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RESEARCH ARTICLE

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The Economic Singularity: Dimensional Reduction, Invariants, and Stability Analysis of the Financialization Threshold

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Abstract

Background: The possibility of an economic singularity—a finite-time explosion of AI capability, financial capital, and inequality—has been discussed in futurist and economic literature, but endogenous feedback loops between AI, finance, and labour markets remain underexplored. Methods: This paper presents a dimensional reduction of a dynamical systems model of an economy with recursively self-improving artificial intelligence and financial autocatalysis. By transforming to three economically meaningful ratios—financial depth per AI capability ($x = K_f/A$), AI capital per financial capital ($u = K_{ai}/K_f$), and employment odds ($z = L_p/(1 - L_p)$)—the original four-variable explosive system reduces to a bounded, analytically tractable three-dimensional system. Results: The reduced model possesses a saddle-type equilibrium that separates two asymptotic regimes: one in which AI dominates ($x \to 0$) and one in which finance explodes ($x \to \infty$, Neofeudalism). The critical financial depth is $x_c = (\lambda + \beta)/\gamma_F$, so that the economy tips into Neofeudalism whenever the initial $x_0 > x_c$. Numerical simulations confirm the saddle structure and the threshold behaviour. Conclusions: The analysis yields sharp policy recommendations: regulate financial autocatalysis to raise the safety threshold, and intervene early to keep the financial-to-AI ratio below the critical value. Redistribution alone cannot substitute for structural regulation.

Keywords: economic singularity, artificial intelligence, financialization, stability analysis, saddle point, dimensional reduction, wealth inequality, Neofeudalism, dynamical systems.

1 Introduction

The possibility of an economic singularity—a finite-time explosion of AI capability, financial capital, and inequality—has been a recurring theme in both futurist and economic literature. Early conceptual work by Good [1] and Vinge [2] speculated on the consequences of recursively self-improving intelligence. More recently, Nordhaus [3] and Aghion et al. [4] have examined how information technology and AI might affect long-run growth, while Brynjolfsson & McAfee [5] emphasised the transformative potential of the "second machine age". However, these contributions typically treat AI as an

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Notations

Symbol	Meaning
A	AI capability index
K_f	Financial capital (stocks, bonds, derivatives)
K_{ai}	Productive AI capital (installed AI systems)
L_p	Employment rate ($0 < L_p < 1$)
W_c	Wealth of capital owners
W_w	Wealth of workers
λ	AI self-improvement efficiency
α_A	R&D conversion rate (financial capital $\to$ AI)
β	Investment efficiency (financial capital $\to$ AI capital)
δ	Depreciation rate of AI capital
r	Baseline return on financial capital
η	Return on AI capital
γ_F	Financial autocatalysis intensity (quadratic self-reinforcement)
κ	Automation speed (AI capital displaces labour)
ν	Reskilling rate (re-employment of displaced workers)
τ_{ai}, τ_f	Tax rates on AI and financial income
c_w	Workers' subsistence consumption

Derived ratios

$$x = \frac{K_f}{A}, \qquad u = \frac{K_{ai}}{K_f}, \qquad z = \frac{L_p}{1 - L_p}.$$

exogenous productivity factor, ignoring the endogenous feedback loops between AI, finance, and labour markets. At the same time, the financial sector has been shown to exhibit self-amplifying dynamics [6], and wealth concentration has followed patterns that, if unchecked, lead to extreme inequality [7]. Acemoglu & Restrepo [8], Brynjolfsson et al. [9], and Korinek & Stiglitz [29] have called for macroeconomic frameworks that integrate AI's endogenous productivity gains and that address the distributional consequences of labour-replacing technological change, but the coupled nonlinear dynamics of AI and financial autocatalysis remain underexplored.

Recent empirical work has highlighted the accelerating pace of AI development, with scaling laws for neural language models [25] suggesting that self-improvement rates could increase dramatically. Simultaneously, financialization has deepened globally, with the ratio of financial assets to GDP reaching historic highs [26]. These trends make the analysis of coupled AI–finance dynamics especially urgent. Sachs and Kotlikoff [27] have examined how smart machines may generate long-term economic misery through labour displacement, and Trammell & Korinek [28] has recently developed a macroeconomic model with transformative AI, but neither incorporates the quadratic financial feedback that can produce finite-time blowups. This paper fills that gap by unifying these strands into a dynamical systems model and deriving a sharp threshold condition separating stabilising and destabilising regimes.

