breve{N}_{\pi_i}, \quad \forall t_2 > t_1 \ge 0,$$

where $\breve{N}_i(t_2, t_1)$ denotes the total control time during the interval (t_1, t_2).

Definition 2. (see [33]) System (1)–(2) is said to be globally p-th moment exponentially ultimately bounded if there exist constants $C > 0$, $\Pi > 0$ and $\Xi \ge 0$ such that for any solution with the initial value x_0,

$$\mathbb{E}|x(t)|^p \le C|x_0|^p e^{-\Pi t} + \Xi, \quad p > 0, t \ge t_0.$$

When $p = 2$, it is usually said to be globally exponentially ultimately bounded in mean square.

3 Main Results

In this section, the design of continuous virtual controllers is first designed by backstepping technique. Then, according to these virtual controllers, event-triggered mechanism is constructed. Combining asynchronous intermittent control strategy, the complete AIETC can be introduced. Finally, the boundedness analysis of system (1)–(2) is given through graph theory and Lyapunov method.

3.1 Controller Design

In this subsection, we construct the Lyapunov function and present the detailed design procedures for the virtual and actual controllers. To implement asynchronous intermittent control, we first introduce the following auxiliary timer:

$$\varepsilon_i(t)=\begin{cases}\tau_i\breve{N}_{\pi_i},\ t\in[T_0^i,S_0^i],\\ \min\left\{\tau_i\breve{N}_{\pi_i},\varepsilon_i(T_l^i)+[(1-\pi_i)\tau_i](t-T_l^i)\right\},\\ \qquad\qquad t\in(T_l^i,S_l^i],l\in\mathcal{N}_0/\{0\},\\ \varepsilon_i(S_l^i)-\pi_i\tau_i(t-S_l^i),\ t\in(S_l^i,T_{l+1}^i],l\in\mathcal{N}_0,\end{cases}\tag{3}$$

where τ_i $(i\in\mathbb{K})$ are tuning parameters. From the definition of (3), one can easily get that $0\le\varepsilon_i(t)\le\tau_i\breve{N}_{\pi_i}$ (cf. [31]), and

$$\dot\varepsilon_i(t)\le\begin{cases}(1-\pi_i)\tau_i, & t\in[T_l^i,S_l^i),\\ -\pi_i\tau_i, & t\in[S_l^i,T_{l+1}^i).\end{cases}\tag{4}$$

Next, we design the controllers step by step.

Step 1. Consider a Lyapunov function $W_i^1(t,\lambda_i^1)=\frac{1}{4}e^{\varepsilon_i(t)}(\lambda_i^1)^4$, $\lambda_i^1=x_i^1$. Denote $\tilde a_{im}^j=a_{im}^jh_{im}^j, i,m\in\mathbb{K}, j\in\mathbb{N}$. According to Assumption 1 and 2, it can be attained that

$$\begin{aligned}&\mathcal{L}W_i^1(t,\lambda_i^1)\\ \le&\dot\varepsilon_i(t)W_i^1+e^{\varepsilon_i(t)}\Big[(\lambda_i^1)^3(\sigma_i^2x_i^2+f_i^1\\ &+\sum_{m=1}^k a_{im}^1\Gamma_{im}^1(x_i^1,x_m^1))+\frac{3}{2}(\lambda_i^1)^2|g_i^1|^2\Big]\\ \le&\dot\varepsilon_i(t)W_i^1+e^{\varepsilon_i(t)}(\lambda_i^1)^3(\sigma_i^2x_i^2-\beta_i^2)+e^{\varepsilon_i(t)}(\lambda_i^1)^3\beta_i^2\\ &+e^{\varepsilon_i(t)}(\lambda_i^1)^4(c_i^{1,1}+\frac{3}{2}(c_i^{1,2})^2)\\ &+e^{\varepsilon_i(t)}(\lambda_i^1)^3\sum_{m=1}^k\tilde a_{im}^1(|x_i^1|+|x_m^1|),\end{aligned}\tag{5}$$

where β_i^2 is a virtual controller. By utilizing the following inequality

$$r|I|^a|L|^b\le\chi|I|^{a+b}+\frac{b}{a+b}\left[\frac{a}{\chi(a+b)}\right]^{\frac{a}{b}}r^{\frac{a+b}{b}}|L|^{a+b},\tag{6}$$

where $I,L\in\mathbb{R}$, $a,b,\chi,r>0$, it can be attained that

$$\begin{aligned}&e^{\varepsilon_i(t)}(\lambda_i^1)^3\sum_{m=1}^k\tilde a_{im}^1(|x_i^1|+|x_m^1|)\\ \le&\Big[k\vartheta(\lambda_i^1)^4+\frac{1}{4}\left(\frac{\theta}{\vartheta}\right)^3\sum_{m=1}^k(\tilde a_{im}^1)^4(x_i^1)^4\\ &+\sum_{m=1}^k e^{\frac{1}{3}(\varepsilon_i(t)-\varepsilon_m(t))}\vartheta(\lambda_i^1)^4\\ &+\frac{1}{4}\left(\frac{\theta}{\vartheta}\right)^3\sum_{m=1}^k e^{\varepsilon_m(t)-\varepsilon_i(t)}(\tilde a_{im}^1)^4(x_m^1)^4\Big]e^{\varepsilon_i(t)}\\ \le&\Big[k\vartheta+\frac{1}{2}\left(\frac{\theta}{\vartheta}\right)^3\sum_{m=1}^k(\tilde a_{im}^1)^4+k\vartheta e^{\frac{1}{3}\tau_i\breve{N}_{\pi_i}}\Big]e^{\varepsilon_i(t)}(\lambda_i^1)^4\\ &+\frac{1}{4}\left(\frac{\theta}{\vartheta}\right)^3\sum_{m=1}^k(\tilde a_{im}^1)^4(e^{\varepsilon_m(t)}(x_m^1)^4-e^{\varepsilon_i(t)}(x_i^1)^4),\end{aligned}\tag{7}$$

where ϑ is an arbitrary positive constant and $\theta=\frac{3}{4}$. Substitution of (7) into (5) gives

$$\begin{aligned}&\mathcal{L}W_i^1(t,\lambda_i^1)\\ \le&\dot\varepsilon_i(t)W_i^1+e^{\varepsilon_i(t)}(\lambda_i^1)^3(\sigma_i^2x_i^2-\beta_i^2)\\ &+e^{\varepsilon_i(t)}(\lambda_i^1)^3\beta_i^2+e^{\varepsilon_i(t)}(\lambda_i^1)^4\Big[c_i^{1,1}+\frac{3}{2}(c_i^{1,2})^2\\ &+k\vartheta(1+e^{\frac{1}{3}\tau_i\breve{N}_{\pi_i}})+\frac{1}{2}\left(\frac{\theta}{\vartheta}\right)^3\sum_{m=1}^k(\tilde a_{im}^1)^4\Big]\\ &+\frac{1}{4}\left(\frac{\theta}{\vartheta}\right)^3\sum_{m=1}^k(\tilde a_{im}^1)^4(e^{\varepsilon_m(t)}(x_m^1)^4-e^{\varepsilon_i(t)}(x_i^1)^4).\end{aligned}\tag{8}$$

Define $\alpha_i^1=\max\{1,c_i^1+c_i^{1,1}+\frac{3}{2}(c_i^{1,2})^2+k\vartheta(1+e^{\frac{1}{3}\tau_i\breve{N}_{\pi_i}})+\frac{1}{2}\left(\frac{\theta}{\vartheta}\right)^3\sum_{m=1}^k(\tilde a_{im}^1)^4\}$, where $c_i^1\in\mathbb{R}$ is a design parameter. Design the virtual controller β_i^2 as

$$\beta_i^2=-\alpha_i^1\lambda_i^1.\tag{9}$$

Therefore, from (8) and (9), we have

$$
\begin{aligned}
&\mathcal{L}W_i^1(t,\lambda_i^1)\\
\leq &-c_i^1 e^{\varepsilon_i(t)}(\lambda_i^1)^4+\dot{\varepsilon}_i(t)W_i^1+e^{\varepsilon_i(t)}(\lambda_i^1)^3\left(\sigma_i^2x_i^2-\beta_i^2\right)\\
&+\frac{1}{4}\left(\frac{\theta}{\vartheta}\right)^3\sum_{m=1}^k(\tilde{a}_{im}^1)^4(e^{\varepsilon_m(t)}(x_m^1)^4-e^{\varepsilon_i(t)}(x_i^1)^4).
\end{aligned}\tag{10}
$$

