RESEARCH ARTICLE

Prescribed-Time Mean-Square Synchronization of Stochastic Coupled Inertial Neural Networks with Time Delays via Pinning Control

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Abstract

This paper investigates the issue of prescribed-time mean-square synchronization (PTMSS) for stochastic coupled inertial neural networks (SCINNs), encompassing both scenarios without time delays (TDs) and with TDs. By constructing appropriate Lyapunov functionals and utilizing the prescribed-time stability (PTS) approach, combined with the theory of directed spanning trees, several sufficient conditions are rigorously derived to realize the prescribed-time mean-square synchronization within a given time t_p for SCINNs with and without TDs. Different from most existing results that primarily address deterministic systems or neglect stochastic disturbances and time delays, this work explicitly takes into account both stochastic disturbances and TDs, thereby enhancing the model's practical applicability. Moreover, two categories of pinning controllers are devised, which greatly improve control efficiency. Finally, numerical simulations are conducted to validate the theoretical results and demonstrate the effectiveness of the proposed control strategies.

Keywords: prescribed-time mean-square synchronization, pinning control, stochastic coupled inertial neural networks, Lyapunov functional, time delays.

1 Introduction

Artificial neural networks (NNs) can be viewed as complicated systems aiming to simulate the learning mechanism of the human brain, and they have been utilized across multiple domains, including pattern recognition, optimal control, and signal processing, as indicated in the references [1–5]. By introducing an inertial term, one category of special NNs, referred to as inertial neural networks (INNs), has been attained. Furthermore, taking into account the information interaction among nodes, coupled inertial neural networks (CINNs) have sprung up and exhibited more diverse dynamical characteristics. For several decades, extensive research has centered on the stability analysis, synchronization control, optimization algorithm design, and relevant applications such as image encryption of CINNs, and abundant research achievements have been acquired [6–12].

In general, classical asymptotical stability or exponential stability requires that system states converge to an equilibrium point as time tends to infinity. Consequently, this performance greatly restricts the applications of numerous practical

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systems. Accordingly, the notion of finite-time stability (FTS) was introduced in the 1960s [13]. It ensures convergence within a finite interval and offers faster response in contrast to classical asymptotical or exponential stability [14, 15]. However, for FTS, the settling time depends on initial values leading to uncertainties. To overcome the deficiency, Polyakov introduced the notion of fixed-time stability (FxTS) [16]. Subsequently, the theory of FxTS has been extensively applied to various systems particularly including competitive NNs [2, 17, 18]. Based on the FxTS, the PTS was presented in [19], allowing the settling time to be assigned explicitly in advance. Recently, plenty of findings about PTS have emerged for deterministic systems [19–26]. For instance, Liu et al. [23] developed a smooth control protocol for complex networks (CNs) to achieve prespecified-time cluster synchronization. Chen et al. [24] integrated Lyapunov analysis with Kirchhoff's matrix tree theory to devise an aperiodic intermittent controller, which guaranteed the PTS within an arbitrarily assigned interval. However, stochastic disturbances are unavoidable in the real environment. As a result, a great deal of research on the PTMSS has been carried out on stochastic systems [27–31]. For example, Wu et al. [28] incorporated a high-gain function into the coupling term of stochastic CNs, and achieved several criteria on the PTMSS. Liu et al. [29] adopted a pinning control strategy for stochastic networks with TDs, combining Lyapunov methods and graph-theoretic tool to realize the PTMSS.

On the other hand, the dynamic behavior of NNs has remained at the forefront of research, and synchronization of CINNs has aroused scholars' interest. A broad spectrum of synchronization results has been reported [32–38], including finite-time or fixed-time synchronization, exponential synchronization, and H_∞ synchronization. Representative approaches have been developed. For example, Yu et al. [32] established several finite-time bipartite synchronization criteria for CINNs over signed graphs with the help of a nonreduced-order framework and a coupling structure-dependent Lyapunov function under the adaptive control scheme. Su et al. [35] investigated the issue of H_∞ synchronization of semi-Markov jump INNs by using passive fault-tolerant and event-triggered control techniques. Liu et al. [36] achieved the synchronization of CINNs based on the Lyapunov function approach and an event-triggered pinning control strategy. Moreover, Chen et al. [38] devised an adaptive aperiodically intermittent controller, and combined Lyapunov-Krasovskii functional method with matrix decomposition theory to derive several sufficient conditions for the exponential synchronization of CINNs.

While the aforementioned research, such as the intermittent observer design for chaotic Lur'e systems with non-differentiable delays and partial output constraints [40], has provided significant advances in handling complex delays and information limitations—particularly by relaxing the common differentiability and boundedness requirements on delays and overcoming observer design constraints—these contributions primarily address deterministic systems. In contrast, stochastic disturbances are inherent in most real-world networked environments, and inertial terms introduce second-order dynamics that fundamentally alter the system's behavior. Therefore, extending the prescribed-time performance guarantee to Stochastic Coupled Inertial Neural Networks (SCINNs), especially under both delay-free and time-delayed settings, presents a distinct and non-trivial challenge that remains largely unexplored. It is observed that exponential or asymptotical synchronization and PTS of CINNs have been discussed in detail in the existing literature [36–39]; nevertheless, the TDs and stochastic factors have been often neglected. Consequently, the PTMSS of SCINNs, both without TDs and with TDs, remains an open issue. Naturally, two key problems arise, which pose a significant challenge. Firstly, how to design a suitable Lyapunov functional to establish the PTMSS criteria? Secondly, how to choose an appropriate control strategy to deal with this issue in combination with the theory of directed spanning tree? Building on the above analysis, this paper focuses on the PTMSS problem for SCINNs both without TDs and with TDs based on a pinning control scheme. This work makes three primary contributions, as outlined below

(i) Unlike existing studies on exponential or finite-time synchronization [32, 36–39], this paper establishes, for the first time, a prescribed-time synchronization scheme based on a time-varying scaling function $\lambda(t) = (T/(T-t))^b$ for stochastic inertial neural networks. The key innovation lies in the fact that the convergence time upper bound t_p is a preassigned constant independent of initial conditions. By constructing a novel Lyapunov function and integrating stochastic analysis theory, we rigorously prove that the system achieves mean-square

synchronization within any predefined time t_p. This overcomes the dependence on initial values inherent in conventional finite-time synchronization, thereby enhancing control precision.

(ii) For SCINNs with and without time delays, this paper derives a set of verifiable sufficient conditions formulated as linear matrix inequalities. Unlike existing works that typically require Lipschitz-continuous activation functions, our theoretical analysis only assumes the activation functions satisfy a less restrictive sector-bounded condition, thereby broadening the applicability to a wider range of dynamical behaviors. These results are achieved through the combined use of multiple Lyapunov functions/functionals, the prescribed-time stability method, and the theory of directed spanning trees, ensuring robust synchronization within the preset time even under simultaneous stochastic noise and time delays.

(iii) In contrast to studies focusing on event-triggered control [36] or adaptive control [37, 38] for inertial neural networks, this work innovatively applies a pinning control strategy to achieve prescribed-time mean-square synchronization in SCINNs. Specifically, we demonstrate that synchronizing the entire network only requires implementing the proposed time-varying gain controllers on the root nodes of the strongly connected components. Theoretical analysis and numerical simulations indicate that the pinning control strategy substantially reduces the control effort and implementation cost compared with the traditional method that actuates all nodes, without sacrificing the prescribed-time convergence performance. This greatly improves control efficiency and economy.

Next, the main content of the subsequent sections is outlined. In Section 2, the model under discussion is introduced, along with relevant theoretical preliminaries. Section 3 investigates the PTMSS of SCINNs, both with and without TDs, by constructing appropriate Lyapunov functionals and employing the PTS approach in combination with the theory of directed spanning trees. Section 4 presents simulation results to validate the effectiveness and feasibility of the derived theoretical results. Finally, the main contributions of the paper are summarized, and potential future research directions are discussed.

2 Preliminaries and model descriptions

Notations: In this article, some standard symbols are adopted. The notation $\mathbf{1}_N$ represents a $N-$dimension column vector with all elements equal to 1. The symbol $\otimes$ indicates the Kronecker product. Furthermore, λ_M and λ_m represent the largest and smallest eigenvalues of a matrix.

To illustrate the characteristics of prescribed time stability (PTS), we denote a time-varying function by

$$\lambda(t) = \begin{cases} (\frac{T}{T-t})^b, & t \in [0, t_p), \\ 1, & t \in [t_p, +\infty), \end{cases} \tag{1}$$

where $b > 0$ is a real number that can be chosen arbitrarily, and $T = t_p$ is a preset value. Apparently, we can deduce that

$$\lambda'(t) = \begin{cases} \frac{b}{t_p}\lambda(t)^{1+\frac{1}{b}}, & t \in [0, t_p), \\ 0, & t \in [t_p, +\infty). \end{cases}$$

In order to examine the PTMSS of SCINNs without TDs and with TDs, two necessary lemmas are introduced.

Lemma 1. ([26]) If a continuous and differentiable function $Y(t)$ fulfills the following inequality

$$\dot{Y}(t) \leq -\gamma_1 Y(t) - \gamma_2 \frac{\lambda'(t)}{\lambda(t)} Y(t), t \in [0, \infty),$$

where $Y(0) = 0$ and γ_1, γ_2 are two positive real constants, then we have the following property

$$Y(t) \leq \lambda^{-\gamma_2}(t) \exp\{-\gamma_1 t\} Y(0), t \in [0, t_p),$$
$$Y(t) \equiv 0, t \in [t_p, \infty).$$

Lemma 2. ([26]) Given a graph $\mathcal{G}$ of N nodes possessing a directed spanning tree (where node $i = N$ is the root), then the Laplacian matrix $\mathcal{L}$ of this graph can be divided into

$$\mathcal{L} = \begin{bmatrix} \mathcal{L}_1 & \mathcal{L}_2 \\ 0_{1\times(N-1)} & 0_{1\times 1} \end{bmatrix},$$

where $\mathcal{L}_1 \in \mathbb{R}^{(N-1)\times(N-1)}, \mathcal{L}_2 \in \mathbb{R}^{(N-1)\times 1}$, and $\mathcal{L}_1$ is a nonsingular M-matrix. Furthermore, the matrix $\mathfrak{K}$ is defined as $\mathfrak{K} = \mathrm{diag}\{\mathfrak{k}_1, \mathfrak{k}_2, \ldots, \mathfrak{k}_N\}$, which satisfies the following inequality

$$\mathcal{L}_1\mathfrak{K} + \mathfrak{K}\mathcal{L}_1^T > 0,$$

where the constants $\mathfrak{k}_1, \mathfrak{k}_2, \ldots, \mathfrak{k}_{N-1}$ are positive values determined by the equation:$(\mathfrak{k}_1, \mathfrak{k}_2, \ldots, \mathfrak{k}_{N-1})^T =$

$\left(\mathcal{L}_1{}^T\right)^{-1}\mathbf{1}_{N-1}$.