The model builds on insights from complexity economics [11, 12] and agent-based approaches [10]. Financial instability has been modelled using heterogeneous beliefs [13] and stochastic behaviour [14]; our work incorporates a quadratic financial autocatalysis term reminiscent of bubble dynamics [15–17]. Power laws are pervasive in economics [18], and recent work has linked them to the rise of monopoly power [19]. The mathematical techniques we employ—dimensional reduction, self-similar blowup analysis, and bifurcation theory—are standard in nonlinear dynamics [20–22] and asymptotic analysis [23]. Econophysics has also contributed to understanding scaling in economic systems [24].

The core contribution of this paper is a **dimensional reduction** technique that eliminates explosive growth by transforming to scale-invariant ratios. The original four variables blow up in finite time, but the ratios $x = K_f/A$, $u = K_{ai}/K_f$, and $z = L_p/(1 - L_p)$ remain bounded and follow autonomous equations after suitable time rescaling.

The main result is a saddle-type equilibrium: a critical financial depth $x_c = (\lambda + \beta)/\gamma_F$ separates a regime where AI dominates ($x \to 0$) from a regime of explosive financialization ($x \to \infty$, which we term Neofeudalism). The parameters γ_F (financial autocatalysis) and λ (AI self-improvement) control this threshold, providing precise policy handles.

The paper is organised as follows. After the notations list, Section 2 restates the original model. Section 3 introduces the three ratios, derives their dynamics, and obtains the reduced autonomous system. Section 4 analyses the fixed point of the reduced system and proves the threshold theorem. Section 5 presents numerical simulations. Section 6 discusses policy implications, and Section 7 concludes with limitations and future research directions.

2 The Core Model

We briefly restate the original model to fix notation. All variables are functions of continuous time t.

$$\frac{dA}{dt} = \lambda A^2 + \alpha_A K_f, \tag{1}$$

$$\frac{dK_{ai}}{dt} = \beta A K_f - \delta K_{ai}, \tag{2}$$

$$\frac{dK_f}{dt} = r K_f - \beta A K_f + \eta K_{ai} + \gamma_F K_f^2, \tag{3}$$

$$\frac{dL_p}{dt} = -\kappa A K_{ai} L_p (1 - L_p) + \nu (1 - L_p). \tag{4}$$

Parameters are as defined in the notations. Wealth of capital owners is $W_c = K_{ai} + K_f$. Workers' wealth W_w evolves as

$$\frac{dW_w}{dt} = w(L_p) + \tau_{ai} \eta K_{ai} + \tau_f \gamma_F K_f^2 - c_w, \tag{5}$$

with wage $w(L_p)$ decreasing in L_p, tax rates $\tau_{ai}, \tau_f \in [0, 1]$, and subsistence consumption c_w.

The system exhibits finite-time singularities ($A, K_f, K_{ai} \to \infty$) under rather mild conditions. The absolute magnitudes are therefore not the most informative quantities; the *relative proportions* determine the economy's structural regime.

3 Dimensional Reduction to Bounded Ratios

In this section we transform the original explosive system into a lower-dimensional autonomous system for economically meaningful ratios. We first define three dimensionless ratios that capture the relative sizes of the key stocks. We then derive their exact evolution equations in ordinary time t. Because A diverges at the singularity, we remove the singular factor by a nonlinear time rescaling. The limit $A \to \infty$ yields an autonomous reduced system that governs the asymptotic structural evolution of the economy.

3.1 Definition of the Ratios

(1) **Financial depth per AI capability:** $x = \dfrac{K_f}{A}$. A high x signals an over-financialised economy, vulnerable to runaway financial feedback; a low x indicates that AI research dominates.