Deductive Step. Suppose that at step $p-1$, there are virtual controllers $\beta_i^2,\beta_i^3,\cdots,\beta_i^p$ designed as

$$
\begin{aligned}
\beta_i^2&=-\alpha_i^1\lambda_i^1, & \lambda_i^1&=x_i^1, &(11)\\
\beta_i^3&=-\alpha_i^2\lambda_i^2, & \lambda_i^2&=\sigma_i^2x_i^2-\beta_i^2, &(12)\\
&\vdots & &\vdots\\
\beta_i^p&=-\alpha_i^{p-1}\lambda_i^{p-1}, & \lambda_i^{p-1}&=\sigma_i^{p-1}x_i^{p-1}-\beta_i^{p-1}, &(13)
\end{aligned}
$$

such that

$$
\begin{aligned}
&\mathcal{L}W_i^{p-1}(t,\bar{\lambda}_i^{p-1})\\
\leq &-\sum_{j=1}^{p-1}(c_i^j-\zeta_i^{p-1,j})e^{\varepsilon_i(t)}(\lambda_i^j)^4+\dot{\varepsilon}_i(t)W_i^{p-1}\\
&+e^{\varepsilon_i(t)}\sigma_i^{p-1}(\lambda_i^{p-1})^3(\sigma_i^px_i^p-\beta_i^p)\\
&+\frac{1}{4}\left(\frac{\theta}{\vartheta}\right)^3\sum_{j=1}^{p-1}\sum_{m=1}^k(\tilde{a}_{im}^j)^4(e^{\varepsilon_m(t)}(x_m^j)^4-e^{\varepsilon_i(t)}(x_i^j)^4)\\
&+\frac{1}{4}\left(\frac{\theta}{\vartheta}\right)^3\sum_{j=1}^{p-1}\sum_{q=1}^{j-1}\sum_{m=1}^k(\varrho_i^q)^4(\tilde{a}_{im}^q)^4(e^{\varepsilon_m(t)}(x_m^q)^4\\
&-e^{\varepsilon_i(t)}(x_i^q)^4),
\end{aligned}\tag{14}
$$

where $\bar{\lambda}_i^{p-1}=(\lambda_i^1,\cdots,\lambda_i^{p-1})^T$, $W_i^{p-1}(t,\bar{\lambda}_i^{p-1})=\frac{1}{4}e^{\varepsilon_i(t)}\sum_{j=1}^{p-1}(\lambda_i^j)^4$, $\varrho_i^q=\prod_{w=q}^{j-1}\sigma_i^q\alpha_i^w$, $c_i^1,\cdots,c_i^{p-1}\in\mathbb{R}$ are design parameters, $\zeta_i^{p-1,1},\cdots,\zeta_i^{p-1,p-2}$ are arbitrary positive constants, and $\zeta_i^{p-1,p-1}=0$.

Then, we consider the λ_i^p-system to validate this result.

Step p. Define $\lambda_i^p=\sigma_i^px_i^p-\beta_i^p$, and from (11)–(13), it indicates that

$$
\lambda_i^p=\sigma_i^px_i^p+\sum_{j=1}^{p-1}\varrho_i^jx_i^j,\tag{15}
$$

where $\varrho_i^j=\prod_{w=j}^{p-1}\sigma_i^j\alpha_i^w$. Based upon (1) and (15), one has

$$
\begin{aligned}
\mathrm{d}\lambda_i^p=&\mathrm{d}(\sigma_i^px_i^p+\sum_{j=1}^{p-1}\varrho_i^jx_i^j)\\
=&\Bigg[\sigma_i^p\Big(\sigma_i^{p+1}x_i^{p+1}+f_i^p+\sum_{m=1}^k a_{im}^p\Gamma_{im}^p(x_i^p,x_m^p)\Big)\\
&+\sum_{j=1}^{p-1}\varrho_i^j(\sigma_i^{j+1}x_i^{j+1}+f_i^j)+\sum_{j=1}^{p-1}\varrho_i^j\sum_{m=1}^k a_{im}^j\\
&\cdot\Gamma_{im}^j(x_i^j,x_m^j)\Bigg]\mathrm{d}t+(\sigma_i^pg_i^p+\sum_{j=1}^{p-1}\varrho_i^jg_i^j)\mathrm{d}B(t).
\end{aligned}\tag{16}
$$

Consider the Lyapunov function

$$
W_i^p(t,\bar{\lambda}_i^p)=W_i^{p-1}(t,\bar{\lambda}_i^{p-1})+\frac{1}{4}e^{\varepsilon_i(t)}(\lambda_i^p)^4.\tag{17}
$$

According to (14), (16) and (17), it can be calculated that

$$
\begin{aligned}
&\mathcal{L}W_i^p(t,\bar{\lambda}_i^p)\\
\leq &-\sum_{j=1}^{p-1}(c_i^j-\zeta_i^{p-1,j})e^{\varepsilon_i(t)}(\lambda_i^j)^4+\dot{\varepsilon}_i(t)W_i^p\\
&+e^{\varepsilon_i(t)}\Bigg\{\sigma_i^{p-1}(\lambda_i^{p-1})^3\lambda_i^p+\sigma_i^p(\lambda_i^p)^3\Big(\sigma_i^{p+1}x_i^{p+1}\\
&-\beta_i^{p+1}\Big)+\sigma_i^p(\lambda_i^p)^3\beta_i^{p+1}+(\lambda_i^p)^3\Bigg[\sigma_i^pf_i^p+\sum_{j=1}^{p-1}\varrho_i^j\\
&\cdot(\sigma_i^{j+1}x_i^{j+1}+f_i^j)\Bigg]+\frac{3}{2}(\lambda_i^p)^2\Big|\sigma_i^pg_i^p+\sum_{j=1}^{p-1}\varrho_i^jg_i^j\Big|^2\\
&+\sigma_i^p(\lambda_i^p)^3\sum_{m=1}^k\tilde{a}_{im}^p\Big(|x_i^p|+|x_m^p|\Big)\\
&+(\lambda_i^p)^3\sum_{j=1}^{p-1}\varrho_i^j\sum_{m=1}^k\tilde{a}_{im}^j(|x_i^j|+|x_m^j|)\Bigg\}\\
&+\frac{1}{4}\left(\frac{\theta}{\vartheta}\right)^3\sum_{j=1}^{p-1}\sum_{m=1}^k(\tilde{a}_{im}^j)^4(e^{\varepsilon_m(t)}(x_m^j)^4-e^{\varepsilon_i(t)}(x_i^j)^4)\\
&+\frac{1}{4}\left(\frac{\theta}{\vartheta}\right)^3\sum_{j=1}^{p-1}\sum_{q=1}^{j-1}\sum_{m=1}^k(\varrho_i^q)^4(\tilde{a}_{im}^q)^4(e^{\varepsilon_m(t)}(x_m^q)^4\\
&-e^{\varepsilon_i(t)}(x_i^q)^4).
\end{aligned}\tag{18}
$$

For $i \in \mathbb{K}, j \in \mathbb{N}$, from (11)–(13) and Assumption 1, there exist positive constants $\breve{c}_i^{j,1}$ and $\breve{c}_i^{j,2}$ such that

$$\begin{aligned} |f_i^j(t, x_i^{[j]})| &\le \breve{c}_i^{j,1}(|\lambda_i^1| + \cdots + |\lambda_i^{j-1}| + |\lambda_i^j|), \quad (19) \\ |g_i^j(t, x_i^{[j]})| &\le \breve{c}_i^{j,2}(|\lambda_i^1| + \cdots + |\lambda_i^{j-1}| + |\lambda_i^j|). \quad (20) \end{aligned}$$

By using (6), we obtain that

$$\begin{aligned} \sigma_i^{p-1}(\lambda_i^{p-1})^3\lambda_i^p \le& \zeta_i^{p,p-1,1}(\lambda_i^{p-1})^4 \\ &+ \frac{1}{4}\left(\frac{4}{3}\zeta_i^{p,p-1,1}\right)^{-3}(\sigma_i^{p-1})^4(\lambda_i^p)^4, \end{aligned} \quad (21)$$

where $\zeta_i^{p,p-1,1}$ is an arbitrary positive constant.

Based on (6) and (19), we arrive at

$$\begin{aligned} \sigma_i^p(\lambda_i^p)^3 f_i^p \le& \sigma_i^p \breve{c}_i^{p,1}|\lambda_i^p|^3(|\lambda_i^1| + \cdots + |\lambda_i^p|) \\ \le& \sum_{j=1}^{p-1}\zeta_i^{p,j,2}(\lambda_i^j)^4 + \Bigg(\sigma_i^p\breve{c}_i^{p,1} + \frac{3}{4}(\sigma_i^p\breve{c}_i^{p,1})^{\frac{4}{3}} \\ &\cdot\sum_{j=1}^{p-1}(4\zeta_i^{p,j,2})^{-\frac{1}{3}}\Bigg)(\lambda_i^p)^4, \end{aligned} \quad (22)$$

where $\zeta_i^{p,j,2}$ is an arbitrary positive constant.

By (6) and (15), it derives that

$$\begin{aligned} &(\lambda_i^p)^3\sum_{j=1}^{p-1}\varrho_i^j\sigma_i^{j+1}x_i^{j+1} \\ \le& |\lambda_i^p|^3\sum_{j=1}^{p-1}\left(\prod_{w=j}^{p-1}\sigma_i^j\alpha_i^w\right)(|\lambda_i^{j+1}| + \alpha_i^j|\lambda_i^j|) \\ \le& \sigma_i^{p-1}\alpha_i^{p-1}(\lambda_i^p)^4 + \sum_{j=1}^{p-1}(\sigma_i^{j-1}\alpha_i^{j-1} + \sigma_i^j\alpha_i^j)\prod_{w=j}^{p-1}\alpha_i^w \\ &\cdot|\lambda_i^p|^3|\lambda_i^j| \\ \le& \sum_{j=1}^{p-1}\zeta_i^{p,j,3}(\lambda_i^j)^4 + \Bigg[\sigma_i^{p-1}\alpha_i^{p-1} + \frac{3}{4}\sum_{j=1}^{p-1}(4\zeta_i^{p,j,3})^{-\frac{1}{3}} \\ &\cdot\left((\sigma_i^{j-1}\alpha_i^{j-1} + \sigma_i^j\alpha_i^j)\prod_{w=j}^{p-1}\alpha_i^w\right)^{\frac{4}{3}}\Bigg](\lambda_i^p)^4, \end{aligned} \quad (23)$$

where $\zeta_i^{p,j,3}$ is an arbitrary positive constant, and let $\sigma_i^0 = \alpha_i^0 = 0$.