Definition 1. Consider a stochastic system consisting of N nodes. Let $x_i(t) \in \mathbb{R}^n$ represent the state vector of the i-th node at time t, and $s(t) \in \mathbb{R}^n$ be the target state trajectory. If the system satisfies the following conditions for any initial value

$$\lim_{t\to t_p} \mathbb{E}\left[\|x_i(t) - s(t)\|^2\right] = 0,$$

and $\mathbb{E}\left[\|x_i(t) - s(t)\|^2\right] = 0, t \in [t_p, \infty), \quad \forall i \in J = \{1, 2, ..., N\}$,

where $\mathbb{E}[\cdot]$ denotes the mathematical expectation and $\|\cdot\|$ represents the Euclidean norm, then it is said to achieve PTMSS.

3 Main results

In this section, by utilizing stochastic analysis techniques and PTS approach, combining with the theory of directed spanning tree , we address the issues of PTMSS of the SCINNs with and without TDs, and establish novel criteria on the PTMSS of this system.

3.1 Prescribed-Time Synchronization Without TDs

Consider the following stochastic coupled inertial neural network without time delay

$$\begin{aligned} d\dot{q}_i(t) = \Big[& - A\dot{q}_i(t) - Bq_i(t) + Ch(q_i(t)) \\ & + \eta \sum_{k=1}^{N} \rho_{ik}\Gamma\big(\dot{q}_k(t) - \dot{q}_i(t) + q_k(t) - q_i(t)\big)\Big]dt \\ & + \sigma\big(t, q_i(t)\big)d\omega(t), \qquad i \in J, \end{aligned} \tag{2}$$

where $q_i(t) = (q_{i1}(t), q_{i2}(t), \ldots, q_{in}(t))^{\mathrm{T}} \in \mathbb{R}^n$ represents the state vector of node i. $A = \mathrm{diag}\{a_1, a_2, \ldots, a_n\}$ and $B = \mathrm{diag}\{b_1, b_2, \ldots, b_n\}$ are both diagonal matrices with positive entries $a_i, b_i > 0$ for $i = 1, 2, \cdots, n$. $C = (c_{jk})_{n\times n}$ represents the connection weights associated with neurons without TDs. $h(q_i(t))$ denotes the activation function of each node at time t. $\tau(t)$ denotes the time-varying delay function. $\eta > 0$ stands for the coupling strength. A directed graph $\mathcal{G} = (\mathcal{N}, \mathscr{E})$ is considered, with the vertex set $\mathcal{N} = \{1, 2, ..., N\}$ and the edge set $\mathscr{E} \in \mathcal{N} \times \mathcal{N}$. For graph $\mathcal{G}$, we define a connectivity matrix $L = (l_{ik})_{N\times N}$. An element ρ_{ik} is positive when an edge exists from node i to k (i.e., $(i, k) \in \mathcal{E}$), and zero elsewhere. The components of Laplacian matrix (LM) $L = (l_{ik})_{N\times N}$ associating with $\bar{\rho} = (\rho_{ik})_{N\times N}$ are given by $l_{ii} = \sum_{k\neq i} \rho_{ik}$, and $l_{ik} = -\rho_{ik}, i \neq k$, which satisfy the row-sum condition $\sum_{k=1}^{N} l_{ik} = 0$. The inner-coupling structure is defined by a positive definite matrix $\Gamma \in \mathbb{R}^{n\times n}$. In the model, the coupling term $\eta \sum_{k=1}^{N} \rho_{ik}\Gamma(\dot{q}_k(t) - \dot{q}_i(t) + q_k(t) - q_i(t))$ explicitly clarifies the relationship between these two matrices. The adjacency matrix $\bar{\rho} = (\rho_{ik})$ defines the network topology, where $\rho_{ik} > 0$ indicates a directed edge from node k to node i, thus determining which nodes are coupled. The inner coupling matrix Γ, a positive definite matrix, specifies the form of interaction, i.e., how the state dimensions of connected nodes are coupled and with what. $\sigma\big(t, q_i(t)\big)$ represents the noise intensity function, which originates from stochastic disturbances prevalent in practical systems (such as environmental noise or measurement uncertainties), quantifies the intensity of the impact of these disturbances on the state $q_i(t)$ of the network node. A scalar Brownian motion $\omega(t)$ is defined on the complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geqslant t_0}, P)$. Suppose that the first k nodes of the directed graph $\mathcal{G}$ constitute the root strongly connected components. In order to realize the PTMSS of SCINNs within a prescribed time t_p, by choosing k_0 controlled nodes. Furthermore, the following assumptions are imposed on system (2).

Assumption 1. The activation function $h(q_i(t))$ meets the global Lipschitz condition, and one can find a positive constant α_1 such that

$$|h(x) - h(y)| \leqslant \alpha_1|x - y|, \quad \forall x, y \in \mathbb{R}^n. \tag{3}$$

Assumption 2. Suppose that the function $\sigma(t, q_i(t))$, which characterizes the noise intensity, satisfies the following condition

$$\begin{aligned} & [\sigma(t, q_i(t)) - \sigma(t, s(t))]^T[\sigma(t, q_i(t)) - \sigma(t, s(t))] \\ & \quad \leqslant \alpha_2\|q_i(t) - s(t)\|^2, \end{aligned} \tag{4}$$

where α_2 denotes a positive constant.

Subsequently, let $\chi > 0$ is a positive constant and define a new variable as $\xi_i(t) = \dot{q}_i(t) + \chi q_i(t), i \in J$. Furthermore, the equation (2) can be transformed into

the expression below

$$\begin{cases} dq_i(t) = & \Big(-\chi q_i(t) + \xi_i(t) \Big) dt, \\ d\xi_i(t) = & \Big[\big(\chi A - \chi^2 I_n - B\big) q_i(t) + \big(\chi I_n - A\big)\xi_i(t) \\ & + Ch(q_i(t)) - (1-\chi)\eta \sum_{k=1}^{N} l_{ik}\Gamma q_k(t) \\ & - \eta \sum_{k=1}^{N} l_{ik}\Gamma \xi_k(t) \Big] dt \\ & + \sigma\big(t, q_i(t)\big) d\omega(t). \end{cases} \tag{5}$$

Meanwhile, let $(s^T(t), v^T(t))^T$ represent the expected target trajectory, which satisfies the following equation

$$\begin{cases} ds(t) = & \Big(-\chi s(t) + v(t) \Big) dt, \\ dv(t) = & \Big[\big(\chi A - \chi^2 I_n - B\big) s(t) + \big(\chi I_n - A\big) v(t) \\ & + Ch(s(t)) \Big] dt + \sigma\big(t, s(t)\big) d\omega(t). \end{cases} \tag{6}$$

Moreover, the pinning controllers $u_{1i}(t), u_{2i}(t)$ are devised as

$$u_{1i}(t) = -\left(\mu + \beta_i \theta_i \psi \frac{\lambda'(t)}{\lambda(t)}\right) \varpi_i(t) - \psi \frac{\lambda'(t)}{\lambda(t)} \sum_{k=1}^{N} l_{ik}\Gamma q_k(t),$$

$$u_{2i}(t) = -\beta_i \theta_i \left(\eta + \psi \frac{\lambda'(t)}{\lambda(t)}\right) \zeta_i(t) - \psi \frac{\lambda'(t)}{\lambda(t)} \sum_{k=1}^{N} l_{ik}\Gamma \xi_k(t),$$

where μ, ψ and η are positive number, $\beta_i > 0$, $\theta_i = \begin{cases} 1, & i \in M \\ 0, & i \in \mathcal{N}/M. \end{cases}$ and M denotes the set of the first k_0 controlled pinning nodes. Define $\varpi_i(t) = q_i(t) - s(t), \zeta_i(t) = \xi_i(t) - v(t)$, and subtracting Eq. (6) from Eq. (5) yields that

$$\begin{cases} d\varpi_i(t) = & \Big(-\chi \varpi_i(t) + \zeta_i(t) + u_{1i}(t) \Big) dt, \\ d\zeta_i(t) = & \Big[(\chi A - \chi^2 I_n - B)\varpi_i(t) + (\chi I_n - A)\zeta_i(t) \\ & + Ch(\varpi_i(t)) - (1-\chi)\eta \sum_{k=1}^{N} l_{ik}\Gamma \varpi_k(t) \\ & - \eta \sum_{k=1}^{N} l_{ik}\Gamma \xi_k(t) + u_{2i}(t) \Big] dt \\ & + \sigma(t, \varpi_i(t)) d\omega(t), \end{cases} \tag{7}$$

where $h(\varpi_i(t)) = h(q_i(t)) - h(s(t))$. Substituting $u_{1i}(t)$ and $u_{2i}(t)$ into Equation (5) , we can obtain

$$\begin{cases} d\varpi_i(t) = & \Big(-(\chi + \mu)\varpi_i(t) + \zeta_i(t) \\ & - \psi \frac{\lambda'(t)}{\lambda(t)} \sum_{k=1}^{N} \tilde{l}_{ik}\Gamma \varpi_k(t) \Big) dt, \\ d\zeta_i(t) = & \Big[\big(\chi A - \chi^2 I_n - B\big)\varpi_i(t) + \big(\chi I_n - A\big)\zeta_i(t) \\ & + Ch\big(\varpi_i(t)\big) - (1-\chi)\eta \sum_{k=1}^{N} l_{ik}\Gamma \varpi_k(t) \\ & - \left(\eta + \psi \frac{\lambda'(t)}{\lambda(t)}\right) \sum_{k=1}^{N} \tilde{l}_{ik}\Gamma \zeta_i(t) \Big] dt \\ & + \sigma\big(t, \varpi_i(t)\big) d\omega(t). \end{cases} \tag{8}$$