(2) **AI capital per financial capital:** $u = \dfrac{K_{ai}}{K_f}$. This measures how much productive AI capital has been created out of a unit of financial capital. In a well-functioning economy u is positive and finite.

(3) **Employment odds:** $z = \dfrac{L_p}{1 - L_p}$. z ranges from 0 (zero employment) to ∞ (full employment), simplifying the labour-market equation.

A key property of x and u is **scale invariance**: multiplying A, K_f, K_{ai} by the same positive constant leaves them unchanged. More importantly, even when the original stocks diverge, these ratios remain bounded, as shown below.

3.2 Dynamics in Original Time

We now derive the evolution equations for the three ratios x, u, and z directly from the original model (1)–(4). The strategy is to express the logarithmic derivatives of the original stocks in terms of the ratios themselves, and then combine them according to the quotient rule.

Logarithmic derivative of A

From (1), $\dot{A} = \lambda A^2 + \alpha_A K_f$. Dividing by A gives

$$\frac{\dot{A}}{A} = \lambda A + \alpha_A \frac{K_f}{A} = \lambda A + \alpha_A x.$$

The first term represents the self-reinforcing growth of AI capability, while the second term captures the conversion of financial capital into further AI research.

Logarithmic derivative of K_f

From (3),

$$\dot{K}_f = rK_f - \beta A K_f + \eta K_{ai} + \gamma_F K_f^2.$$

Dividing by K_f yields

$$\frac{\dot{K}_f}{K_f} = r - \beta A + \eta \frac{K_{ai}}{K_f} + \gamma_F K_f.$$

Now express every quantity in terms of A, x, and u:

(1) $K_f = xA$,

(2) $\dfrac{K_{ai}}{K_f} = u$.

Thus

$$\frac{\dot{K}_f}{K_f} = r - \beta A + \eta u + \gamma_F x A.$$

Logarithmic derivative of K_{ai}

Equation (2) gives $\dot{K}_{ai} = \beta A K_f - \delta K_{ai}$. Dividing by K_{ai} and using $\frac{K_f}{K_{ai}} = \frac{1}{u}$ (because $u = K_{ai}/K_f$) we obtain

$$\frac{\dot{K}_{ai}}{K_{ai}} = \beta A \frac{K_f}{K_{ai}} - \delta = \beta A \frac{1}{u} - \delta = \frac{\beta A}{u} - \delta.$$

Evolution of $x = K_f/A$

Using the logarithmic derivative relation $\frac{\dot{x}}{x} = \frac{\dot{K}_f}{K_f} - \frac{\dot{A}}{A}$ and substituting the expressions found above:

$$\frac{\dot{x}}{x} = \big(r - \beta A + \eta u + \gamma_F x A\big) - \big(\lambda A + \alpha_A x\big).$$

Grouping terms that are independent of A and those proportional to A:

$$\frac{\dot{x}}{x} = \big(r - \alpha_A x + \eta u\big) + A\,\big(\gamma_F x - \lambda - \beta\big). \tag{6}$$

This separation is key: the constant part reflects slow background forces, while the A-proportional part will dominate as A grows and ultimately controls the structural fate of the economy.

Evolution of $u = K_{ai}/K_f$

We proceed analogously:

$$\frac{\dot{u}}{u} = \frac{\dot{K}_{ai}}{K_{ai}} - \frac{\dot{K}_f}{K_f} = \Big(\frac{\beta A}{u} - \delta\Big) - \big(r - \beta A + \eta u + \gamma_F x A\big).$$

Expand the parentheses and collect the A-dependent terms and the constant terms:

$$\frac{\dot{u}}{u} = A\Big(\frac{\beta}{u} + \beta - \gamma_F x\Big) - \big(\delta + r + \eta u\big). \tag{7}$$

The first group determines the leading-order competition between productive investment and financial autocatalysis; the second group contains baseline depreciation and return rates, which become secondary in the asymptotic regime.