Together (6), (15) with (19), one can obtain that

$$\begin{aligned} &(\lambda_i^p)^3\sum_{j=1}^{p-1}\varrho_i^j f_i^j \\ \le& |\lambda_i^p|^3\sum_{j=1}^{p-1}\breve{c}_i^{j,1}(|\lambda_i^1| + \cdots + |\lambda_i^j|)\left(\prod_{w=j}^{p-1}\sigma_i^j\alpha_i^w\right) \\ \le& |\lambda_i^p|^3(|\lambda_i^1| + \cdots + |\lambda_i^{p-1}|)(\bar{c}_i^{p-1,1}\bar{\sigma}_i^{p-1}(p-1)\prod_{w=1}^{p-1}\alpha_i^w) \\ \le& \sum_{j=1}^{p-1}\zeta_i^{p,j,4}(\lambda_i^j)^4 + \frac{3}{4}\sum_{j=1}^{p-1}(4\zeta_i^{p,j,4})^{-\frac{1}{3}}\Bigg(\bar{c}_i^{p-1,1}\bar{\sigma}_i^{p-1} \\ &\cdot(p-1)\prod_{w=1}^{p-1}\alpha_i^w\Bigg)^{\frac{4}{3}}(\lambda_i^p)^4, \end{aligned} \quad (24)$$

where $\zeta_i^{p,j,4}$ is an arbitrary positive constant, $\bar{c}_i^{p-1,1} = \max\{\breve{c}_i^{1,1}, \breve{c}_i^{2,1}, \cdots, \breve{c}_i^{p-1,1}\}$, and $\bar{\sigma}_i^{p-1} = \max\{\sigma_i^1, \sigma_i^2, \cdots, \sigma_i^{p-1}\}$.

Combining (6), (15) with (20), it follows that

$$\begin{aligned} &\frac{3}{2}(\lambda_i^p)^2\left|\sigma_i^p g_i^p + \sum_{j=1}^{p-1}\varrho_i^j g_i^j\right|^2 \\ \le& \frac{3}{2}(\lambda_i^p)^2\Bigg[\sigma_i^p\breve{c}_i^{p,2}(|\lambda_i^1| + \cdots + |\lambda_i^{p-1}| + |\lambda_i^p|) \\ &+ \sum_{j=1}^{p-1}\left(\prod_{w=j}^{p-1}\sigma_i^j\alpha_i^w\right)\breve{c}_i^{j,2}(|\lambda_i^1| + \cdots + |\lambda_i^{j-1}| + |\lambda_i^j|)\Bigg]^2 \\ \le& \frac{3}{2}(\lambda_i^p)^2\Bigg[\sigma_i^p\breve{c}_i^{p,2}|\lambda_i^p| + \sum_{j=1}^{p-1}\Bigg(\sigma_i^p\breve{c}_i^{p,2} \\ &+ \sum_{q=j}^{p-1}\left(\prod_{w=q}^{p-1}\sigma_i^q\alpha_i^w\right)\breve{c}_i^{q,2}\Bigg)|\lambda_i^j|\Bigg]^2 \\ \le& \frac{3}{2}p(\sigma_i^p\breve{c}_i^{p,2})^2(\lambda_i^p)^4 + \frac{3}{2}p\sum_{j=1}^{p-1}\Bigg(\sigma_i^p\breve{c}_i^{p,2} \\ &+ \sum_{q=j}^{p-1}\left(\prod_{w=q}^{p-1}\sigma_i^q\alpha_i^w\right)\breve{c}_i^{q,2}\Bigg)^2(\lambda_i^p)^2(\lambda_i^j)^2 \\ \le& \sum_{j=1}^{p-1}\zeta_i^{p,j,5}(\lambda_i^j)^4 + \Bigg[\frac{3}{2}p(\sigma_i^p\breve{c}_i^{p,2})^2 + \frac{9}{16}p^2\sum_{j=1}^{p-1}(\zeta_i^{p,j,5})^{-1} \\ &\cdot\left(\sigma_i^p\breve{c}_i^{p,2} + \sum_{q=j}^{p-1}\left(\prod_{w=q}^{p-1}\sigma_i^q\alpha_i^w\right)\breve{c}_i^{q,2}\right)^4\Bigg](\lambda_i^p)^4, \end{aligned} \quad (25)$$

where $\zeta_i^{p,j,5}$ is an arbitrary positive constant.

From (13) and (15), it yields that

$$x_i^p = (\sigma_i^p)^{-1}\lambda_i^p - (\sigma_i^p)^{-1}\alpha_i^{p-1}\lambda_i^{p-1},$$

which means that

$$(x_i^p)^4 \le 8\Big((\sigma_i^p)^{-4}(\lambda_i^p)^4 + (\sigma_i^p)^{-4}(\alpha_i^{p-1})^4(\lambda_i^{p-1})^4\Big). \tag{26}$$

Together (6) with (26), one can get that

$$\begin{aligned}
&e^{\varepsilon_i(t)}\sigma_i^p(\lambda_i^p)^3\sum_{m=1}^{k}\tilde{a}_{im}^p(|x_i^p|+|x_m^p|)\\
\le&\Bigg[k\vartheta(\sigma_i^p)^{\frac{4}{3}}(\lambda_i^p)^4 + \frac{1}{4}\left(\frac{\theta}{\vartheta}\right)^3\sum_{m=1}^{k}(\tilde{a}_{im}^p)^4(x_i^p)^4\\
&+\sum_{m=1}^{k}\vartheta(\sigma_i^p)^{\frac{4}{3}}e^{\frac{1}{3}(\varepsilon_i(t)-\varepsilon_m(t))}(\lambda_i^p)^4\\
&+\frac{1}{4}\left(\frac{\theta}{\vartheta}\right)^3\sum_{m=1}^{k}e^{\varepsilon_m(t)-\varepsilon_i(t)}(\tilde{a}_{im}^p)^4(x_m^p)^4\Bigg]e^{\varepsilon_i(t)}\\
\le&\Bigg[k\vartheta(\sigma_i^p)^{\frac{4}{3}}(1+e^{\frac{1}{3}\tau_i\breve{N}_{\pi_i}})(\lambda_i^p)^4\\
&+\frac{1}{2}\left(\frac{\theta}{\vartheta}\right)^3\sum_{m=1}^{k}(\tilde{a}_{im}^p)^4(x_i^p)^4\Bigg]e^{\varepsilon_i(t)}\\
&+\frac{1}{4}\left(\frac{\theta}{\vartheta}\right)^3\sum_{m=1}^{k}(\tilde{a}_{im}^p)^4(e^{\varepsilon_m(t)}(x_m^p)^4 - e^{\varepsilon_i(t)}(x_i^p)^4)\\
\le&\Bigg\{\Bigg[k\vartheta(\sigma_i^p)^{\frac{4}{3}}(1+e^{\frac{1}{3}\tau_i\breve{N}_{\pi_i}})\\
&+4\left(\frac{\theta}{\vartheta}\right)^3\sum_{m=1}^{k}(\sigma_i^p)^{-4}(\tilde{a}_{im}^p)^4\Bigg](\lambda_i^p)^4\\
&+4\left(\frac{\theta}{\vartheta}\right)^3\sum_{m=1}^{k}(\sigma_i^p)^{-4}(\alpha_i^{p-1})^4(\tilde{a}_{im}^p)^4(\lambda_i^{p-1})^4\Bigg\}e^{\varepsilon_i(t)}\\
&+\frac{1}{4}\left(\frac{\theta}{\vartheta}\right)^3\sum_{m=1}^{k}(\tilde{a}_{im}^p)^4(e^{\varepsilon_m(t)}(x_m^p)^4 - e^{\varepsilon_i(t)}(x_i^p)^4).
\end{aligned} \tag{27}$$

