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Journal of Nonlinear Dynamics and Applications
http://dx.doi.org/10.62762/JNDA.2026.507142

LETTER

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Fixed-Time Stabilization for a Class of Dynamical Systems and Its Application to a Chaotic System

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Abstract

In this letter, the fixed-time stabilization of a class of dynamical systems is investigated. By using a fixed-time stability lemma and feedback control, new algebraic criteria ensuring the fixed-time stabilization of the dynamical systems under consideration are established. The obtained results are then applied to stabilize a chaotic system, and numerical simulations demonstrate the effectiveness of the theoretical results.

Keywords: fixed-time stability, dynamical system, chaotic system, feedback control.

1 Introduction

As is well known, stability of linear and nonlinear dynamical systems is important, since it is a prerequisite for the practical applications of such systems. In 2019, Li *et al.* [1] discussed the exponential stability of nonlinear systems with delayed impulses. In 2023, Chang *et al.* [2] investigated the global exponential stability of delayed inertial neural networks; however, such exponentially stable systems only converge asymptotically. Bhat and Bernstein [3] established the fundamental theory of finite-time stability for continuous autonomous systems. However, exponential stability and finite-time stability both have drawbacks in the estimation of convergence time: an exponentially stable system converges only asymptotically, while the settling time of a finite-time stable system depends on the initial values, which may be unknown in advance.

Fixed-time stability has the advantage that the upper bound of the convergence time is a fixed positive constant independent of the initial values [4]. Therefore, fixed-time stability has become a hot topic in recent years; see, e.g., [5–8]. In 2023, Zhang and Cao [6] obtained fixed/predefined-time synchronization results for delayed fuzzy inertial discontinuous neural networks. Qin *et al.* [7] studied fixed/prescribed-time synchronization of quaternion-valued fuzzy BAM neural networks under aperiodic intermittent pinning control, and Hu *et al.* [8] proposed a fixed-/preassigned-time stabilization approach for discontinuous systems based on strictly intermittent control.

However, to achieve fixed-time stability and stabilization, most of the controllers designed in [5–8] contain two power-index terms. In this letter, without using two power-index terms in the control scheme, we investigate the fixed-time stabilization of a class of general dynamical systems based on the exponential function used in [9].

Submitted: 17 July 2026
Revised: 24 September 2026
Accepted: 26 September 2026
Published: 28 September 2026

Vol. 2, **No.** 3, 2026.
10.62762/JNDA.2026.507142

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Citation
Han, J., Fan, Y., & Huang, J. (2026). Fixed-Time Stabilization for a Class of Dynamical Systems and Its Application to a Chaotic System. *Journal of Nonlinear Dynamics and Applications*, 2(3), 217–221.

The rest of this letter is organized as follows. Preliminaries are given in Section 2. The main results are presented in Section 3. In Section 4, the fixed-time stabilization result is applied to stabilize a chaotic system. Finally, conclusions are drawn in Section 5.

2 Preliminaries

Here, we consider the following dynamical system

$$\frac{\mathrm{d}p(t)}{\mathrm{d}t} = Ap(t) + Bg(p(t)), \quad t \geqslant 0, \tag{1}$$

in which $p(t) = (p_1(t), p_2(t), \dots, p_n(t))^T$ stands for the n-dimensional state vector, $A = (a_{ij})_{n\times n}$ and $B = (b_{ij})_{n\times n}$ are real-valued matrices, and $g(p(t)) = (g_1(p_1(t)), g_2(p_2(t)), \dots, g_n(p_n(t)))^T$ is a continuous vector function. The initial value of system (1) is $p(0) = (p_1(0), p_2(0), \dots, p_n(0))^T$.

Assumption 1. For system (1), each continuous function $g_l(\cdot)$ satisfies $g_l(0) = 0$ and, for all $x_1, x_2 \in \mathbb{R}$,

$$|g_l(x_1) - g_l(x_2)| \leqslant \mathcal{M}_l |x_1 - x_2|, \tag{2}$$

where $\mathcal{M}_l > 0$, $l = 1, 2, \dots, n$.