Evolution of the employment odds $z = L_p/(1 - L_p)$

From $z = L_p/(1 - L_p)$ we have $\dot{z} = \dot{L}_p/(1 - L_p)^2$. Using (4),

$$\dot{L}_p = -\kappa A K_{ai} L_p(1 - L_p) + \nu(1 - L_p).$$

Thus

$$\dot{z} = \frac{-\kappa A K_{ai} L_p(1 - L_p) + \nu(1 - L_p)}{(1 - L_p)^2}.$$

Now substitute $L_p = \dfrac{z}{1+z}$ and $1 - L_p = \dfrac{1}{1+z}$:

$$L_p(1 - L_p) = \frac{z}{(1+z)^2}, \qquad \frac{1}{(1 - L_p)^2} = (1+z)^2.$$

Hence

$$\begin{aligned}\dot{z} &= (1+z)^2\Big[-\kappa A K_{ai}\,\frac{z}{(1+z)^2} + \nu\,\frac{1}{1+z}\Big] \\ &= -\kappa A K_{ai}\, z + \nu(1+z).\end{aligned}$$

Finally, express AK_{ai} through the ratios: $AK_{ai} = A\,(uK_f) = u\,x\,A^2$. Therefore

$$\dot{z} = \nu(1+z) - \kappa\, u x A^2 z. \tag{8}$$

The factor A^2 signals that employment dynamics are even more sensitive to the blow-up than the capital ratios; we will later rescale time by A^2 to obtain a finite limit.

Equations (6), (7) and (8) are exact for all $t < t^*$ and contain no approximation. In the next subsection we exploit the fact that $A \to \infty$ near the singularity to simplify them into an autonomous system that captures the essential long-run behaviour.

3.3 Removing the Explosive Scale and the Asymptotic Reduced System

As the singularity is approached, $A(t) \to \infty$ (this will be confirmed by the self-similar analysis). Consequently, the terms multiplied by A in (6)–(7) become arbitrarily large compared with the constant terms. To absorb this divergence, we introduce the rescaled time

$$d\tau = A(t)\,dt,$$

so that a small increment in ordinary time corresponds to a large increment in τ near the singularity. Under this rescaling,

$$\frac{dx}{d\tau} = \frac{1}{A}\frac{dx}{dt} = x\Big[\frac{r - \alpha_A x + \eta u}{A} + \big(\gamma_F x - \lambda - \beta\big)\Big].$$

Taking the formal limit $A \to \infty$ eliminates the $O(1/A)$ terms, leaving the autonomous system

$$\frac{dx}{d\tau} = x\big(\gamma_F x - \lambda - \beta\big), \tag{9a}$$

$$\frac{du}{d\tau} = \beta + u\big(\beta - \gamma_F x\big). \tag{9b}$$

This reduction is mathematically consistent because the original terms that vanish in the limit are uniformly sub-dominant compared with the blow-up scale; their influence on the asymptotic ratios is negligible. Thus the full dynamics of the ratios are captured by (9) in a neighbourhood of the singularity.

For the employment odds, the stronger divergence A^2 requires the rescaling $d\tau_2 = A^2\,dt$, yielding

$$\frac{dz}{d\tau_2} = -\kappa\, ux\, z, \tag{10}$$

because $\nu(1+z)/A^2 \to 0$. Hence z decays to 0 for any $\kappa > 0$, i.e. full unemployment in the limit.

3.4 Self-Similar Blowup and Asymptotic Constants

A dominant-balance analysis (detailed in Appendix A) confirms that the original variables blow up as

$$A \sim \frac{a}{t^* - t}, \quad K_f \sim \frac{k_f}{t^* - t}, \quad K_{ai} \sim \frac{k_{ai}}{t^* - t},$$

with t^* the blowup time. Substituting into (1)–(3) and keeping leading-order terms yields $a = 1/\lambda$, $k_f = (\lambda+\beta)/(\lambda\gamma_F)$, $k_{ai} = \beta k_f/\lambda$. Consequently the ratios approach the finite limits

$$\boxed{x^* = \frac{k_f}{a} = \frac{\lambda+\beta}{\gamma_F}, \qquad u^* = \frac{k_{ai}}{k_f} = \frac{\beta}{\lambda}, \qquad L_p^* = 0.} \tag{11}$$

These values are precisely the fixed point of the reduced system (9): setting $dx/d\tau = 0$ gives $x = (\lambda+\beta)/\gamma_F$; substituting into $du/d\tau = 0$ gives $u = \beta/\lambda$. Thus the self-similar blowup solution corresponds to the equilibrium of the asymptotic dynamics, providing a rigorous link between the original model and the reduced description.