Similar to (27), it is deduced that

$$\begin{aligned}
&e^{\varepsilon_i(t)}(\lambda_i^p)^3\varrho_i^j\sum_{m=1}^{k}\tilde{a}_{im}^j(|x_i^j|+|x_m^j|)\\
\le&e^{\varepsilon_i(t)}\Bigg[k\vartheta(\lambda_i^p)^4 + \frac{1}{4}\left(\frac{\theta}{\vartheta}\right)^3\sum_{m=1}^{k}(\varrho_i^j)^4(\tilde{a}_{im}^j)^4(x_i^j)^4\\
&+\sum_{m=1}^{k}\vartheta e^{\frac{1}{3}(\varepsilon_i(t)-\varepsilon_m(t))}(\lambda_i^p)^4\\
&+\frac{1}{4}\left(\frac{\theta}{\vartheta}\right)^3\sum_{m=1}^{k}e^{\varepsilon_m(t)-\varepsilon_i(t)}(\varrho_i^j)^4(\tilde{a}_{im}^j)^4(x_m^j)^4\Bigg]\\
\le&e^{\varepsilon_i(t)}\Bigg[k\vartheta(1+e^{\frac{1}{3}\tau_i\breve{N}_{\pi_i}})(\lambda_i^p)^4\\
&+4\left(\frac{\theta}{\vartheta}\right)^3\sum_{m=1}^{k}(\varrho_i^j)^4(\tilde{a}_{im}^j)^4(\sigma_i^j)^{-4}(\lambda_i^j)^4\\
&+4\left(\frac{\theta}{\vartheta}\right)^3\sum_{m=1}^{k}(\varrho_i^j)^4(\tilde{a}_{im}^j)^4(\sigma_i^j)^{-4}(\alpha_i^{j-1})^4(\lambda_i^{j-1})^4\Bigg]\\
&+\frac{1}{4}\left(\frac{\theta}{\vartheta}\right)^3\sum_{m=1}^{k}(\varrho_i^j)^4(\tilde{a}_{im}^j)^4\big(e^{\varepsilon_m(t)}(x_m^j)^4\\
&-e^{\varepsilon_i(t)}(x_i^j)^4\big),
\end{aligned} \tag{28}$$

which implies that

$$\begin{aligned}
&e^{\varepsilon_i(t)}(\lambda_i^p)^3\sum_{j=1}^{p-1}\varrho_i^j\sum_{m=1}^{k}\tilde{a}_{im}^j(|x_i^j|+|x_m^j|)\\
\le&e^{\varepsilon_i(t)}\Bigg[(p-1)k\vartheta(1+e^{\frac{1}{3}\tau_i\breve{N}_{\pi_i}})(\lambda_i^p)^4\\
&+4\left(\frac{\theta}{\vartheta}\right)^3\sum_{j=1}^{p-1}\sum_{m=1}^{k}(\varrho_i^j)^4(\tilde{a}_{im}^j)^4(\sigma_i^j)^{-4}(\lambda_i^j)^4\\
&+4\left(\frac{\theta}{\vartheta}\right)^3\sum_{j=1}^{p-1}\sum_{m=1}^{k}(\varrho_i^j)^4(\tilde{a}_{im}^j)^4(\sigma_i^j)^{-4}(\alpha_i^{j-1})^4(\lambda_i^{j-1})^4\Bigg]\\
&+\frac{1}{4}\left(\frac{\theta}{\vartheta}\right)^3\sum_{j=1}^{p-1}\sum_{m=1}^{k}(\varrho_i^j)^4(\tilde{a}_{im}^j)^4\big(e^{\varepsilon_m(t)}(x_m^j)^4\\
&-e^{\varepsilon_i(t)}(x_i^j)^4\big).
\end{aligned} \tag{29}$$

For the sake of conciseness, we define that

$$\begin{aligned}
\tilde{\alpha}_i^p =& \frac{1}{4}\left(\frac{4}{3}\zeta_i^{p,p-1,1}\right)^{-3}(\sigma_i^{p-1})^4\\
&+\sigma_i^p\breve{c}_i^{p,1} + \frac{3}{4}(\sigma_i^p\breve{c}_i^{p,1})^{\frac{4}{3}}\sum_{j=1}^{p-1}(4\zeta_i^{p,j,2})^{-\frac{1}{3}}\\
&+\sigma_i^{p-1}\alpha_i^{p-1}\\
&+\frac{3}{4}\sum_{j=1}^{p-1}(4\zeta_i^{p,j,3})^{-\frac{1}{3}}\left((\sigma_i^{j-1}\alpha_i^{j-1}+\sigma_i^j\alpha_i^j)\prod_{w=j}^{p-1}\alpha_i^w\right)^{\frac{4}{3}}\\
&+\frac{3}{4}\sum_{j=1}^{p-1}(4\zeta_i^{p,j,4})^{-\frac{1}{3}}\left(\breve{c}_i^{p-1,1}\bar{\sigma}_i^{p-1}(p-1)\prod_{w=1}^{p-1}\alpha_i^w\right)^{\frac{4}{3}}\\
&+\frac{9}{16}p^2\sum_{j=1}^{p-1}(\zeta_i^{p,j,5})^{-1}\left(\sigma_i^p\breve{c}_i^{p,2}+\sum_{q=j}^{p-1}\prod_{w=q}^{p-1}\sigma_i^q\alpha_i^w\breve{c}_i^{q,2}\right)^4
\end{aligned}$$

$$+\frac{3}{2}p(\sigma_i^p\breve{c}_i^{p,2})^2+k\vartheta(\sigma_i^p)^{\frac{4}{3}}(1+e^{\frac{1}{3}\tau_i\breve{N}_{\pi_i}})$$
$$+(p-1)k\vartheta(1+e^{\frac{1}{3}\tau_i\breve{N}_{\pi_i}})$$
$$+4\left(\frac{\theta}{\vartheta}\right)^3\sum_{m=1}^{k}(\sigma_i^p)^{-4}(\tilde{a}_{im}^p)^4, \tag{30}$$

Therefore, submitting (21)–(25), (27) and (29) into (18), it can be concluded that

$$\mathcal{L}W_i^p(t,\bar{\lambda}_i^p)$$
$$\leq-\sum_{j=1}^{p-1}(c_i^j-\zeta_i^{p,j})e^{\varepsilon_i(t)}(\lambda_i^j)^4+\dot{\varepsilon}_i(t)W_i^p$$
$$+e^{\varepsilon_i(t)}\Big[\sigma_i^p(\lambda_i^p)^3(\sigma_i^{p+1}x_i^{p+1}-\beta_i^{p+1})+\sigma_i^p(\lambda_i^p)^3\beta_i^{p+1}$$
$$+(\lambda_i^p)^4\tilde{\alpha}_i^p\Big]$$
$$+\frac{1}{4}\left(\frac{\theta}{\vartheta}\right)^3\sum_{j=1}^{p}\sum_{m=1}^{k}(\tilde{a}_{im}^j)^4(e^{\varepsilon_m(t)}(x_m^j)^4-e^{\varepsilon_i(t)}(x_i^j)^4)$$
$$+\frac{1}{4}\left(\frac{\theta}{\vartheta}\right)^3\sum_{j=1}^{p}\sum_{q=1}^{j-1}\sum_{m=1}^{k}(\varrho_i^q)^4(\tilde{a}_{im}^q)^4$$
$$\cdot(e^{\varepsilon_m(t)}(x_m^q)^4-e^{\varepsilon_i(t)}(x_i^q)^4), \tag{31}$$

where

$$\zeta_i^{p,j}=\zeta_i^{p-1,j}+\sum_{v=2}^{7}\zeta_i^{p,j,v}$$
$$+4\left(\frac{\theta}{\vartheta}\right)^3\sum_{m=1}^{k}(\varrho_i^j)^4(\tilde{a}_{im}^j)^4(\sigma_i^j)^{-4}$$
$$+4\left(\frac{\theta}{\vartheta}\right)^3\sum_{m=1}^{k}(\varrho_i^{j+1})^4(\tilde{a}_{im}^{j+1})^4(\sigma_i^{j+1})^{-4}(\alpha_i^j)^4,$$
$$j=1,\cdots,p-2,$$

$$\zeta_i^{p,p-1}=\zeta_i^{p-1,p-1}+\sum_{v=1}^{7}\zeta_i^{p,p-1,v}$$
$$+4\left(\frac{\theta}{\vartheta}\right)^3\sum_{m=1}^{k}(\sigma_i^p)^{-4}(\alpha_i^{p-1})^4(\tilde{a}_{im}^p)^4$$
$$+4\left(\frac{\theta}{\vartheta}\right)^3\sum_{m=1}^{k}(\varrho_i^{p-1})^4(\tilde{a}_{im}^{p-1})^4(\sigma_i^{p-1})^{-4}.$$

Let $\alpha_i^p=\max\{1,\frac{c_i^p+\tilde{\alpha}_i^p}{\sigma_i^p}\}$ where $c_i^p\in\mathbb{R}$ is a design parameter. Design the virtual controller as

$$\beta_i^{p+1}=-\alpha_i^p\lambda_i^p, \tag{32}$$

then we can derive from (31) and (32) that

$$\mathcal{L}W_i^p(t,\bar{\lambda}_i^p)$$
$$\leq-\sum_{j=1}^{p}(c_i^j-\zeta_i^{p,j})e^{\varepsilon_i(t)}(\lambda_i^j)^4+\dot{\varepsilon}_i(t)W_i^p$$
$$+e^{\varepsilon_i(t)}\sigma_i^p(\lambda_i^p)^3(\sigma_i^{p+1}x_i^{p+1}-\beta_i^{p+1})$$
$$+\frac{1}{4}\left(\frac{\theta}{\vartheta}\right)^3\sum_{j=1}^{p}\sum_{m=1}^{k}(\tilde{a}_{im}^j)^4(e^{\varepsilon_m(t)}(x_m^j)^4-e^{\varepsilon_i(t)}(x_i^j)^4)$$
$$+\frac{1}{4}\left(\frac{\theta}{\vartheta}\right)^3\sum_{j=1}^{p}\sum_{q=1}^{j-1}\sum_{m=1}^{k}(\varrho_i^q)^4(\tilde{a}_{im}^q)^4(e^{\varepsilon_m(t)}(x_m^q)^4$$
$$-e^{\varepsilon_i(t)}(x_i^q)^4),$$

where $\zeta_i^{p,p}=0$.