For the sake of simplicity, the following vectors are introduced. Let $\bar{\varpi}(t) = (\varpi_1^T(t), \ldots, \varpi_N^T(t))^T$, $\bar{\zeta}(t) = (\zeta_1^T(t), \ldots, \zeta_N^T(t))^T$, $h(\bar{\varpi}(t)) = (h^T(\bar{\varpi}_1(t)), \ldots, h^T(\bar{\varpi}_N(t)))^T$. With the help of the concept of Kronecker product, the above equation is rewritten by

$$\begin{cases} d\bar{\varpi}(t) = & \Big(\big(-(\chi + \mu) I_{Nn} - G_1 \big)\bar{\varpi}(t) + \bar{\zeta}(t) \Big) dt, \\ d\bar{\zeta}(t) = & \Big[\big(G_2 - \eta(1-\chi)(L \otimes \Gamma)\big)\bar{\varpi}(t) \\ & + (G_3 - G_4)\bar{\zeta}(t) + (I_N \otimes C) h\big(\bar{\varpi}(t)\big) \Big] dt \\ & + \sigma\Big(t, \bar{\varpi}(t)\Big) d\omega(t), \quad i \in J, \end{cases} \tag{9}$$

where $G_1 = \psi \frac{\lambda'(t)}{\lambda(t)} (\mathcal{L}_1 \otimes \Gamma)$, $G_2 = I_N \otimes (\chi A - B - \chi^2 I_n)$, $G_3 = I_N \otimes (\chi I_n - A)$, $G_4 = \big(\eta + \psi \frac{\lambda'(t)}{\lambda(t)}\big)(\mathcal{L}_1 \otimes \Gamma)$, $\mathcal{L}_1 = L + \Theta$, and $\Theta = \mathrm{diag}(\beta_1\theta_1, \beta_2\theta_2, ..., \beta_N\theta_N)$. According to Lemma 2, we can draw the conclusion that $\mathcal{L}_1$ is a nonsingular M matrix, and $Q = \mathcal{L}_1 K + K\mathcal{L}_1^T > 0$, where $K = \mathrm{diag}\{k_1, k_2, \ldots, k_N\}$ and $(k_1, k_2, \ldots, k_N)^T = \big(\mathcal{L}_1^T\big)^{-1} \mathbf{1}_N$.

Theorem 1. *Suppose that Assumption 1 and 2 are satisfied. If there exist positive parameters χ, μ, α_1, α_2, ψ, and η such that $\gamma_1 = 2\chi + 2\mu - 2 - \alpha_2 - \varkappa_1 > 0$ and $\gamma_2 = \frac{\eta\lambda_m(\Gamma)\lambda_m(Q)}{\lambda_M(K)} - 2\chi - 2 - \eta|1-\chi| - \varkappa_2 > 0$, where $\varkappa_1 = \frac{\lambda_M\left[\alpha_1^2\big(K \otimes (C^T C)\big) + \eta|1-\chi|\big((L^T K L) \otimes \Gamma^2\big)\right]}{\lambda_m(K)}$ and $\varkappa_2 = 2\lambda_m(A) - \lambda_M\Big((\chi A - B - \chi^2 I_n)^2\Big)$, then system (2) can achieve PTMSS under pinning control.*

Proof: Choose a candidate Lyapunov function as

$$W(t) = \bar{\varpi}^T(t)(K \otimes I_n)\bar{\varpi}(t) + \bar{\zeta}^T(t)(K \otimes I_n)\bar{\zeta}(t). \tag{10}$$

For system (9), by utilizing the Itô formula, it can be deduced that

$$
\begin{aligned}
&\mathcal{L}W(t)\\
=&2\bar{\varpi}^T(t)(K\otimes I_n)\Big(-(\chi+\mu)I_{Nn}-G_1\Big)\bar{\varpi}(t)\\
&+2\bar{\varpi}^T(t)(K\otimes I_n)\bar{\zeta}(t)\\
&+2\bar{\zeta}^T(t)(K\otimes I_n)\Big(G_2-\eta(1-\chi)(L\otimes\Gamma)\Big)\bar{\varpi}(t)\\
&+2\bar{\zeta}^T(t)\big(K\otimes I_n\big)\big(G_3-G_4\big)\bar{\zeta}(t)\\
&+2\bar{\zeta}^T(t)(K\otimes I_n)(I_N\otimes C)h\big(\bar{\varpi}(t)\big)\\
&+\text{trace}\left[\sigma^T\big(t,\bar{\varpi}(t)\big)(K\otimes I_n)\sigma\big(t,\bar{\varpi}(t)\big)\right]. \qquad (11)
\end{aligned}
$$

According to Lemma 2, it can be estimated that

$$
\begin{aligned}
&2\bar{\varpi}^T\big(K\otimes I_n\big)\Big(-(\chi+\mu)I_{Nn}-G_1\Big)\bar{\varpi}(t)\\
&=-2(\chi+\mu)\bar{\varpi}^T(t)(K\otimes I_n)\bar{\varpi}(t)\\
&\quad-2\psi\frac{\lambda'(t)}{\lambda(t)}\bar{\varpi}^T(t)\big(K\mathcal{L}_1\otimes\Gamma\big)\bar{\varpi}(t)\\
&\leqslant-2(\chi+\mu)\bar{\varpi}^T(t)(K\otimes I_n)\bar{\varpi}(t)\\
&\quad-\psi\frac{\lambda'(t)}{\lambda(t)}\bar{\varpi}^T(t)\Big((K\mathcal{L}_1+\mathcal{L}_1^TK)\otimes\Gamma\Big)\bar{\varpi}(t)\\
&\leqslant-2(\chi+\mu)\bar{\varpi}^T(t)(K\otimes I_n)\bar{\varpi}(t)\\
&\quad-\frac{\lambda_m(\Gamma)\lambda_m(Q)\psi}{\lambda_M(K)}\frac{\lambda'(t)}{\lambda(t)}\bar{\varpi}^T(t)(K\otimes I_n)\bar{\varpi}(t), \qquad (12)
\end{aligned}
$$

and

$$
\begin{aligned}
&2\bar{\zeta}^T(t)(K\otimes I_n)(G_3-G_4)\bar{\zeta}(t)\\
&=2\bar{\zeta}^T(t)\Big(K\otimes(\chi I_n-A)\Big)\bar{\zeta}(t)\\
&\quad-\big(\eta+\psi\frac{\lambda'(t)}{\lambda(t)}\big)\bar{\zeta}^T(t)\Big((K\mathcal{L}_1+\mathcal{L}_1^TK)\otimes\Gamma\Big)\bar{\zeta}(t)\\
&\leqslant2\bar{\zeta}^T(t)\Big(K\otimes(\chi I_n-A)\Big)\bar{\zeta}(t)\\
&\quad-\frac{\lambda_m(\Gamma)\lambda_m(Q)}{\lambda_M(K)}\Big(\eta+\psi\frac{\lambda'(t)}{\lambda(t)}\Big)\bar{\zeta}^T(t)(K\otimes I_n)\bar{\zeta}(t). \qquad (13)
\end{aligned}
$$

Based on the Young's inequality, we can infer that

$$
\begin{aligned}
&2\bar{\varpi}^T(t)(K\otimes I_n)\bar{\zeta}(t)\\
&\leqslant\bar{\varpi}^T(t)(K\otimes I_n)\bar{\varpi}(t)+\bar{\zeta}^T(t)(K\otimes I_n)\bar{\zeta}(t), \qquad (14)
\end{aligned}
$$

and

$$
\begin{aligned}
&2\bar{\zeta}^T(t)(K\otimes I_n)\Big(G_2-\eta(1-\chi)(L\otimes\Gamma)\Big)\bar{\varpi}(t)\\
&=2\bar{\zeta}^T(t)\Big(K\otimes(\chi A-B-\chi^2I_n)\Big)\bar{\varpi}(t)\\
&\quad-2\eta(1-\chi)\bar{\zeta}^T(t)\big(KL\otimes\Gamma\big)\bar{\varpi}(t)\\
&\leqslant\bar{\zeta}^T(t)\Big(K\otimes(\chi A-B-\chi^2I_n)^2\Big)\bar{\zeta}(t)\\
&\quad+\bar{\varpi}^T(t)(K\otimes I_n)\bar{\varpi}(t)+\eta|1-\chi|\bar{\zeta}^T(t)\big(K\otimes I_n\big)\bar{\zeta}(t)\\
&\quad+\eta|1-\chi|\bar{\varpi}^T(t)(L^TKL\otimes\Gamma^2)\bar{\varpi}(t). \qquad (15)
\end{aligned}
$$

Under Assumption 1, we have that

$$
\begin{aligned}
&2\bar{\zeta}^T(t)(K\otimes I_n)(I_N\otimes C)h\big(\bar{\varpi}(t)\big)\\
&\leqslant\alpha_1^2\bar{\varpi}^T(t)(K\otimes C^TC)\bar{\varpi}(t)+\bar{\zeta}^T(t)(K\otimes I_n)\bar{\zeta}(t). \qquad (16)
\end{aligned}
$$

On the other hand, combining Assumption 2, we can obtain that

$$
\begin{aligned}
&\text{trace}\left[\sigma^T\big(t,\bar{\varpi}(t)\big)(K\otimes I_n)\sigma\big(t,\bar{\varpi}(t)\big)\right]\\
&=\sum_{i=1}^{N}k_i|\sigma(t,\varpi_i(t))|^2\\
&=\sum_{i=1}^{N}k_i|\sigma(t,q_i(t))-\sigma(t,s(t))|^2\\
&\leqslant\alpha_2\sum_{i=1}^{N}\varpi_i^T(t)k_i\varpi_i(t)\\
&=\alpha_2\bar{\varpi}^T(t)(K\otimes I_n)\bar{\varpi}(t). \qquad (17)
\end{aligned}
$$