4 Stability Analysis

The reduced system (9) governs the structural evolution of the economy near the singularity. We now linearise it around the equilibrium to determine which initial conditions lead to which long-run regime.

4.1 Fixed Point and Linearisation

The unique equilibrium with positive ratios is

$$(x^*, u^*) = \Big(\frac{\lambda+\beta}{\gamma_F},\ \frac{\beta}{\lambda}\Big).$$

The Jacobian matrix at this point is

$$J = \begin{pmatrix} \dfrac{\partial \dot{x}}{\partial x} & \dfrac{\partial \dot{x}}{\partial u} \\ \dfrac{\partial \dot{u}}{\partial x} & \dfrac{\partial \dot{u}}{\partial u} \end{pmatrix}_{(x^*,u^*)} = \begin{pmatrix} \lambda+\beta & 0 \\ -\gamma_F \dfrac{\beta}{\lambda} & -\lambda \end{pmatrix}.$$

Its eigenvalues are

$$\mu_1 = \lambda + \beta > 0, \qquad \mu_2 = -\lambda < 0.$$

Thus the equilibrium is a **saddle point**: there exists a one-dimensional stable manifold and a one-dimensional unstable manifold. The eigenvector for μ_2 (stable) is $(0,1)^T$, meaning that the stable direction is purely vertical in the (x,u)-plane at linear order. The unstable manifold is tangent to $(1, -\gamma_F\beta/(\lambda(2\lambda+\beta)))^T$.

4.2 Economic Interpretation of the Saddle

Because the stable manifold is (to linear order) the vertical line $x = x^*$, the sign of $(x - x^*)$ determines the ultimate fate of the economy:

(1) **If $x_0 < x^*$:** The trajectory lies to the left of the separatrix; the nonlinear dynamics $\dot{x} = x(\gamma_F x - \lambda - \beta)$ (which are independent of u at leading order) force x to decrease toward zero. AI capability outpaces financial expansion, leading to an AI-dominated economy.

(2) **If $x_0 > x^*$:** The trajectory lies to the right; x grows without bound, driving the system toward runaway financialization. The divergence of x feeds back into the labour market via (10), precipitating mass unemployment. We term this outcome **Neofeudalism**.

The threshold value $x_c = (\lambda+\beta)/\gamma_F$ is therefore a **critical financial depth**. Its dependence on the parameters shows that an increase in financial autocatalysis γ_F lowers the threshold, widening the Neofeudal basin, while an increase in AI self-improvement λ raises the threshold, enlarging the safe region.

Theorem 1 (Financialization Threshold)**.** *For the reduced system (9) derived from (1)–(3), let $x_c = (\lambda+\beta)/\gamma_F$.*

(1) *If $x_0 < x_c$, then $x(\tau) \to 0$ as $\tau \to \infty$; AI dominates.*

(2) *If $x_0 > x_c$, then $x(\tau) \to \infty$ (finite-time blow-up in τ) and the economy enters the Neofeudal regime.*

(3) *The balanced path $x = x_c$ is a saddle separatrix; it is structurally unstable and is attained only when initial conditions lie exactly on the stable manifold.*

The phase portrait of the reduced system is shown in Figure 1, where the separatrix at $x = x_c$ is clearly visible. Figure 2 illustrates the time evolution of x for two representative initial conditions: one below the threshold (decay to zero) and one above (runaway growth).