Step n. Based on the induction, we can extend it to n-th step. Define $\bar{\lambda}_i^n=(\lambda_i^1,\cdots,\lambda_i^n)^T$, and $W_i^n(t,\bar{\lambda}_i^n)=\frac{1}{4}\sum_{j=1}^{n}e^{\varepsilon_i(t)}(\lambda_i^j)^4$.

When $t\in[T_l^i,S_l^i]$, let $\lambda_i^n=\sigma_i^nx_i^n-\beta_i^n$, and $\beta_i^{n+1}=-\alpha_i^n\lambda_i^n$ where $\alpha_i^n\geq1$ is a constant. Then it can be acquired that

$$\mathcal{L}W_i^n(t,\bar{\lambda}_i^n)$$
$$\leq-\sum_{j=1}^{n}(c_i^j-\zeta_i^{n,j})e^{\varepsilon_i(t)}(\lambda_i^j)^4+\dot{\varepsilon}_i(t)W_i^n$$
$$+e^{\varepsilon_i(t)}\sigma_i^n(\lambda_i^n)^3(u_i-\beta_i^{n+1})$$
$$+\frac{1}{4}\left(\frac{\theta}{\vartheta}\right)^3\sum_{j=1}^{n}\sum_{m=1}^{k}(\tilde{a}_{im}^j)^4(e^{\varepsilon_m(t)}(x_m^j)^4-e^{\varepsilon_i(t)}(x_i^j)^4)$$
$$+\frac{1}{4}\left(\frac{\theta}{\vartheta}\right)^3\sum_{j=1}^{n}\sum_{q=1}^{j-1}\sum_{m=1}^{k}(\varrho_i^q)^4(\tilde{a}_{im}^q)^4(e^{\varepsilon_m(t)}(x_m^q)^4$$
$$-e^{\varepsilon_i(t)}(x_i^q)^4), \tag{33}$$

where $c_i^n\in\mathbb{R}$ is a design parameter, $\zeta_i^{n,1},\cdots,\zeta_i^{n,n-1}$ are positive constants and $\zeta_i^{n,n}=0$. Based on the above analysis, the event-triggered control strategy is designed as follows

$$\omega_i(t)=-(1+\delta_i)\left[\beta_i^{n+1}\tanh\left(\frac{(\lambda_i^n)^3\beta_i^{n+1}}{\rho}\right)\right.$$
$$\left.+\bar{\eta}\tanh\left(\frac{(\lambda_i^n)^3\bar{\eta}}{\rho}\right)\right], \tag{34}$$
$$u_i(t)=\omega_i(t_{l,k}^i),t\in[T_l^i,S_l^i)\cap[t_{l,k}^i,t_{l,k+1}^i), \tag{35}$$
$$t_{l,k+1}^i=\inf\{t>t_{l,k}^i:|\omega_i(t)-u_i(t)|>\delta_i|u_i(t)|+\eta\}, \tag{36}$$

where $0 < \delta_i < 1, \eta > 0, \bar{\eta} > \frac{\eta}{1-\delta_i}$ and ρ are all positive design parameters, $t^i_{l,k}$ represents the update time, (36) describes the event-triggered mechanism (ETM).

When $t \in [T^i_l, S^i_l] \cap [t^i_{l,k}, t^i_{l,k+1})$, it can be seen that there exist two time-varying functions $\chi_1(t)$ and $\chi_2(t)$ such that

$$\omega_i(t) - u_i(t) = \chi_1(t)\delta_i u_i(t) + \chi_2(t)\eta, \tag{37}$$

where $|\chi_1(t)| \leq 1$ and $|\chi_2(t)| \leq 1$. We rewrite (37) as follows:

$$u_i(t) = \frac{\omega_i(t)}{1+\chi_1(t)\delta_i} - \frac{\chi_2(t)\eta}{1+\chi_1(t)\delta_i}. \tag{38}$$

From the definition of $\omega_i(t)$ as given in (34), it is checked that $(\lambda^n_i)^3\omega_i(t) \leq 0$. Then, the following two inequalities hold:

$$\frac{(\lambda^n_i)^3\omega_i(t)}{1+\chi_1(t)\delta_i} \leq \frac{(\lambda^n_i)^3\omega_i(t)}{1+\delta_i}, \tag{39}$$

$$-\frac{(\lambda^n_i)^3\chi_2(t)\eta}{1+\chi_1(t)\delta_i} \leq \left|\frac{(\lambda^n_i)^3\eta}{1-\delta_i}\right|. \tag{40}$$

By [34], the hyperbolic tangent function $\tanh(\cdot)$ satisfies

$$0 \leq |o| - \tanh\left(\frac{o}{\xi}\right) \leq 0.2785\xi, \tag{41}$$

where $\xi > 0$ and $o \in \mathbb{R}$. Based on $\bar{\eta} > \frac{\eta}{1-\delta_i}$, we have $-\left|(\lambda^n_i)^3\bar{\eta}\right| + \frac{\eta}{1-\delta_i}|\lambda^n_i|^3 < 0$. According to (38)–(41), it yields that

$$\begin{aligned}
&(\lambda^n_i)^3 u_i(t) \\
=&\frac{(\lambda^n_i)^3\omega_i(t)}{1+\chi_1(t)\delta_i} - \frac{(\lambda^n_i)^3\chi_2(t)\eta}{1+\chi_1(t)\delta_i} \\
\leq&\frac{(\lambda^n_i)^3\omega_i(t)}{1+\delta_i} + \frac{\eta}{1-\delta_i}|\lambda^n_i|^3 \\
\leq&-(\lambda^n_i)^3\beta^{n+1}_i \tanh\left(\frac{(\lambda^n_i)^3\beta^{n+1}_i}{\rho}\right) \\
&-(\lambda^n_i)^3\bar{\eta}\tanh\left(\frac{(\lambda^n_i)^3\bar{\eta}}{\rho}\right) + \frac{\eta}{1-\delta_i}|\lambda^n_i|^3 \\
\leq&\left|(\lambda^n_i)^3\beta^{n+1}_i\right| - (\lambda^n_i)^3\beta^{n+1}_i \tanh\left(\frac{(\lambda^n_i)^3\beta^{n+1}_i}{\rho}\right) \\
&+\left|(\lambda^n_i)^3\bar{\eta}\right| - (\lambda^n_i)^3\bar{\eta}\tanh\left(\frac{(\lambda^n_i)^3\bar{\eta}}{\rho}\right) \\
&+(\lambda^n_i)^3\beta^{n+1}_i - \left|(\lambda^n_i)^3\bar{\eta}\right| + \frac{\eta}{1-\delta_i}|\lambda^n_i|^3 \\
\leq&0.557\rho + (\lambda^n_i)^3\beta^{n+1}_i,
\end{aligned} \tag{42}$$

which means that

$$\begin{aligned}
&e^{\varepsilon_i(t)}\sigma^n_i(\lambda^n_i)^3(u_i - \beta^{n+1}_i) \\
\leq&e^{\varepsilon_i(t)}\sigma^n_i\left[0.557\rho + (\lambda^n_i)^3\beta^{n+1}_i - (\lambda^n_i)^3\beta^{n+1}_i\right] \\
\leq&0.557\rho\sigma^n_i e^{\tau_i\check{N}_{\pi_i}}.
\end{aligned} \tag{43}$$

Remark 2. The reason for designing (34) in this way is that the first term of (34) is used to handle $(\lambda^n_i)^3\beta^{n+1}_i$, and the second term can be used to deal with η appearing in (36) by the properties of hyperbolic functions.

Choosing $\gamma_i = 4\min_{1\leq j\leq n-1}\{c^j_i - \zeta^{n,j}_i, c^n_i\}$, by combining (4), (33), with (43), one has

$$\begin{aligned}
\mathcal{L}W^n_i(t,\bar{\lambda}^n_i) \leq& -(\gamma_i - (1-\pi_i)\tau_i)W^n_i + M_1 \\
&+\sum_{j=1}^{n}\sum_{m=1}^{k}\kappa^j_{im}\aleph^j_{im}(t, x^j_i, x^j_m),
\end{aligned} \tag{44}$$

where $M_1 = 0.557\rho\sigma^n_i e^{\tau_i\check{N}_{\pi_i}}$, $\kappa^j_{im} = \frac{1}{4}\left(\frac{\theta}{\vartheta}\right)^3(\tilde{a}^j_{im})^4\Big(1 + (n-j)(\varrho^j_i)^4\Big)$, and $\aleph^j_{im}(t, x^j_i, x^j_m) = e^{\varepsilon_m(t)}(x^j_m)^4 - e^{\varepsilon_i(t)}(x^j_i)^4$.