Substituting inequalities (12) - (17) into equation (11), we have

$$
\begin{aligned}
&\mathcal{L}W(t)\\
&\leqslant-\bar{\varpi}^T(t)\Big[\Big(K\otimes\big((2\chi+2\mu-2-\alpha_2)I_n\\
&\quad-\alpha_1^2C^TC\big)\Big)-\eta|1-\chi|(L^TKL\otimes\Gamma^2)\Big]\bar{\varpi}(t)\\
&\quad-\bar{\zeta}^T(t)\Big[K\otimes\Big(2A+\big(\frac{\eta\lambda_m(\Gamma)\lambda_m(Q)}{\lambda_M(K)}-2\chi\\
&\quad-2-\eta|1-\chi|\big)I_n-(\chi A-B-\chi^2I_n)^2\Big)\Big]\bar{\zeta}(t)\\
&\quad-\frac{\lambda_m(\Gamma)\lambda_m(Q)\psi}{\lambda_M(K)}\frac{\lambda'(t)}{\lambda(t)}\bar{\varpi}^T(t)(K\otimes I_n)\bar{\varpi}(t)\\
&\quad-\frac{\lambda_m(\Gamma)\lambda_m(Q)\psi}{\lambda_M(K)}\frac{\lambda'(t)}{\lambda(t)}\bar{\zeta}^T(t)(K\otimes I_n)\bar{\zeta}(t)\\
&\leqslant-\Big(2\chi+2\mu-2-\alpha_2-\varkappa_1\Big)\bar{\varpi}^T(t)(K\otimes I_n)\bar{\varpi}(t)\\
&\quad-\big(\frac{\eta\lambda_m(\Gamma)\lambda_m(Q)}{\lambda_M(K)}-2\chi-2-\eta|1-\chi|\\
&\quad-\varkappa_2\Big)\bar{\zeta}^T(t)(K\otimes I_n)\bar{\zeta}(t)\\
&\quad-\frac{\lambda_m(\Gamma)\lambda_m(Q)\psi}{\lambda_M(K)}\frac{\lambda'(t)}{\lambda(t)}\bar{\varpi}^T(t)(K\otimes I_n)\bar{\varpi}(t)
\end{aligned}
$$

$$- \frac{\lambda_m(\Gamma)\lambda_m(Q)\psi}{\lambda_M(K)}\frac{\lambda'(t)}{\lambda(t)}\bar{\zeta}^T(t)(K\otimes I_n)\bar{\zeta}(t), \tag{18}$$

where $\varkappa_1 = \frac{\lambda_M\left[\alpha_1^2\left(K\otimes(C^TC)\right)+\eta|1-\chi|\left((L^TKL)\otimes\Gamma^2\right)\right]}{\lambda_m(K)}$, $\varkappa_2 = 2\lambda_m(A) - \lambda_M\Big((\chi A - B - \chi^2 I_n)^2\Big)$. Let $\gamma_1 = 2\chi + 2\mu - 2 - \alpha_2 - \varkappa_1$, $\gamma_2 = \frac{\eta\lambda_m(\Gamma)\lambda_m(Q)}{\lambda_M(K)} - 2\chi - 2 - \eta|1-\chi| - \varkappa_2$, $\gamma_3 = \frac{\lambda_m(\Gamma)\lambda_m(Q)\psi}{\lambda_M(K)}$, and $\gamma_0 = \min\{\gamma_1, \gamma_2\}$. Considering the conditions in Theorem 1, we can deduce that

$$\mathcal{L}W(t) \leqslant -\gamma_0 W(t) - \gamma_3\frac{\lambda'(t)}{\lambda(t)}W(t).$$

According to Lemma 1, one obtains

$$\mathbb{E}(W(t)) \leq \lambda^{-\gamma_3}(t)\exp\{-\gamma_0 t\}\mathbb{E}(W(0)), t \in [0, t_p). \tag{19}$$

Thus, it can deduce that

$$\begin{aligned}
&\lambda_m(K)\mathbb{E}(\| \left(\bar{\varpi}^T(t), \bar{\zeta}^T(t)\right)^T \|^2)\\
&\leqslant \lambda^{-\gamma_3}(t)\exp\{-\gamma_0 t\}\mathbb{E}\big(W(0)\big)\\
&\leqslant \lambda^{-\gamma_3}(t)\exp\{-\gamma_0 t\}\lambda_M(K)\mathbb{E}(\| \left(\bar{\varpi}^T(0), \bar{\zeta}^T(0)\right) \|^2)\\
&= \lambda^{-\gamma_3}(t)\exp\{-\gamma_0 t\}\lambda_M(K)\mathbb{E}(\| \left(E(0)\right) \|^2),
\end{aligned} \tag{20}$$

where $E(t) = \left(\bar{\varpi}^T(t), \bar{\zeta}^T(t)\right)^T$ and $E(0) = \left(\bar{\varpi}^T(0), \bar{\zeta}^T(0)\right)^T$. Therefore, we can deduce that $\mathbb{E}(\| \left(E(t)\right) \|^2) \to 0$ as $t \to t_p$. Subsequently, we will elucidate the scenario in which $t \in [t_p, +\infty)$. By utilizing the Itô formular, similarly, we have that

$$\mathcal{L}W(t) \leqslant -\gamma_0 W(t). \tag{21}$$

Applying the expectation operator to both sides of the above inequality results in

$$\mathbb{E}(W(t)) \leq \exp\{-\gamma_0(t-t_p)\}\mathbb{E}(W(t_p)).$$

Apparently, we are in a position to conclude that

$$\lambda_m(K)\mathbb{E}(\|E(t) \|^2) \leq \exp\{-\gamma_0(t-t_p)\}\mathbb{E}(W(t_p)) = 0. \tag{22}$$

It is not challenging to observe that $\mathbb{E}(|E(t)|^2) = 0$ when $t \in [t_p, +\infty)$. In conclusion, we assert that the system can realize PTMSS within the given time t_p under pinning control.

3.2 Prescribed-Time Synchronization With TDs

In this part, we address an issue about the PTMSS with TDs. The model of a stochastic coupled inertial neural network with TDs is depicted by

$$\begin{aligned}
d\dot{q}_i(t) = \Big[&- A\dot{q}_i(t) - Bq_i(t) + Ch(q_i(t))\\
&+ Dh(q_i(t-\tau(t))) + \eta\sum_{k=1}^{N}\rho_{ik}\Gamma\Big(\dot{q}_k(t) - \dot{q}_i(t)\\
&+ q_k(t) - q_i(t)\Big)\Big]dt\\
&+ \sigma\big(t, q_i(t), q_i(t-\tau(t))\big)d\omega(t), \qquad i \in J,
\end{aligned} \tag{23}$$

where $D = (d_{jk})_{n\times n}$ represents the connection weights associated with neurons with TDs. Furthermore, the following assumptions are imposed on system (23).

Assumption 3. Assume that the time-varying delay $\tau(t)$ is differentiable and meets

$$0 \leq \tau(t) \leq \tau_0, \dot{\tau}(t) \leqslant \tau_1 < 1,$$

where τ_0 and τ_1 are known positive constants.

Assumption 4. Suppose that the function $\sigma(t, q_i(t), q_i(t-\tau(t)))$ satisfies a condition involving positive scalars α_2 and α_3 such that

$$\begin{aligned}
&[\sigma(t, q_i(t), q_i(t-\tau(t))) - \sigma(t, s(t), s(t-\tau(t))]^T\\
&\cdot[\sigma(t, q_i(t), q_i(t-\tau(t))) - \sigma(t, s(t), s(t-\tau(t))]\\
&\leqslant \alpha_2\|q_i(t) - s(t)\|^2 + \alpha_3\|q(t-\tau(t)) - s(t-\tau(t)\|^2.
\end{aligned} \tag{24}$$

Similar to Eq. (5) , by conducting variable substitution, we can derive that

$$\begin{cases}
dq_i(t) = & \Big(-\chi q_i(t) + \xi_i(t)\Big)dt,\\
d\xi_i(t) = & \Big[\big(\chi A - \chi^2 I_n - B\big)q_i(t) + \big(\chi I_n - A\big)\xi_i(t)\\
& +Ch(q_i(t)) + Dh(q_i(t-\tau(t)))\\
& -(1-\chi)\eta\sum_{k=1}^{N} l_{ik}\Gamma q_k(t) - \eta\sum_{k=1}^{N} l_{ik}\Gamma\xi_k(t)\Big]dt\\
& +\sigma\big(t, q_i(t), q_i(t-\tau(t))\big)d\omega(t).
\end{cases}$$

The target orbit $(s^T(t), v^T(t))^T$ is represented that

$$\begin{cases}
ds(t) = & \Big(-\chi s(t) + v(t)\Big)dt,\\
dv(t) = & \Big[\big(\chi A - \chi^2 I_n - B\big)s(t) + \big(\chi I_n - A\big)v(t)\\
& +Ch(s(t)) + Dh(s(t-\tau(t)))\Big]dt\\
& +\sigma\big(t, s(t), s(t-\tau(t))\big)d\omega(t).
\end{cases} \tag{25}$$

The error system is subsequently derived as shown

below

$$\begin{cases} d\varpi_i(t) = & \Big(-\chi\varpi_i(t) + \zeta_i(t) + u_{1i}(t)\Big)dt, \\ d\zeta_i(t) = & \Big[(\chi A - \chi^2 I_n - B)\varpi_i(t) + (\chi I_n - A)\zeta_i(t) \\ & +Ch(\varpi_i(t)) + Dh(\varpi_i(t-\tau(t))) \\ & -(1-\chi)\eta\sum_{k=1}^{N} l_{ik}\Gamma\varpi_k(t) - \eta\sum_{k=1}^{N} l_{ik}\Gamma\xi_k(t) \\ & +u_{2i}(t)\Big]dt \\ & +\sigma(t,\varpi_i(t),\varpi_i(t-\tau(t)))d\omega(t). \end{cases} \tag{26}$$

where $h(\varpi_i(t-\tau(t))) = h(q_i(t-\tau(t))) - h(s(t-\tau(t)))$, $\sigma(t,\varpi_i(t),\varpi_i(t-\tau(t))) = \sigma(t,q_i(t),q_i(t-\tau(t))) - \sigma(t,s(t),s(t-\tau(t)))$. Consequently, the controllers $u_{1i}(t)$ and $u_{2i}(t)$ are designed as follows

$$\begin{aligned} u_{1i}(t) = &-\Big(\mu + \beta_i\theta_i\psi\frac{\lambda'(t)}{\lambda(t)}\Big)\varpi_i(t) - \psi\frac{\lambda'(t)}{\lambda(t)}\sum_{k=1}^{N} l_{ik}\Gamma q_k(t) \\ &-\frac{1}{2}\Big(\delta_1 + \delta_3\frac{\lambda'(t)}{\lambda(t)}\Big)\delta_2\int_{t-\tau(t)}^{t}\varpi_i^T(s)k_i\varpi_i(s)ds \\ &\cdot\frac{\varpi_i(t)}{\varpi_i^T(t)k_i\varpi_i(t)}, \\ u_{2i}(t) = &-\beta_i\theta_i\Big(\eta + \psi\frac{\lambda'(t)}{\lambda(t)}\Big)\zeta_i(t) - \psi\frac{\lambda'(t)}{\lambda(t)}\sum_{k=1}^{N} l_{ik}\Gamma\xi_k(t). \end{aligned}$$

where δ_1, δ_2 and δ_3 denote positive constants to be determined. Substitute $u_{1i}(t)$ and $u_{2i}(t)$ into Equation (26), we have that

$$\begin{cases} d\varpi_i(t) = & \Big(-(\chi+\mu)\varpi_i(t) + \zeta_i(t) - \psi\frac{\lambda'(t)}{\lambda(t)}\sum_{k=1}^{N}\tilde{l}_{ik}\Gamma\varpi_k(t) \\ & -\frac{1}{2}\big(\delta_1 + \delta_3\frac{\lambda'(t)}{\lambda(t)}\big)\delta_2\int_{t-\tau(t)}^{t}\varpi_i^T(s)k_i\varpi_i(s)ds \\ & \cdot\frac{\varpi_i(t)}{\varpi_i^T(t)k_i\varpi_i(t)}\Big)dt, \\ d\zeta_i(t) = & \Big[\big(\chi A - \chi^2 I_n - B\big)\varpi_i(t) + \big(\chi I_n - A\big)\zeta_i(t) \\ & +Ch\big(\varpi_i(t)\big) + Dh\big(\varpi_i(t-\tau(t))\big) \\ & -(1-\chi)\eta\sum_{k=1}^{N} l_{ik}\Gamma\varpi_k(t) \\ & -\Big(\eta + \psi\frac{\lambda'(t)}{\lambda(t)}\Big)\sum_{k=1}^{N}\tilde{l}_{ik}\Gamma\zeta_i\Big]dt \\ & +\sigma\big(t,\varpi_i(t),\varpi_i(t-\tau(t))\big)d\omega(t). \end{cases} \tag{27}$$