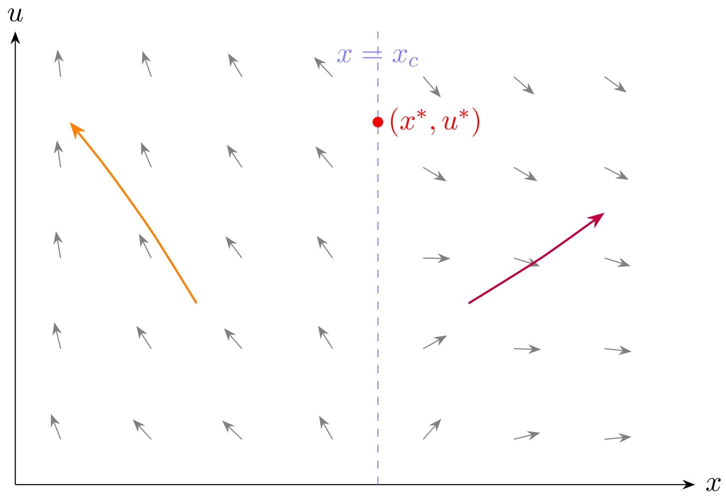

Figure 1. Phase portrait of the reduced system (9) for $\lambda = 0.2$, $\gamma_F = 0.25$, $\beta = 0.8$. The saddle equilibrium is at $(x^*, u^*) = (4, 4)$. Trajectories starting with $x_0 < 4$ move leftward ($x \to 0$), while those with $x_0 > 4$ move rightward ($x \to \infty$).

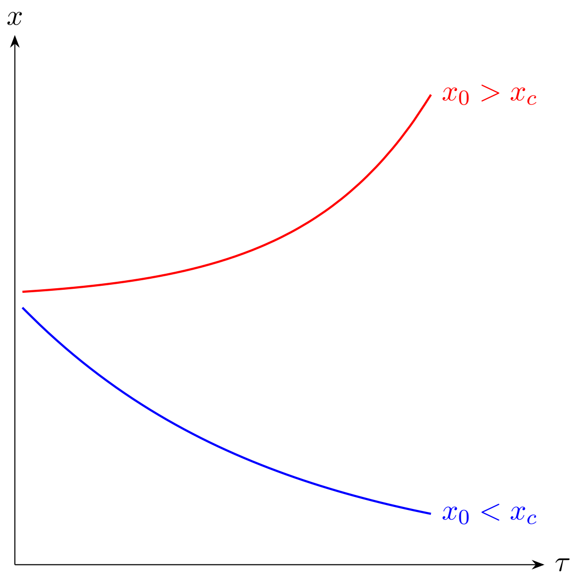

Figure 2. Time evolution of x in the reduced system for two initial conditions. When $x_0 < x_c$, x decays toward zero (AI dominance). When $x_0 > x_c$, x grows rapidly, signalling the onset of Neofeudalism.

5 Numerical Simulations

To validate the theoretical predictions, we integrate the full four-dimensional system (1)–(4) using a fourth-order Runge–Kutta method with adaptive step size. The baseline parameters are $\alpha_A = 0.01$, $\beta = 0.8$, $\delta = 0.05$, $r = 0.03$, $\eta = 0.02$, $\kappa = 0.1$, $\nu = 0.02$, while λ and γ_F are varied to illustrate the threshold behaviour.

6 Policy Implications

The saddle-threshold structure translates directly into actionable policy recommendations.

6.1 Regulate γ_F to Raise the Critical Threshold

Since $x_c = (\lambda + \beta)/\gamma_F$, reducing γ_F directly increases the safe basin, making the economy more resilient. Specific instruments include:

(1) Financial transaction taxes (FTT) that dampen high-frequency speculative trading.

(2) Leverage caps and margin requirements that limit the multiplicative effect of reinvested returns.

(3) Structural separation of commercial and investment banking.

(4) Progressive taxation of short-term capital gains.

Redistributive taxes alone cannot alter γ_F; they only redistribute wealth after the singularity. In the regime $x_0 > x_c$, no tax rate can prevent the tipping—regulation of the financial feedback loop itself is essential.

6.2 Boost λ Through AI Research

Increasing AI self-improvement efficiency widens the safe region. Public investment in foundational AI R&D, open-source models, and international research consortia can raise λ, albeit with natural upper bounds imposed by algorithmic and physical limits.