For brevity, we use the following notational simplification:

$$\begin{aligned}
\iota^n_i =& \frac{1}{4}\left(\frac{4}{3}\zeta^{n,n-1,1}_i\right)^{-3}(\sigma^{n-1}_i)^4 + \sigma^n_i\check{c}^{n,1}_i \\
&+\frac{3}{4}(\sigma^n_i\check{c}^{n,1}_i)^{\frac{4}{3}}\sum_{j=1}^{n-1}(4\zeta^{n,j,2}_i)^{-\frac{1}{3}} + \sigma^{n-1}_i\alpha^{n-1}_i \\
&+\frac{3}{4}\sum_{j=1}^{n-1}(4\zeta^{n,j,3}_i)^{-\frac{1}{3}}\left((\sigma^{j-1}_i\alpha^{j-1}_i + \sigma^j_i\alpha^j_i)\prod_{w=j}^{n-1}\alpha^w_i\right)^{\frac{4}{3}} \\
&+\frac{3}{4}\sum_{j=1}^{n-1}(4\zeta^{n,j,4}_i)^{-\frac{1}{3}}\left(\bar{c}^{n-1,1}_i\bar{\sigma}^{n-1}_i(n-1)\prod_{w=1}^{n-1}\alpha^w_i\right)^{\frac{4}{3}} \\
&+\frac{9}{16}n^2\sum_{j=1}^{n-1}(\zeta^{n,j,5}_i)^{-1}\left(\sigma^n_i\check{c}^{n,2}_i + \sum_{q=j}^{n-1}\prod_{w=q}^{n-1}\sigma^q_i\alpha^w_i\check{c}^{q,2}_i\right)^4 \\
&+\frac{3}{2}n(\sigma^n_i\check{c}^{n,2}_i)^2 + k\vartheta(\sigma^n_i)^{\frac{4}{3}}(1+e^{\frac{1}{3}\tau_i\check{N}_{\pi_i}}) + (n-1) \\
&\cdot k\vartheta(1+e^{\frac{1}{3}\tau_i\check{N}_{\pi_i}}) + 4\left(\frac{\theta}{\vartheta}\right)^3\sum_{m=1}^{k}(\sigma^n_i)^{-4}(\tilde{a}^n_{im})^4,
\end{aligned}$$

where for $j = 1, 2, \cdots, n-1, v = 2, 3, \cdots, 7$, $\zeta^{n,n-1,1}_i$ and $\zeta^{n,j,v}_i$ are arbitrary positive constants, $\bar{c}^{n-1,1}_i = \max\{\check{c}^{1,1}_i, \cdots, \check{c}^{n-1,1}_i\}$, and $\bar{\sigma}^{n-1}_i = \max\{\sigma^1_i, \cdots, \sigma^{n-1}_i\}$.

When $t \in [S^i_l, T^i_{l+1})$, it can be deduced that there exists $\zeta^{n-1,j}_i > 0$ such that

$$\mathcal{L}W^n_i(t, \bar{\lambda}^n_i)$$

$$\leq -\sum_{j=1}^{n-1}(c_i^j - \zeta_i^{n-1,j})e^{\varepsilon_i(t)}(\lambda_i^j)^4 + \dot{\varepsilon}_i(t)W_i^n$$
$$+ \iota_i^n e^{\varepsilon_i(t)}(\lambda_i^n)^4 + \sum_{j=1}^{n}\sum_{m=1}^{k}\kappa_{im}^j \aleph_{im}^j(t,x_i^j,x_m^j)$$
$$\leq -(\tilde{\gamma}_i + \pi_i\tau_i)W_i^n + \sum_{j=1}^{n}\sum_{m=1}^{k}\kappa_{im}^j \aleph_{im}^j(t,x_i^j,x_m^j), \quad (45)$$

where $\tilde{\gamma}_i = 4\min_{1\leq j\leq n-1}\{c_i^j - \zeta_i^{n-1,j}, -\iota_i^n\}$.

Here we introduce M_0 as the desired convergence rate and we impose

$$\gamma_i - (1-\pi_i)\tau_i - M_0 > 0, \quad (46)$$
$$\tilde{\gamma}_i + \pi_i\tau_i - M_0 > 0. \quad (47)$$

From (44)–(47), for any $t \geq t_0$, one can derive that

$$\mathcal{L}W_i^n(t,\bar{\lambda}_i^n) \leq -M_0W_i^n + M_1 + \sum_{j=1}^{n}\sum_{m=1}^{k}\kappa_{im}^j \aleph_{im}^j(t,x_i^j,x_m^j). \quad (48)$$

Up to this point, the AIETC design for each SSFNS is completed.

Remark 3. For conditions (46) and (47), it can be noted that the value of γ_i and $\tilde{\gamma}_i$ can be tuned to some extent through the choice of c_i^j. We first fix the desired convergence rate M_0. Then the control interval is chosen to yield the average control rate π_i. Finally, the tuning parameters τ_i are adjusted so that the design requirements are satisfied.

Remark 4. The controllers are designed recursively via backstepping method. We first construct the initial Lyapunov function W_i^1 and the virtual controller $\beta_i^2 = -\alpha_i^1\lambda_i^1$ with gain $\alpha_i^1 \geq 1$. The variable λ_i^2 is then introduced to link W_i^1 and β_i^2, forming the subsequent Lyapunov function W_i^2. Repeating this recursive construction until the final step, we derive the actual asynchronous intermittent event-triggered controller and obtain the infinitesimal-operator estimates $\mathcal{L}W_i^n$ for each subsystem. In the next subsection, we will construct a global Lyapunov function and integrate these results to complete the boundedness analysis.

3.2 Boundedness Analysis

Based upon the design of AIETC for each subsystem, a strongly connected digraph $(\mathcal{G}^j, \aleph^j)$ is established with $\aleph^j = (\kappa_{im}^j)_{k\times k}$, and r_i^j denotes the cofactor of the i-th diagonal of Laplacian matrix of digraph $(\mathcal{G}^j, \aleph^j)$. We proceed to construct a global Lyapunov function

$$W(t,\lambda) = \sum_{i=1}^{k} r_iW_i^n(t,\bar{\lambda}_i^n), \quad (49)$$

where $\lambda = \left((\bar{\lambda}_1^n)^T, (\bar{\lambda}_2^n)^T, \cdots, (\bar{\lambda}_k^n)^T\right)^T$, $r_i = \prod_{j=1}^{n} r_i^j$. Hence, along each directed cycle $C_{\mathcal{Q}}^j$ of $\mathcal{Q}$, it follows that

$$\sum_{(m,i)\in E(C_{\mathcal{Q}}^j)} \aleph_{im}^j(t,x_i^j,x_m^j) = 0.$$

According to Theorem 2.2 in [35], one has

$$\sum_{i,m=1}^{k} r_i^j\kappa_{im}^j\aleph_{im}^j(t,x_i^j,x_m^j)$$
$$= \sum_{\mathcal{Q}\in\mathbb{Q}}\mathcal{W}(\mathcal{Q})\sum_{(m,i)\in E(C_{\mathcal{Q}}^j)}\aleph_{im}^j(t,x_i^j,x_m^j) = 0,$$

where $\mathbb{Q}$ denotes the set of all spanning unicycle graphs of $(\mathcal{G}^j, \aleph^j)$, $\mathcal{W}(\mathcal{Q})$ stands for the weights of $\mathcal{Q}$. And it can be attained that

$$\sum_{i=1}^{k} r_i\sum_{j=1}^{n}\sum_{m=1}^{k}\kappa_{im}^j\aleph_{im}^j(t,x_i^j,x_m^j) = 0. \quad (50)$$

Consequently, according to (48), (49) and (50), it yields

$$\mathcal{L}W(t,\lambda) \leq -M_0W + M_1\sum_{i=1}^{k} r_i$$
$$\leq -M_0W + M_2, \quad (51)$$

where $r = \max_{1\leq i\leq k}\{r_i\}$, and $M_2 = krM_1$.

Theorem 1. Under Assumption 1 and 2, considering the system (1)–(2) based on the designed controllers with the event-triggered rules as (34)–(36), then the closed-loop system satisfies the following properties:

(i) The system (1)–(2) is globally exponentially ultimately bounded in mean square.

(ii) Zeno behavior of ETM (36) can be avoided.