Let $h(\bar{\varpi}(t-\tau(t))) = (h^T(\bar{\varpi}_1(t-\tau(t))),\ldots,h^T(\bar{\varpi}_N(t-\tau(t))))^T$, $\sigma(t,\bar{\varpi}(t),\bar{\varpi}(t-\tau(t))) = (\sigma^T(t,\bar{\varpi}_1(t),\bar{\varpi}_1(t-\tau(t))),\ldots,\sigma^T(t,\bar{\varpi}_N(t),\bar{\varpi}_N(t-\tau(t))))^T$, and we can obtain that

$$\begin{cases} d\bar{\varpi}(t) = & \Big(\big(-(\chi+\mu)I_{Nn} - G_1\big)\bar{\varpi}(t) + \bar{\zeta}(t) \\ & -\frac{1}{2}\big(\delta_1 + \delta_3\frac{\lambda'(t)}{\lambda(t)}\big)\delta_2\int_{t-\tau(t)}^{t}\bar{\varpi}^T(s)(K\otimes I_n)\bar{\varpi}(s)ds \\ & \cdot\frac{\bar{\varpi}(t)}{\bar{\varpi}^T(t)(K\otimes I_n)\bar{\varpi}(t)}\Big)dt, \\ d\bar{\zeta}(t) = & \Big[\big(G_2 - \eta(1-\chi)(L\otimes\Gamma)\big)\bar{\varpi}(t) \\ & +(G_3 - G_4)\bar{\zeta}(t) + (I_N\otimes C)h\big(\bar{\varpi}(t)\big) \\ & +(I_N\otimes D)h\big(\bar{\varpi}(t-\tau(t))\big)\Big]dt \\ & +\sigma\big(t,\bar{\varpi}(t),\bar{\varpi}(t-\tau(t))\big)d\omega(t), \quad i\in J, \end{cases} \tag{28}$$

Theorem 2. *Suppose that Assumption 1, 3 and 4 are satisfied. If there exist positive parameters χ, μ, α_1, α_2, ψ, and η such that $\gamma_1 = 2\chi + 2\mu - 2 - \alpha_2 - \delta_2 - \varkappa_1 > 0$, $\gamma_2 = \frac{\eta\lambda_m(\Gamma)\lambda_m(Q)}{\lambda_M(K)} - 2\chi - 2 - \eta|1-\chi| - \varkappa_2 > 0$, and $\delta_2(1-\tau_1) - (\alpha_1^2 + \alpha_2) > 0$, where $\varkappa_1 = \frac{\lambda_M\left[\alpha_1^2\left(K\otimes(C^TC)\right)+\eta|1-\chi|\left((L^TKL)\otimes\Gamma^2\right)\right]}{\lambda_m(K)}$, $\varkappa_2 = 2\lambda_m(A) - \lambda_M\Big(\big(\chi A - B - \chi^2 I_n\big)^2 - D^TD\Big)$, then system (23) can achieve PTMSS under pinning control.*

Proof: Choose a candidate Lyapunov functional as

$$\begin{aligned} W(t) =&\bar{\varpi}^T(t)(K\otimes I_n)\bar{\varpi}(t) + \bar{\zeta}^T(t)(K\otimes I_n)\bar{\zeta}(t) \\ &+\delta_2\int_{t-\tau(t)}^{t}\bar{\varpi}^T(s)(K\otimes I_n)\bar{\varpi}(s)ds. \end{aligned} \tag{29}$$

For system (28), by utilizing the Itô formula and the Leibnitz formula, it can be deduced as shown below

$$\begin{aligned} &\mathcal{L}W(t) \\ =&2\bar{\varpi}^T(t)(K\otimes I_n)\big(-(\chi+\mu)I_{Nn} - G_1\big)\bar{\varpi}(t) \\ &+2\bar{\varpi}^T(t)(K\otimes I_n)\bar{\zeta}(t) \\ &-\big(\delta_1 + \delta_3\frac{\lambda'(t)}{\lambda(t)}\big)\delta_2\int_{t-\tau(t)}^{t}\bar{\varpi}^T(s)(K\otimes I_n)\bar{\varpi}(s)ds \\ &+2\bar{\zeta}^T(t)(K\otimes I_n)\big(G_2 - \eta(1-\chi)(L\otimes\Gamma)\big)\bar{\varpi}(t) \\ &+2\bar{\zeta}^T(t)(K\otimes I_n)(G_3 - G_4)\bar{\zeta}(t) \\ &+2\bar{\zeta}^T(t)(K\otimes I_n)(I_N\otimes C)h\big(\bar{\varpi}(t)\big) \\ &+2\bar{\zeta}^T(t)(K\otimes I_n)(I_N\otimes D)h\big(\bar{\varpi}(t-\tau(t))\big) \\ &+\mathrm{trace}\left[\sigma^T\big(t,\bar{\varpi}(t),\bar{\varpi}(t-\tau(t))(K\otimes I_n)\right. \\ &\left.\cdot\,\sigma\big(t,\bar{\varpi}(t),\bar{\varpi}(t-\tau(t))\big)\right] \\ &+\delta_2\bar{\varpi}^T(t)(K\otimes I_n)\bar{\varpi}(t) \\ &-\delta_2(1-\dot{\tau}(t))\bar{\varpi}^T(t-\tau(t))(K\otimes I_n)\bar{\varpi}(t-\tau(t)). \end{aligned} \tag{30}$$

Subsequently, similar to (16), together with Assumption 1, it follows that

$$2\bar{\zeta}^T(t)(K\otimes I_n)(I_N\otimes D)h\big(\bar{\varpi}(t-\tau(t))\big)$$

$$\leqslant \bar{\zeta}^T(t)(K\otimes D^TD)\bar{\zeta}(t) + \alpha_1^2\bar{\varpi}^T(t-\tau(t))(K\otimes I_n)\bar{\varpi}(t-\tau(t)). \quad (31)$$

Similar to the approach in Theorem 1, based on Assumption 4, we can derive that

$$\begin{aligned}
&\text{trace}\left[\sigma^T\left(t,\bar{\varpi}(t),\bar{\varpi}(t-\tau(t))(K\otimes I_n)\sigma\left(t,\bar{\varpi}(t),\bar{\varpi}(t-\tau(t))\right)\right]\right.\\
&=\sum_{i=1}^{N}k_i\left|\sigma(t,\varpi_i(t),\varpi_i(t-\tau(t))\right|^2\\
&=\sum_{i=1}^{N}k_i\left|\sigma\Big(t,q_i(t),q_i\big(t-\tau(t)\big)\Big)-\sigma\Big(t,s(t),s\big(t-\tau(t)\big)\Big)\right|^2\\
&\leqslant\alpha_2\sum_{i=1}^{N}\varpi_i^T(t)k_i\varpi_i(t)+\alpha_3\sum_{i=1}^{N}\varpi_i^T(t-\tau(t))k_i\varpi_i(t-\tau(t))\\
&=\alpha_2\bar{\varpi}^T(t)(K\otimes I_n)\bar{\varpi}(t)\\
&\quad+\alpha_3\bar{\varpi}^T(t-\tau(t))(K\otimes I_n)\bar{\varpi}(t-\tau(t)). \quad (32)
\end{aligned}$$

Substituting inequalities (12) - (16) as well as (31) and (32) into (30), we can obtain as shown below results

$$\begin{aligned}
\mathcal{L}W(t)\leqslant&-\bar{\varpi}^T(t)\Bigg[\Big(K\otimes\big((2\chi+2\mu-2-\alpha_2-\delta_2)I_n\\
&-\alpha_1^2C^TC\big)\Big)-\eta|1-\chi|(L^TKL\otimes\Gamma^2)\Bigg]\bar{\varpi}(t)\\
&-\bar{\zeta}^T(t)\Bigg[K\otimes\Bigg(2A+\Big(\frac{\eta\lambda_m(\Gamma)\lambda_m(Q)}{\lambda_M(K)}-2\chi-2\\
&-\eta|1-\chi|\Big)I_n-(\chi A-B-\chi^2I_n)^2-D^TD\Bigg)\Bigg]\bar{\zeta}(t)\\
&-\delta_1\delta_2\int_{t-\tau(t)}^{t}\bar{\varpi}^T(s)(K\otimes I_n)\bar{\varpi}(s)ds\\
&-\frac{\lambda_m(\Gamma)\lambda_m(Q)\psi}{\lambda_M(K)}\frac{\lambda'(t)}{\lambda(t)}\bar{\varpi}^T(t)(K\otimes I_n)\bar{\varpi}(t)\\
&-\frac{\lambda_m(\Gamma)\lambda_m(Q)\psi}{\lambda_M(K)}\frac{\lambda'(t)}{\lambda(t)}\bar{\zeta}^T(t)(K\otimes I_n)\bar{\zeta}(t)\\
&-\delta_3\delta_2\frac{\lambda'(t)}{\lambda(t)}\int_{t-\tau(t)}^{t}\bar{\varpi}^T(s)(K\otimes I_n)\bar{\varpi}(s)ds\\
&+(\alpha_1^2+\alpha_3)\bar{\varpi}^T(t-\tau(t))\big(K\otimes I_n\big)\bar{\varpi}(t-\tau(t))\\
&-\delta_2(1-\dot{\tau}(t))\bar{\varpi}^T(t-\tau(t))(K\otimes I_n)\bar{\varpi}(t-\tau(t))\\
\leqslant&-(2\chi+2\mu-2-\alpha_2-\delta_2-\varkappa_1)\bar{\varpi}^T(t)(K\otimes I_n)\bar{\varpi}(t)\\
&-\Big(\frac{\eta\lambda_m(\Gamma)\lambda_m(Q)}{\lambda_M(K)}-2\chi-2-\eta|1-\chi|\\
&-\varkappa_2\Big)\bar{\zeta}^T(t)(K\otimes I_n)\bar{\zeta}(t)\\
&-\delta_1\delta_2\int_{t-\tau(t)}^{t}\bar{\varpi}^T(s)(K\otimes I_n)\bar{\varpi}(s)ds\\
&-\frac{\lambda_m(\Gamma)\lambda_m(Q)\psi}{\lambda_M(K)}\frac{\lambda'(t)}{\lambda(t)}\bar{\varpi}^T(t)(K\otimes I_n)\bar{\varpi}(t)\\
&-\frac{\lambda_m(\Gamma)\lambda_m(Q)\psi}{\lambda_M(K)}\frac{\lambda'(t)}{\lambda(t)}\bar{\zeta}^T(t)(K\otimes I_n)\bar{\zeta}(t)\\
&-\delta_3\delta_2\frac{\lambda'(t)}{\lambda(t)}\int_{t-\tau(t)}^{t}\bar{\varpi}^T(s)(K\otimes I_n)\bar{\varpi}(s)ds\\
&+\big(\alpha_1^2+\alpha_3\big)\bar{\varpi}^T(t-\tau(t))\big(K\otimes I_n\big)\bar{\varpi}(t-\tau(t))\\
&-\delta_2(1-\tau_1)\bar{\varpi}^T(t-\tau(t))(K\otimes I_n)\bar{\varpi}(t-\tau(t))
\end{aligned}$$