Definition 1. ([4, 8]) The origin of system (1) is said to be fixed-time stable if it is globally finite-time stable, i.e., it is Lyapunov stable and there exists a settling-time function $\mathcal{T}(p(0))$ such that $\lim_{t\to\mathcal{T}(p(0))} \|p(t)\| = 0$ and $p(t) \equiv 0$ for all $t \geqslant \mathcal{T}(p(0))$, and, moreover, there is a constant $L > 0$ such that $\mathcal{T}(p(0)) \leqslant L$ for any initial value $p(0)$. Here, $\|p(t)\| = \sqrt{\sum_{l=1}^{n} p_l^2(t)}$, and L is called an upper bound of the settling time.

Lemma 1. ([9]) Suppose that the continuous function $\mathbf{V}(\cdot)$ is positive definite and radially unbounded, and that along the solutions of the considered system it satisfies

$$\frac{\mathrm{d}\mathbf{V}(t)}{\mathrm{d}t} \leqslant -c\, b^{\mathbf{V}^q(t)} \mathbf{V}^{1-q}(t), \tag{3}$$

where $b > 1$, $c > 0$ and $0 < q \leqslant 1$. Then the system is fixed-time stable, and the settling time satisfies $\mathcal{T}(p(0)) \leqslant L = \frac{1}{qc\ln b}$.

3 Main results

If dynamical system (1) is unstable, we consider the following controlled system

$$\frac{\mathrm{d}p(t)}{\mathrm{d}t} = Ap(t) + Bg(p(t)) + U(t), \quad t \geqslant 0, \tag{4}$$

where the control input $U(t)$ in (4) is designed as

$$U(t) = \begin{cases} -\gamma p(t) - \eta\, b^{\mathbf{V}^q(t)} \mathbf{V}^{-q}(t) p(t), & p(t) \neq 0, \\ 0, & p(t) = 0, \end{cases} \tag{5}$$

in which $\gamma, \eta > 0$, $b > 1$, $0 < q \leqslant 1$, $\mathbf{V}(t) = p^T(t)Qp(t)$, and $Q = \mathrm{diag}\{\lambda_1, \lambda_2, \dots, \lambda_n\}$ is a positive definite diagonal matrix.

Theorem 1. Suppose that Assumption 1 holds. Under controller (5), if there exists a positive definite diagonal matrix Q such that

$$QA + A^TQ + 2\|QB^+\mathcal{M}\|I - 2\gamma Q \leqslant 0, \tag{6}$$

then the controlled system (4) is fixed-time stable, i.e., system (1) is fixed-time stabilized, and the settling time satisfies $\mathcal{T}(p(0)) \leqslant L = \frac{1}{2\eta q \ln b}$, where $\|QB^+\mathcal{M}\| = \sqrt{\lambda_{\max}\big((QB^+\mathcal{M})^T(QB^+\mathcal{M})\big)}$, $\mathcal{M} = \mathrm{diag}\{\mathcal{M}_1, \mathcal{M}_2, \dots, \mathcal{M}_n\}$ and $B^+ = (|b_{ij}|)_{n\times n}$.

Proof. Consider the Lyapunov function

$$\mathbf{V}(t) = p^T(t)Qp(t). \tag{7}$$

For $p(t) \neq 0$, the derivative of $\mathbf{V}(t)$ along the solutions of system (4) is

$$\begin{aligned} \frac{\mathrm{d}\mathbf{V}(t)}{\mathrm{d}t}\Big|_{(4)} =& 2p^T(t)Q\Big(Ap(t) + Bg(p(t)) + U(t)\Big) \\ =& p^T(t)\big(QA + A^TQ\big)p(t) + 2p^T(t)QBg(p(t)) \\ & - 2\gamma p^T(t)Qp(t) - 2\eta\, b^{\mathbf{V}^q(t)} \mathbf{V}^{-q}(t) p^T(t)Qp(t). \end{aligned} \tag{8}$$

By Assumption 1 with $g_l(0) = 0$, one has $|g_l(p_l(t))| \leqslant \mathcal{M}_l |p_l(t)|$, $l = 1, 2, \dots, n$. Denote $|p(t)| = (|p_1(t)|, |p_2(t)|, \dots, |p_n(t)|)^T$. Since Q is a positive diagonal matrix, it follows from the Cauchy–Schwarz inequality that

$$\begin{aligned} 2p^T(t)QBg(p(t)) \leqslant& 2\sum_{i=1}^{n}\sum_{j=1}^{n} \lambda_i |p_i(t)||b_{ij}|\mathcal{M}_j|p_j(t)| \\ =& 2|p(t)|^T QB^+\mathcal{M}|p(t)| \\ \leqslant& 2\|QB^+\mathcal{M}\|\, p^T(t)p(t), \end{aligned} \tag{9}$$