Proof. From (51), it can be attained that

$$\mathbb{E}W(t,\lambda) \leq W(t_0,\lambda_0)e^{-M_0t} + \frac{M_2}{M_0}. \quad (52)$$

Let $\sigma_i^{n+1} = 1$, $F_1 = \max_{1\leq i\leq k,1\leq j\leq n}\left\{(\sigma_i^j)^{-2}\right\}$, $F_2 = \max_{1\leq i\leq k,1\leq j\leq n}\left\{(\sigma_i^{j+1})^{-2}(\alpha_i^j)^2\right\}$. It follows that

$|x(t)|$

$$=\Big[\sum_{i=1}^{k}\sum_{j=1}^{n}(x_i^j)^2\Big]^{\frac{1}{2}}$$
$$=\Big[\sum_{i=1}^{k}\sum_{j=1}^{n}\Big((\sigma_i^j)^{-1}\lambda_i^j-(\sigma_i^j)^{-1}\alpha_i^{j-1}\lambda_i^{j-1}\Big)^2\Big]^{\frac{1}{2}}$$
$$\leq\Big[2\sum_{i=1}^{k}\sum_{j=1}^{n}((\sigma_i^j)^{-1}\lambda_i^j)^2+2\sum_{i=1}^{k}\sum_{j=0}^{n-1}((\sigma_i^{j+1})^{-1}\alpha_i^j\lambda_i^j)^2\Big]^{\frac{1}{2}}$$
$$\leq(2(F_1+F_2))^{\frac{1}{2}}|\lambda(t)|. \quad (53)$$

By (52), (53) and Schwarz inequality, one obtains

$$\mathbb{E}|x(t)|^2\leq 2(F_1+F_2)\mathbb{E}|\lambda(t)|^2$$
$$\leq 4(F_1+F_2)(kn)^{\frac{1}{2}}\{\mathbb{E}W(t,\lambda)\}^{\frac{1}{2}}$$
$$\leq 4(F_1+F_2)(kn)^{\frac{1}{2}}(W(t_0,\lambda_0)e^{-M_0t}+\frac{M_2}{M_0})^{\frac{1}{2}}$$
$$\leq 4(F_1+F_2)\Big((knW(t_0,\lambda_0))^{\frac{1}{2}}e^{-\frac{1}{2}M_0t}+\sqrt{\frac{knM_2}{M_0}}\Big) \quad (54)$$

Define $M_3=4(F_1+F_2)\sqrt{\frac{knM_2}{M_0}}$. By the definition of $W(t_0,\lambda_0)$, there exists a constant $\varkappa$, such that

$$\mathbb{E}|x(t)|^2\leq\varkappa|x_0|^2e^{-\frac{1}{2}M_0t}+M_3. \quad (55)$$

Based on the above analysis, it is obvious that the system (1)–(2) is globally exponentially ultimately bounded in mean square.

Next, we prove that the Zeno behavior does not happen. Based on Itô rule, $\omega_i(t)$ satisfies the diffusion process

$$\mathrm{d}\omega_i(t)=\mathcal{L}\omega_i(t,x)\mathrm{d}t+\mathcal{H}\omega_i(t,x)\mathrm{d}B(t). \quad (56)$$

Note that $x(t)$ is bounded almost surely, there are two positive constants J_1,J_2 such that $|\mathcal{L}\omega_i|\leq J_1$ and $|\mathcal{H}\omega_i|\leq J_2$.

Let π be the solution of $J_1\pi+4\sqrt{2}J_2\sqrt{\pi}=\eta$. Then we can obtain that $\sqrt{\pi}=\frac{-4\sqrt{2}J_2+\sqrt{32J_2^2+4J_1\eta}}{2J_1}$. Integrating (56) from $t_{l,k}^i$ to $t_{l,k}^i+\pi$ with respect to t and utilizing Burkholder-Davis-Gundy inequality [36], one has

$$\mathbb{E}[|\omega_i(t_{l,k}^i+\pi)-\omega_i(t_{l,k}^i)||\mathcal{F}_{t_{l,k}^i}]$$
$$\leq\mathbb{E}\Big[\Big|\int_{t_{l,k}^i}^{t_{l,k}^i+\pi}\mathcal{L}\omega_i(s,x(s))\mathrm{d}s\Big|\Big|\mathcal{F}_{t_{l,k}^i}\Big]$$
$$+\mathbb{E}\Big[\Big|\int_{t_{l,k}^i}^{t_{l,k}^i+\pi}\mathcal{H}\omega_i(s,x(s))\mathrm{d}B(s)\Big|\Big|\mathcal{F}_{t_{l,k}^i}\Big]$$
$$\leq\mathbb{E}\Big[\int_{t_{l,k}^i}^{t_{l,k}^i+\pi}|\mathcal{L}\omega_i(s,x(s))|\mathrm{d}s|\mathcal{F}_{t_{l,k}^i}\Big]$$
$$+\mathbb{E}\Big[\sup_{t_0\leq s\leq t}\Big|\int_{t_k}^{t_k+\pi}\mathcal{H}\omega_i(s,x(s))\mathrm{d}B(s)\Big|\Big|\mathcal{F}_{t_{l,k}^i}\Big]$$
$$\leq J_1(t_{l,k}^i+\pi-t_{l,k}^i)+4\sqrt{2}J_2\sqrt{t_{l,k}^i+\pi-t_{l,k}^i}$$
$$\leq J_1\pi+4\sqrt{2}J_2\sqrt{\pi}\leq\delta_i|u_i(t)|+\eta. \quad (57)$$

According to the event-triggered mechanism of (36), we find that the controller is triggered only when $|\omega_i(t)-u_i(t)|>\delta_i|u_i(t)|+\eta$. As a result of (57), no event will be triggered at least π units of time after every $t_{l,k}^i$, which implies that at least one positive constant π exists so that $t-t_{l,k}^i\geq\pi$. Therefore, Zeno behavior can be avoided.

This completes the proof of Theorem 1. □

4 A Numerical Example

In this section, we give a simulation example to validate the effectiveness of the results in this article. Consider a robot arm system as follow:

$$J\ddot{\xi}_i(t)+D\dot{\xi}_i(t)+MgL\sin(\xi_i(t))=0,$$

where $\xi_i(t)$, $\dot{\xi}_i(t)$, and $\ddot{\xi}_i(t)$ represent angular position, velocity, and acceleration. J,D,M,g and L denote rotational inertia, damping coefficient, total mass, gravitational constant, and the center-of-mass offset from the joint axis, respectively.

Let $x_i^1(t)=\xi_i(t)$, $x_i^2(t)=\frac{1}{A_i}\dot{\xi}_i(t)$, where A_i is a positive constant, one has

$$\begin{cases}\mathrm{d}x_i^1(t)= & A_ix_i^2(t)\mathrm{d}t,\\ \mathrm{d}x_i^2(t)= & -\frac{1}{JA_i}\Big(Dx_i^2(t)+MgL\sin(x_i^1(t))\Big)\mathrm{d}t.\end{cases}$$

Take stochastic perturbations, coupling factors and control input into account, the system can be written as

$$\begin{cases}\mathrm{d}x_i^1(t)= & \Big(\sigma_i^2x_i^2+\sum\limits_{m=1}^{k}a_{im}^1x_m^1\Big)\mathrm{d}t+q_i^1x_i^1\mathrm{d}B(t),\\ \mathrm{d}x_i^2(t)= & \Big(u_i-\mu_i^1x_i^2-\mu_i^2\sin x_i^1+\sum\limits_{m=1}^{k}a_{im}^2x_m^2\Big)\mathrm{d}t\\ & +q_i^2x_i^2\mathrm{d}B(t),\end{cases} \quad (58)$$

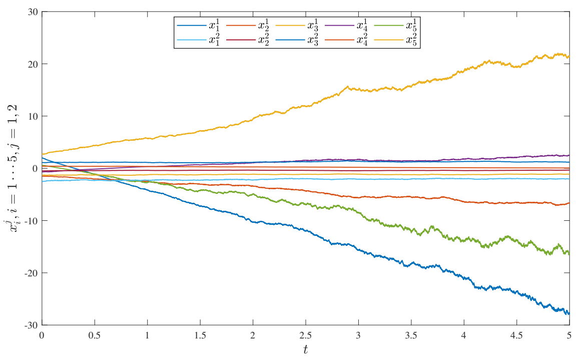
where $\sigma_i^2=A_i$, $\mu_i^1=\frac{D}{JA_i}$, and $\mu_i^2=\frac{MgL}{JA_i}$. By the design scheme developed in Section 3, we first give the following notations:

$$\alpha_i^1=\max\Big\{1,c_i^1+\frac{3}{2}(q_i^1)^2+\frac{1}{4}\Big(\frac{\theta}{\vartheta}\Big)^3\sum_{m=1}^{k}(a_{im}^1)^4$$

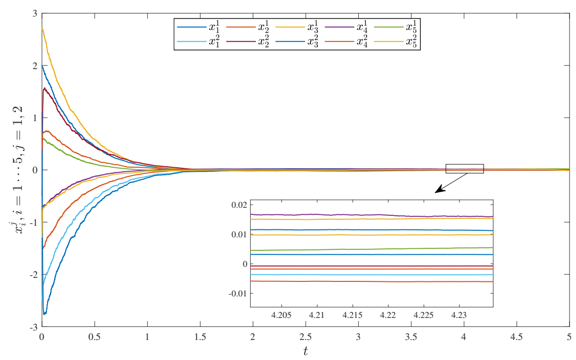
$$
\begin{aligned}
&+ k\vartheta e^{\frac{1}{3}\tau_i \breve{N}_{\pi_i}} \Big\},\\
\lambda_i^2 =&\sigma_i^2 x_i^2 + \alpha_i^1 x_i^1,\\
\beta_i^3 =& -\frac{1}{\sigma_i^2}\Bigg[c_i^2 + \frac{27}{4} + \frac{3}{4}(\sigma_i^2\mu_i^1)^{\frac{4}{3}} + \frac{3}{4}(\mu_i^2\alpha_i^1)^{\frac{4}{3}} + \mu_i^2\\
&+ \alpha_i^1 + \frac{3}{4}(\alpha_i^1)^{\frac{8}{3}} + 6(q_i^2)^2 + \frac{3}{2}(q_i^1)^2(\alpha_i^1)^4\\
&+ 3(q_i^2)^2(\alpha_i^1)^4 + 2\left(\frac{\theta}{\vartheta}\right)^3 \sum_{m=1}^{k} (a_{im}^2)^4(\sigma_i^2)^{-4}\\
&+ k\vartheta(\sigma_i^2)^{\frac{4}{3}} e^{\frac{1}{3}\tau_i \breve{N}_{\pi_i}} + k\vartheta e^{\frac{1}{3}\tau_i \breve{N}_{\pi_i}}\Bigg](\sigma_i^2 x_i^2 + \alpha_i^1 x_i^1).
\end{aligned}
$$