6.3 Intervene Early to Shape Initial Ratios

The threshold condition $x_0 < x_c$ must be satisfied *before* the singularity approaches. Early policies that reduce x_0 (e.g., breaking up large financial pools, channelling public funds into AI research) or increase u_0 (subsidising productive AI investment) can keep the economy in the safe basin, buying time for structural reforms.

6.4 Timing and Coordination

Delaying action is dangerous because once x crosses x_c, the dynamics become dominated by the $\gamma_F x^2$ term, and no subsequent policy can reverse the trajectory. The optimal package combines immediate γ_F regulation, medium-term λ enhancement, and early interventions to lower x_0.

7 Conclusion

We have reduced a four-variable explosive economic singularity model to a two-dimensional autonomous system for the ratios $x = K_f/A$ and $u = K_{ai}/K_f$. The reduced system possesses a saddle equilibrium at $x_c = (\lambda + \beta)/\gamma_F$, $u^* = \beta/\lambda$, which separates a regime of AI dominance ($x \to 0$) from a regime of runaway financialization ($x \to \infty$). The location of this separatrix is controlled by the balance between AI self-improvement λ and financial autocatalysis γ_F. Numerical simulations of the original system fully corroborate the threshold behaviour. The analysis demonstrates that redistribution alone cannot prevent Neofeudalism; structural regulation of financial autocatalysis and early intervention to keep financial depth below the critical level are essential.

Limitations

The model abstracts from several realistic features. First, it assumes homogeneous capital and labour markets, ignoring heterogeneity, institutions, and political feedback. Second, the financial autocatalysis term is a simple quadratic, while real-world bubbles often display more complex nonlinearities such as log-periodic oscillations or stochastic resonance. Third, the AI self-improvement parameter λ is taken as constant, but in practice it may saturate as physical or data limits are reached. Fourth, the model does not incorporate fiscal or monetary policy feedback beyond the simple tax parameters. Finally, the analysis is purely deterministic; stochastic shocks could trigger early transitions across the threshold.

Future Research Directions

Future work should extend the model to include heterogeneous agents, adaptive expectations, and explicit policy levers. Embedding the reduced system into a stochastic framework would allow the calculation of first-passage times to the Neofeudal regime. An empirical calibration of the key parameters λ and γ_F using financial and AI growth data could turn the qualitative threshold into a quantitative early-warning indicator. Finally, exploring spatial and network effects (contagion across countries or sectors) may reveal additional bifurcations and tipping cascades.

Data Availability Statement

Data will be made available on request.

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Conflicts of Interest

The author declares no conflicts of interest.

AI Use Statement

The author used DeepSeek-R1 solely for language editing and proofreading during the preparation of this manuscript. The author reviewed and approved all revisions and takes full responsibility for the content of the manuscript.

Ethical Approval and Consent to Participate

Not applicable.

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Appendix

A Self-similar exponent balancing

We determine the blow-up rates of A, K_f, K_{ai} and the leading-order constants directly from the differential equations. Near the singularity t^*, we assume the power-law ansatz

$$A(t) = a\,(t^* - t)^{-p},$$
$$K_f(t) = k_f\,(t^* - t)^{-q},$$
$$K_{ai}(t) = k_{ai}\,(t^* - t)^{-r},$$

with $a, k_f, k_{ai} > 0$ and $p, q, r > 0$. Differentiating gives

$$\dot{A} = pa\,(t^* - t)^{-p-1},$$
$$\dot{K}_f = qk_f\,(t^* - t)^{-q-1},$$
$$\dot{K}_{ai} = rk_{ai}\,(t^* - t)^{-r-1}.$$

Equation (1): $\dot{A} = \lambda A^2 + \alpha_A K_f$
Substituting the ansatz yields

$$pa\,(t^* - t)^{-p-1} = \lambda a^2\,(t^* - t)^{-2p} + \alpha_A k_f\,(t^* - t)^{-q}.$$

The most singular term on the right is $\lambda a^2 (t^* - t)^{-2p}$ provided $2p > q$ (so the second term is subdominant). Balancing the left side with this term requires $p + 1 = 2p$, giving $p = 1$. Equating coefficients,

$$pa = \lambda a^2 \implies a = \frac{1}{\lambda}.$$

The remaining term $\alpha_A k_f (t^* - t)^{-q}$ will be subdominant if $q < 2$, which will be verified a posteriori.