where $\varkappa_1 = \frac{\lambda_M\left[\alpha_1^2\left(K\otimes(C^TC)\right)+\eta|1-\chi|\left((L^TKL)\otimes\Gamma^2\right)\right]}{\lambda_m(K)}$, $\varkappa_2 = 2\lambda_m(A)-\lambda_M\Big((\chi A-B-\chi^2I_n)^2-D^TD\Big)$. Let $\gamma_1 = 2\chi+2\mu-2-\alpha_2-\delta_2-\varkappa_1$, $\gamma_2 = \frac{\eta\lambda_m(\Gamma)\lambda_m(Q)}{\lambda_M(K)}-2\chi-2-\eta|1-\chi|-\varkappa_2$, $\gamma_3 = \min\{\frac{\lambda_m(\Gamma)\lambda_m(Q)\psi}{\lambda_M(K)},\delta_3\}$, and $\gamma_0=\min\{\gamma_1,\gamma_2,\delta_1\}$. According to the conditions given in Theorem 2, we can deduce that

$$\mathcal{L}W(t)\leqslant-\gamma_0W(t)-\gamma_3\frac{\lambda'(t)}{\lambda(t)}W(t),$$

As previously demonstrated in (19)–(22), for $t\in[0,t_p)$, immediately, we can get $\mathbb{E}(\|\ (E(t))\ \|^2)\to 0$ as $t\to t_p$. Furthermore, similarly, we also claim that $\mathbb{E}(|E(t)|^2)=0$ when $t\in[t_p,+\infty)$. Consequently, the PTMSS of SCINNs with TDs can be achieved within the given time t_p and maintain in $[t_p,+\infty)$. The proof is accomplished.

Remark 1. The designed controllers ensure that the system achieves mean-square synchronization before a prespecified time t_p via the time-varying gain $\psi\lambda'(t)/\lambda(t)$, offering the theoretical advantage of explicitly assignable convergence time. However, two practical considerations should be noted. First, the gain tends to infinity as $t\to t_p^-$, which must be bounded in real implementations via saturation or a switching mechanism. Second, the choice of parameters t_p and b involves a trade-off between convergence speed and control energy—faster convergence demands higher actuator output. Moreover, the controller's effectiveness relies on accurate system models; in the presence of parameter uncertainties, adaptive strategies may be required to maintain robustness.

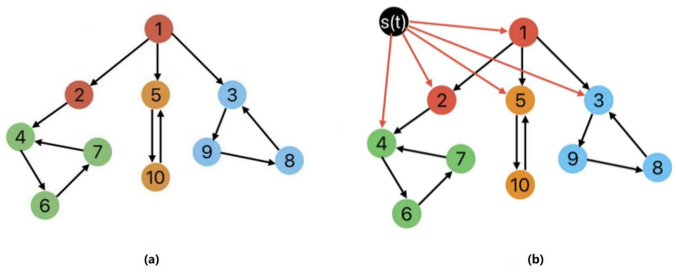

Figure 1. (a) Topology structure of the directed network with ten nodes. (b) Directed graph with root node $s(t)$.

4 Experiments

In this section, we present the following two numerical examples to verify the effectiveness of our theoretical findings.

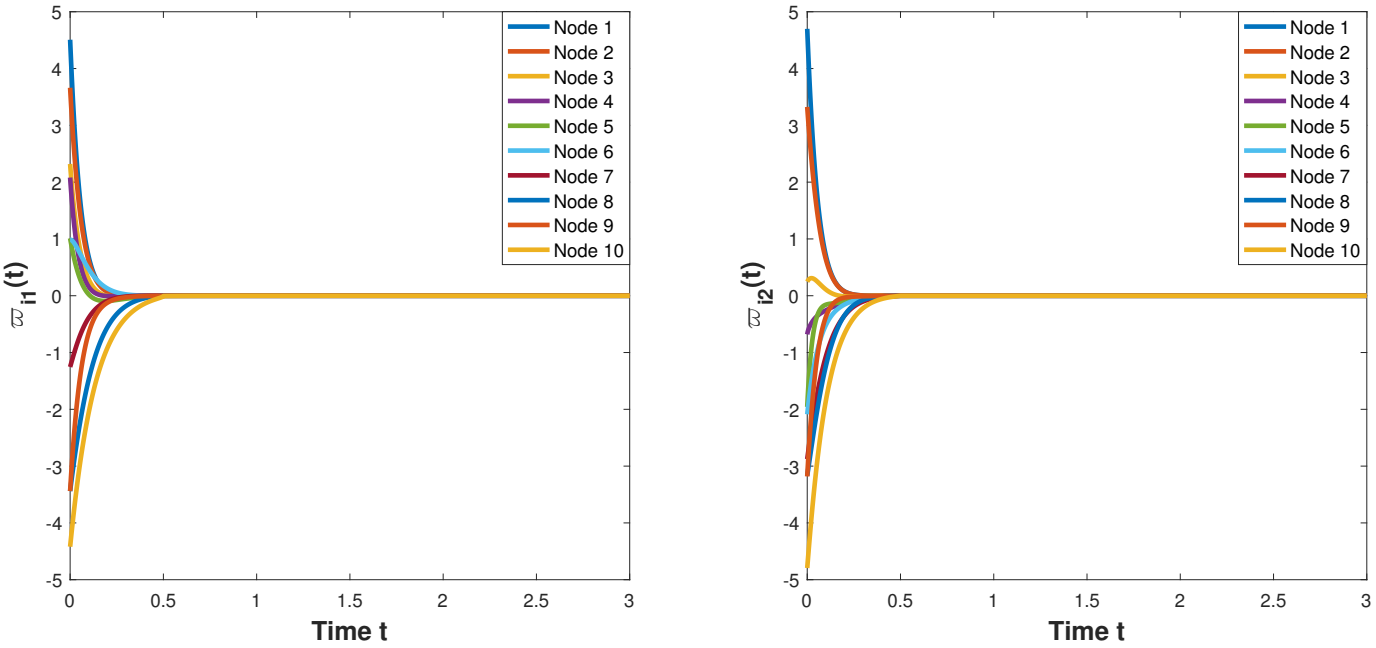

Figure 2. Error vector $\varpi_i(t)$, $i \in J$, for SCINNs with the prescribed-time $t_p = 0.5$ under pinning control.

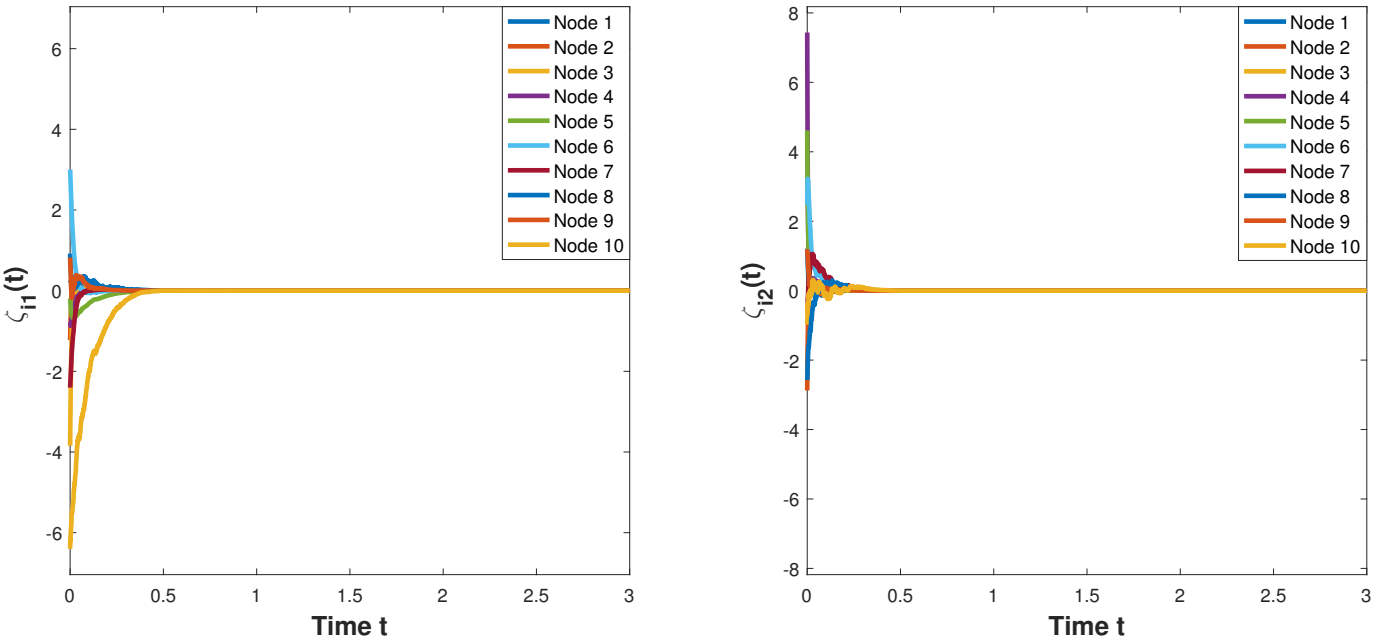

Figure 3. Error vector $\zeta_i(t)$, $i \in J$, for SCINNs with the prescribed-time $t_p = 0.5$ under pinning control.