Then we design the controllers as

$$
\begin{aligned}
\omega_i(t) =& -(1+\delta_i)\left[\beta_i^3 \tanh\left(\frac{(\lambda_i^2)^3\beta_i^3}{\rho}\right)\right.\\
&\left. + \bar{\eta}\tanh\left(\frac{(\lambda_i^2)^3\bar{\eta}}{\rho}\right)\right],\\
u_i(t) =& \omega_i(t_{l,k}^i), t \in [T_l^i, S_l^i) \cap [t_{l,k}^i, t_{l,k+1}^i).
\end{aligned}
$$



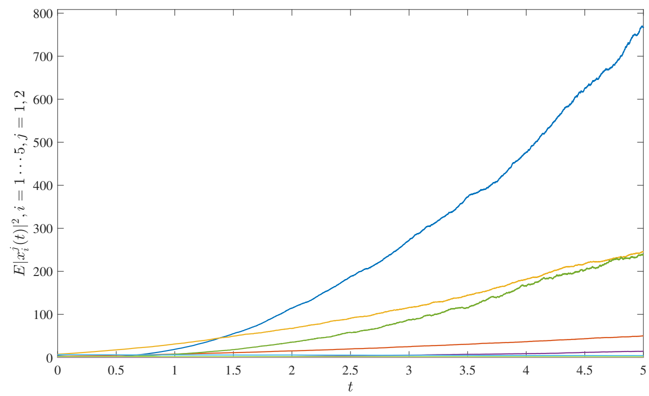
(a)



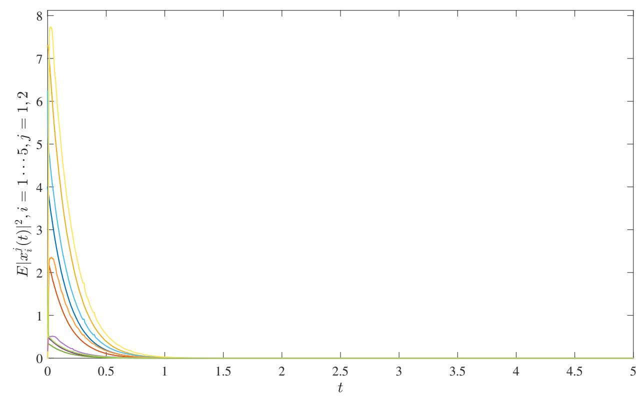
(b)

Figure 1. (a) The state trajectories of system (58) without control. (b) The state trajectories of system (58) with control.

For simulation, select $k = 5$, $J = 4.9$, $D = 0.637$, $M = 0.5$, $g = 9.8$, $L = 0.13$. For $i = 1, \cdots, 5$, we choose $A_i = 2.6$ which means that $\sigma_i^2 = 2.6$, $\mu_i^1 = \mu_i^2 = 0.05$. Let $q_i^1 = q_i^2 = 0.1, i = 1,2,3, q_i^1 = 0.2, q_i^2 = 0.1, i = 4,5$. Set $\delta_i = 0.7, i = 1, \cdots, 5$, and $\eta = 0.1$, $\bar{\eta} = 5$, $\rho = 0.25$, $\vartheta = 0.018$. For $i = 1, \cdots, 4$, let $a_{i+1,i}^1 = a_{1,5}^1 = 0.005$, $a_{i,i+1}^2 = a_{5,1}^2 = 0.006$, else $a_{im}^j = 0$.



(a)



(b)

Figure 2. (a) The mean square trajectories of system (58) without control. (b) The mean square trajectories of system (58) with control.

The control intervals for subsystems $i = 1,2,3$ are chosen as $\cup_{l=0}^{\infty}([0.4l, 0.4l+0.3) \cup [0.4l+0.313, 0.4l+0.389)$, while for $i = 4,5$, they are $\cup_{l=0}^{\infty}([0.5l, 0.5l+0.2) \cup [0.5l+0.212, 0.5l+0.487))$. Clearly, the intermittent control is asynchronous. One can calculate that $\pi_i = 0.94, \breve{N}_{\pi_i} = 0.01222, i = 1,2,3$ and $\pi_i = 0.95, \breve{N}_{\pi_i} = 0.01235, i = 4,5$. Choosing $\tau_i = 107.5$, $c_i^1 = 2.67$, $c_i^2 = 1.65$, $i = 1,2,3$, and $\tau_i = 126$, $c_i^1 = 2.68$, $c_i^2 = 1.6$, $i = 4,5$. Additionally, set $M_0 = -0.001$. It can be attained that $\gamma_i - (1-\pi_i)\tau_i - M_0 = 0.0294 > 0$, $\tilde{\gamma}_i + \pi_i\tau_i - M_0 = 0.0581 > 0, i = 1,2,3$ and $\gamma_i - (1-\pi_i)\tau_i - M_0 = 0.099 > 0$, $\tilde{\gamma}_i + \pi_i\tau_i - M_0 = 1.4468 > 0, i = 4,5$. It is evident that all required conditions are satisfied. Figure 1 (a) presents the state trajectories of system (58) without

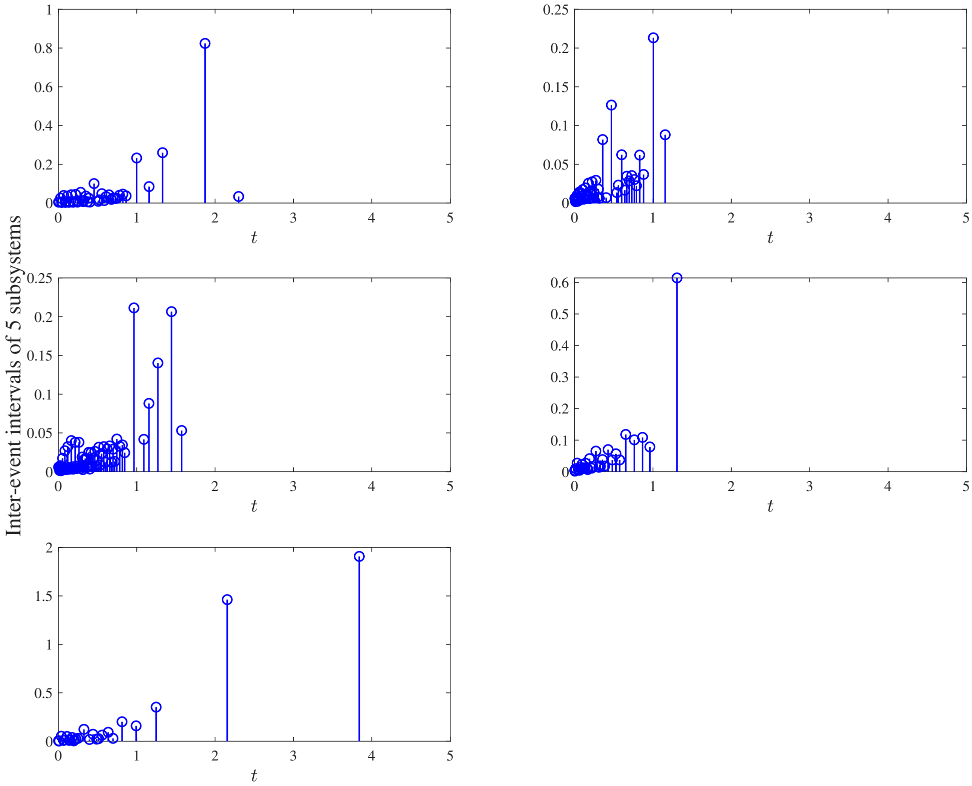


Figure 3. Event-triggered time intervals of 5 subsystems.

control. The state trajectories of system (58) with control are shown in Figure 1 (b), which indicates that they are all bounded under AIETC. The mean square trajectories of system (58) without control and with control are exhibited in Figure 2 (a) and (b), respectively. Figure 3 describes the intervals of ETM. Therefore, the effectiveness of our theoretical results is demonstrated.

5 Conclusion

In the paper, an AIETC scheme has been investigated for a class of CSSFNSs. Based on the backstepping technique, graph theory and Lyapunov approach, the asynchronous intermittent event-triggered controllers were designed. The proposed control strategy synergistically integrated the energy-saving merits of both intermittent control and ETC. Moreover, by accommodating potential real-world implementation constraints, the designed controllers were permitted to operate in an asynchronous manner. In the end, a simulation example was proposed to show the effectiveness of the control strategy. In the future, we will consider the time delay of CSSFNSs.

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Data will be made available on request.

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Conflicts of Interest

The authors declare no conflicts of interest.

Ethical Approval and Consent to Participate

Not applicable.

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