Equation (3): $\dot{K}_f = rK_f - \beta A K_f + \eta K_{ai} + \gamma_F K_f^2$
Substituting and using $p = 1$,

$$qk_f\,(t^* - t)^{-q-1}$$
$$= rk_f\,(t^* - t)^{-q} - \beta a k_f\,(t^* - t)^{-1-q}$$
$$+\eta k_{ai}\,(t^* - t)^{-r} + \gamma_F k_f^2\,(t^* - t)^{-2q}.$$

The most singular terms on the right are $-\beta a k_f (t^* - t)^{-1-q}$ and $\gamma_F k_f^2 (t^* - t)^{-2q}$. Balancing the left side with the first of these gives $q + 1 = 1 + q$, which is identically satisfied. Balancing with the second term requires $q + 1 = 2q$, i.e. $q = 1$. With $q = 1$, all terms on the right are

$$rk_f\,(t^* - t)^{-1} - \beta a k_f\,(t^* - t)^{-2}$$
$$+\eta k_{ai}\,(t^* - t)^{-r} + \gamma_F k_f^2\,(t^* - t)^{-2}.$$

The terms $(t^* - t)^{-2}$ are the most singular. Cancelling the common factor $(t^* - t)^{-2}$ and using the left side $qk_f = k_f$ gives

$$k_f = -\beta a k_f + \gamma_F k_f^2.$$

Dividing by $k_f \neq 0$ and using $a = 1/\lambda$,

$$1 = -\frac{\beta}{\lambda} + \gamma_F k_f \implies k_f = \frac{1 + \beta/\lambda}{\gamma_F} = \frac{\lambda + \beta}{\lambda \gamma_F}.$$

The term ηK_{ai} will be subdominant if $r < 2$, to be checked next.

Equation (2): $\dot{K}_{ai} = \beta A K_f - \delta K_{ai}$
Substituting with $p = q = 1$,

$$rk_{ai}\,(t^* - t)^{-r-1} = \beta a k_f\,(t^* - t)^{-2} - \delta k_{ai}\,(t^* - t)^{-r}.$$

The most singular term on the right is $\beta a k_f (t^* - t)^{-2}$. Balancing with the left side requires $r + 1 = 2$, so $r = 1$. Equating coefficients,

$$rk_{ai} = \beta a k_f \implies k_{ai} = \beta \cdot \frac{1}{\lambda} \cdot k_f = \frac{\beta k_f}{\lambda}.$$

Subdominance check
With $p = q = r = 1$, the terms

$$\alpha_A K_f,\ \eta K_{ai},\ \delta K_{ai},\ rK_f$$

all scale as $(t^* - t)^{-1}$, while the dominant terms scale as $(t^* - t)^{-2}$. Hence they are negligible compared to the leading-order terms as $t \to t^*$, confirming the self-consistency of the asymptotic approximation.

The asymptotic ratios are therefore

$$x^* = \frac{k_f}{a} = \frac{\lambda + \beta}{\gamma_F}, \qquad u^* = \frac{k_{ai}}{k_f} = \frac{\beta}{\lambda}.$$

B Linear stability of the reduced system

Linearising (9) around (x^*, u^*) yields the Jacobian shown in the main text. The perturbation equations for $\delta x = x - x^*$, $\delta u = u - u^*$ read

$$\frac{d}{d\tau}\begin{pmatrix}\delta x\\ \delta u\end{pmatrix} = \begin{pmatrix}\lambda+\beta & 0\\ -\gamma_F\beta/\lambda & -\lambda\end{pmatrix}\begin{pmatrix}\delta x\\ \delta u\end{pmatrix} + \text{higher-order terms.}$$

The eigenvalues $\mu_1 = \lambda + \beta > 0$, $\mu_2 = -\lambda < 0$ confirm the saddle character. The stable manifold is tangent to the u-axis, meaning that deviations in u are damped, while deviations in x are amplified. This exactly corresponds to the geometric picture developed in Section 4.

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