Case 1: Consider a class of SCINNs consisting of ten nodes as shown below

$$\begin{cases} dq_i(t) = \Big(-\chi q_i(t) + \xi_i(t)\Big)dt, \\ d\xi_i(t) = \Big[\big(\chi A - \chi^2 I_n - B\big)q_i(t) + \big(\chi I_n - A\big)\xi_i(t) \\ \qquad +Ch(q_i(t)) - (1-\chi)\eta \sum\limits_{k=1}^{N} l_{ik}\Gamma q_k(t) \\ \qquad -\eta \sum\limits_{k=1}^{N} l_{ik}\Gamma\xi_k(t)\Big]dt + \sigma\big(t, q_i(t)\big)d\omega(t). \end{cases} \tag{33}$$

and

$$u_{1i}(t) = -\Big(\mu + \beta_i\theta_i\psi\frac{\lambda'(t)}{\lambda(t)}\Big)\varpi_i(t) - \psi\frac{\lambda'(t)}{\lambda(t)}\sum_{k=1}^{N} l_{ik}\Gamma q_k(t),$$

$$u_{2i}(t) = -\beta_i\theta_i\Big(\eta + \psi\frac{\lambda'(t)}{\lambda(t)}\Big)\zeta_i(t) - \psi\frac{\lambda'(t)}{\lambda(t)}\sum_{k=1}^{N} l_{ik}\Gamma\xi_k(t),$$

in which $\varpi_i(t) = q_i(t) - s(t)$, $\zeta_i(t) = \xi_i(t) - v(t)$, $\chi = 0.9$ and $\Gamma = \mathrm{diag}\{1, 2\}$. The parameters of the system

dynamics are delineated as shown below

$$A=\begin{bmatrix}0.8 & 0\\ 0 & 0.9\end{bmatrix}, B=\begin{bmatrix}1.2 & 0\\ 0 & 1.5\end{bmatrix}, C=\begin{bmatrix}0.5 & -0.3\\ 0.2 & 0.4\end{bmatrix}.$$

$h(q_i(t)) = \begin{bmatrix}\tanh(q_{i1}(t)) & \tanh(q_{i2}(t))\end{bmatrix}^T$. It can be verified that this function satisfies the global Lipschitz condition, and the calculation indicates that the Lipschitz coefficient is $\alpha_1 = 1$. Noise intensity function is $\sigma\big(t, q_i(t)\big) = [0.4q_{i1}(t),\ 0.6q_{i2}(t)]^T$. This function satisfies Assumption 2 with the coefficient $\alpha_2 = 0.36$.

The network topology of the system is depicted in Figure 1 presents a directed graph consistinng of several nodes, which constitutes a directed spanning tree, which have a root node $s(t)$. it is obvious that the number of strongly connected components of the root equals $k_0 = 5$. The controlled node set can be expressed as $M = \{1, 2, 3, 4, 5\}$.

The Laplacian matrix of the system is $L = (l_{ij})_{10\times 10}$, in which $l_{21} = -0.5$, $l_{31} = -0.2$, $l_{38} = -0.1$, $l_{42} = -0.2$, $l_{47} = -0.4$, $l_{51} = -0.3$, $l_{5,10} = -0.2$, $l_{64} = -0.4$, $l_{76} = -0.2$, $l_{89} = -0.2$, $l_{93} = -0.5$, $l_{10,5} = -0.1$, $l_{22} = 0.5$, $l_{33} = 0.3$, $l_{44} = 0.6$, $l_{55} = 0.5$, $l_{66} = 0.4$, $l_{77} = 0.2$, $l_{88} = 0.2$, $l_{99} = 0.5$, $l_{10,10} = 0.1$, and other elements are 0. Combining with Lemma 2, there exists a positive definite matrix $K = \mathrm{diag}\{k_1, k_2, \ldots, k_N\}$ satisfying $Q = \mathcal{L}_1 K + K\mathcal{L}_1^T > 0$, where $(k_1, k_2, \ldots, k_N)^T = \big(\mathcal{L}_1{}^T\big)^{-1}\mathbf{1}_N$ and $\mathcal{L}_1 = L + \Theta$. We set $\Theta = \mathrm{diag}(\beta_1\theta_1, \beta_2\theta_2, ..., \beta_N\theta_N) = \mathrm{diag}(1, 1, 1, 1, 1, 0, 0, 0, 0, 0)$. Through straightforward computation, we obtain that $K = \mathrm{diag}\{2.4615, 1, 2.5, 2.5, 1.5385, 7.5, 10, 6.25, 4.5, 13.0769\}$, $\lambda_M(K) = 13.0769$, $\lambda_m(K) = 1$, and $\lambda_m(Q) = 0.6203$.

By selecting parameters $b = 5$ and $t_p = 0.5\,\mathrm{s}$, $\psi = 1$, Given that $\gamma_2 > 0$, after calculation, we can take $\eta = 89.7087$. At this point, γ_2 satisfies the inequality in Theorem 1. Furthermore, given that $\gamma_1 > 0$, it has been verified that $\mu = 6.7138$. At this point, γ_1 satisfies the conditions of the theorem. we may calculate that $\varkappa_1 = 11.341112$, $\varkappa_2 = -0.925239$, $\gamma_1 = 0.5$, $\gamma_2 = 0.3$, and $\gamma_3 = 0.0474$. We can calculate and verify to obtain that the conditions in Theorem 1 are satisfied. Consequently, the PTMSS of the system (33) without TDs can be achieved under controllers $u_{1i}(t)$ and $u_{2i}(t)$. Figures 2 and 3 respectively show the state errors of the system (33) without TDs under controllers $u_{1i}(t)$ and $u_{2i}(t)$. From Figures 2 and 3, it can be seen that system (33) achieves the PTMSS with the prescribed-time $t_p = 0.5\,\mathrm{s}$. Figure 4 illustrates the state error of the system (33) in the absence of a controller and time delays, from which we find that the system (33) without a controller and time delays fails to achieve synchronization. In contrast to Figures 2 and 3, the errors of each node in Figure 4 exhibit distinct fluctuations within 0-3 seconds and do not converge to zero vector even after 3 seconds.

Case 2: Consider a class of SCINNs with time-varying delay consisting of ten nodes as below

$$\begin{cases} dq_i(t) = & \Big(-\chi q_i(t) + \xi_i(t)\Big)dt,\\ d\xi_i(t) = & \Big[(\chi A - \chi^2 I_n - B)q_i(t) + (\chi I_n - A)\xi_i(t)\\ & + Ch(q_i(t)) + Dh(q_i(t-\tau(t)))\\ & -(1-\chi)\eta\sum_{k=1}^{N} l_{ik}\Gamma q_k(t) - \eta\sum_{k=1}^{N} l_{ik}\Gamma\xi_k(t)\Big]dt\\ & + \sigma\big(t, q_i(t), q_i(t-\tau(t))\big)d\omega(t). \end{cases} \tag{34}$$

and

$$\begin{aligned} u_{1i}(t) = & -\Big(\mu + \beta_i\theta_i\psi\frac{\lambda'(t)}{\lambda(t)}\Big)\varpi_i(t) - \psi\frac{\lambda'(t)}{\lambda(t)}\sum_{k=1}^{N} l_{ik}\Gamma q_k(t)\\ & - \frac{1}{2}\Big(\delta_1 + \delta_3\frac{\lambda'(t)}{\lambda(t)}\Big)\delta_2\int_{t-\tau(t)}^{t}\varpi_i^T(s)k_i\varpi_i(s)ds\\ & \cdot\frac{\varpi_i(t)}{\varpi_i^T(t)k_i\varpi_i(t)},\\ u_{2i}(t) = & -\beta_i\theta_i\Big(\eta + \psi\frac{\lambda'(t)}{\lambda(t)}\Big)\zeta_i(t) - \psi\frac{\lambda'(t)}{\lambda(t)}\sum_{k=1}^{N} l_{ik}\Gamma\xi_k(t). \end{aligned}$$

Except for the matrix $D = \begin{bmatrix}0.3 & -0.1\\ 0.1 & 0.2\end{bmatrix}$, all parameters of system (34) are retained as same as those in (33). Let the time-varying delay $\tau(t) = 0.01\cos(t)$, which indicates that $\tau_0 = \tau_1 = 0.01$. The noise intensity function is given by $\sigma\big(t, q_i(t), q_i(t-\tau(t))\big) = \big[0.4q_{i1}(t) + 0.3q_{i1}(t-\tau(t)),\ 0.6q_{i2}(t) + 0.2q_{i2}(t-\tau(t))\big]^T$. It follows that $\alpha_2 = 0.52$ and $\alpha_3 = 0.13$ from Assumption 4. By choosing the parameters $\psi = 1$, $\eta = 85.698$, $\mu = 7.168$, $\delta_1 = 0.1$, $\delta_2 = 1.6463$, $\delta_3 = 0.03$, $b = 0.1$, $t_p = 2\,\mathrm{s}$, we further obtain that $\gamma_1 = 0.3$, $\gamma_2 = 0.1$, $\gamma_3 = 0.0474$, $\varkappa_1 = 11.85$, $\varkappa_2 = -0.88$ through calculation. It can be confirmed that all conditions of Theorem 2 hold. Therefore, the PTMSS for the system (34) with time-varying delay can be attained by designing the appropriate controllers $u_{1i}(t)$ and $u_{2i}(t)$. The state errors of the system (33) with TDs under controllers $u_{1i}(t)$ and $u_{2i}(t)$ have been simulated in Figures 5 and 6, respectively. From Figures 5 and 6, it can be observed that system (33) achieves the PTMSS with the prescribed-time $t_p = 2\,\mathrm{s}$. Figure 7 illustrates the state error of the time-delayed system (34) in the absence of controller, from which we find that the system (34) without a controller fails to

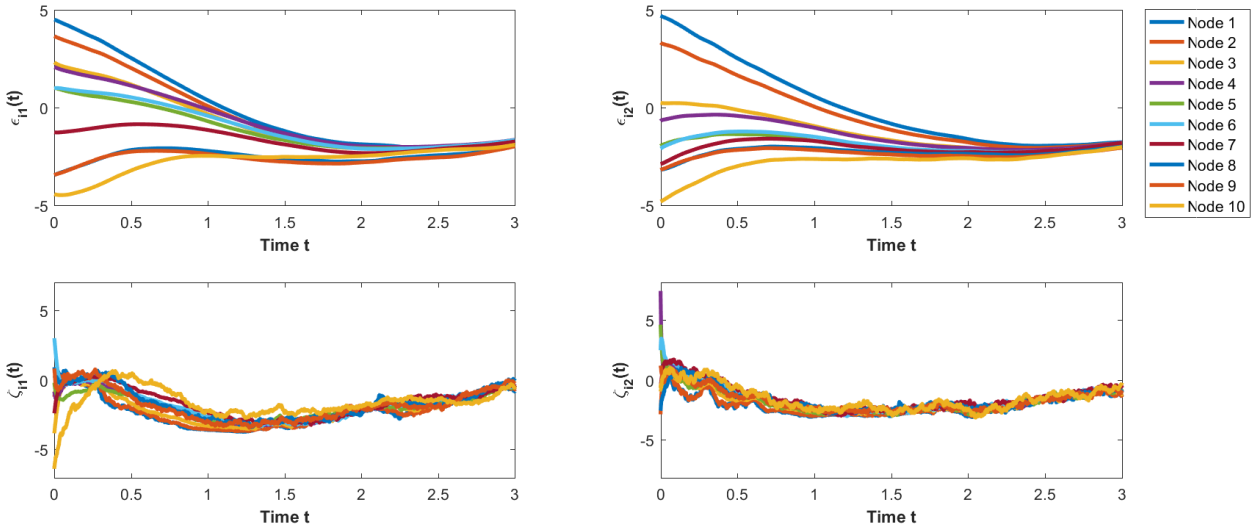

Figure 4. Error vector $\varpi_i(t), \zeta_i(t), i \in J$, for SCINNs without time delay under uncontrolled pinning control.

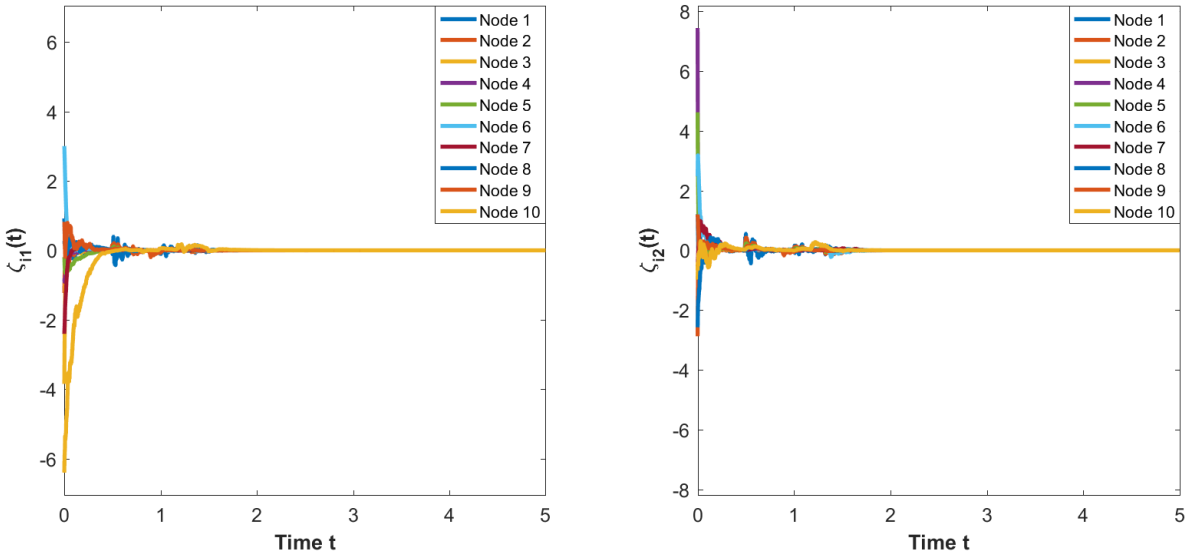

Figure 5. Error vector $\varpi_i(t), i \in J$, for SCINNs with time delay under parameters $t_p = 2$ and pinning control.

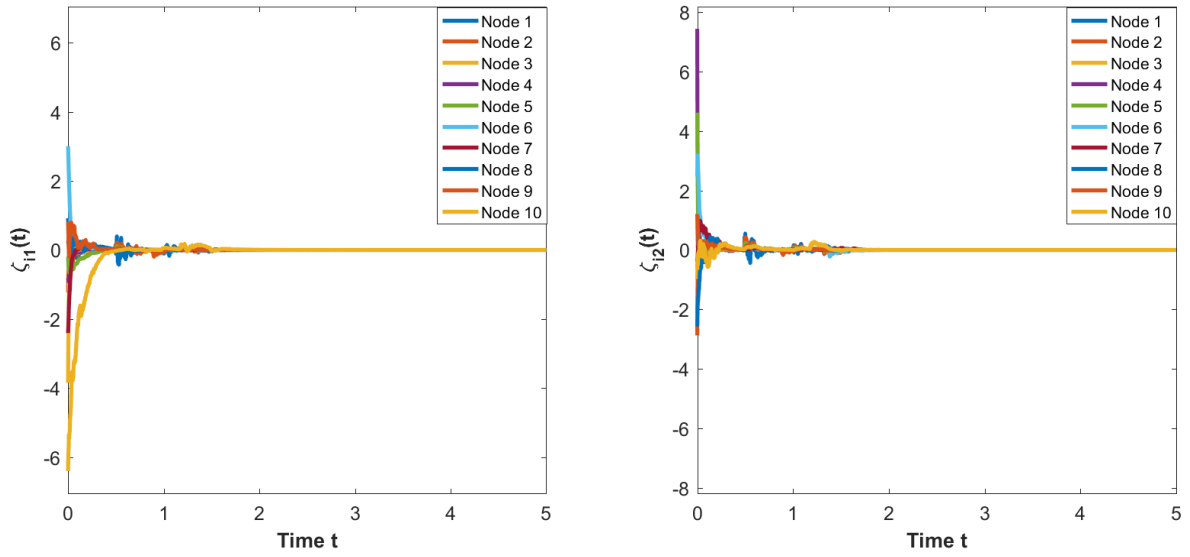

Figure 6. Error vector $\zeta_i(t), i \in J$, for SCINNs with time delay under parameters $t_p = 2$ and pinning control.

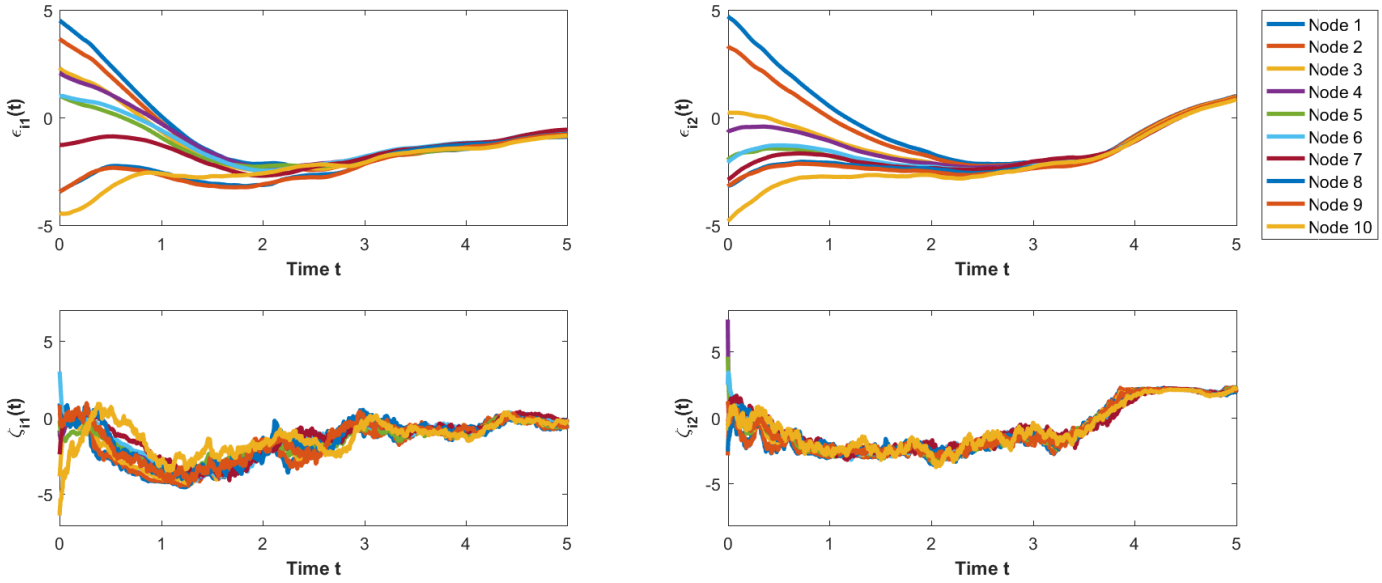

Figure 7. Error vector $\varpi_i(t), \zeta_i(t), i \in J$, for SCINNs with time delay under uncontrolled pinning control.

achieve synchronization. In contrast to Figures 5 and 6, the errors of each node in Figure 7 exhibit distinct fluctuations within 0-5 seconds and do not converge to zero vector even after 5 seconds.

5 Conclusion

This paper has investigated the issue of PTMSS for SCINNs, encompassing both scenarios without TDs and with TDs. By constructing appropriate Lyapunov functionals and utilizing the PTS approach, combined with the theory of directed spanning trees, several sufficient conditions have been rigorously presented to realize the prescribed-time mean-square synchronization within a given time t_p for SCINNs with TDs. Different from existing findings [36–39], factors such as stochastic disturbances and TDs have been explicitly taken into account, thereby enhancing the model's practical applicability. Additionally, the PTMSS of SCINNs has been discussed, and two categories of pinning controllers have been devised, which greatly improves control efficiency. In future work, we will further investigate the practical prescribed-time mean-square synchronization (PTMSS) of diverse complex networks. Promising research directions include extending the proposed framework to networks with switching topologies or mixed coupling configurations, as well as integrating the prescribed-time strategy with more advanced control paradigms—such as event-triggered control and distributed impulsive control—to mitigate communication overhead and actuation effort. Moreover, examining PTMSS under more sophisticated stochastic settings (e.g., systems subject to Lévy noise) and addressing practical implementation constraints—including actuator saturation—will be essential for narrowing the gap between theoretical design and real-world deployment.

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Data will be made available on request.

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Conflicts of Interest

The authors declare no conflicts of interest.

AI Use Statement

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Ethical Approval and Consent to Participate

Not applicable.

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