where the last inequality follows from $x^TMx \leqslant \|M\|_2\|x\|^2$ for any $M \in \mathbb{R}^{n\times n}$ and $x \in \mathbb{R}^n$, with $\|M\|_2 = \sqrt{\lambda_{\max}(M^TM)}$ denoting the spectral norm of M. Noting that $p^T(t)Qp(t) = \mathbf{V}(t)$ and substituting (9) into (8), one obtains from (6) that

$$\frac{\mathrm{d}\mathbf{V}(t)}{\mathrm{d}t}\Big|_{(4)} \leqslant p^T(t)\Big(QA + A^TQ + 2\|QB^+\mathcal{M}\|I - 2\gamma Q\Big)p(t)$$

$$
\begin{aligned}
&- 2\eta\, b^{\mathbf{V}^{q}(t)}\mathbf{V}^{1-q}(t)\\
\leqslant &- 2\eta\, b^{\mathbf{V}^{q}(t)}\mathbf{V}^{1-q}(t). \qquad (10)
\end{aligned}
$$

Based on Lemma 1 with $c = 2\eta$, the controlled system (4) is fixed-time stable under controller (5), and the settling time satisfies $\mathcal{T}(p(0)) \leqslant L = \frac{1}{2\eta q \ln b}$. This completes the proof of Theorem 1.

Remark 1. When $0 < q < 1/2$, controller (5) is continuous at $p(t) = 0$. Indeed, since Q is positive definite, there exist constants $\underline{\lambda}, \overline{\lambda} > 0$ such that $\underline{\lambda}\|p(t)\|^2 \leqslant \mathbf{V}(t) \leqslant \overline{\lambda}\|p(t)\|^2$. Hence

$$
\eta\, b^{\mathbf{V}^{q}(t)}\mathbf{V}^{-q}(t)\|p(t)\| \leqslant \eta\, b^{\overline{\lambda}^{q}\|p(t)\|^{2q}}\, \underline{\lambda}^{-q}\|p(t)\|^{1-2q},
$$

which tends to zero as $p(t) \to 0$ whenever $1 - 2q > 0$, i.e., $q < 1/2$. Therefore, $\eta\, b^{\mathbf{V}^{q}(t)}\mathbf{V}^{-q}(t)p(t) \to 0$ as $p(t) \to 0$, confirming that controller (5) is continuous and sign-function-free in this case.

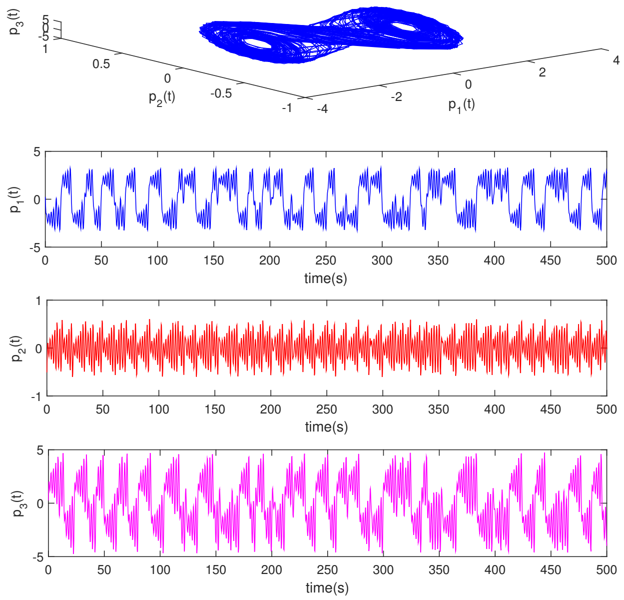

Figure 1. Chaotic behaviors and state trajectories of dynamical system (11) without control.

4 Application to the stabilization of a chaotic system

In this section, based on Chua's circuit [10] as discussed in [1], we consider the following three-dimensional chaotic system

$$
\frac{\mathrm{d}p(t)}{\mathrm{d}t} = A_{3\times 3}p(t) + B_{3\times 3}g(p(t)), \quad t \geqslant 0, \qquad (11)
$$

where

$$
A_{3\times3} = \begin{pmatrix} -2 & 9 & 0 \\ 1 & -1 & 1 \\ 0 & -15 & 0 \end{pmatrix},
$$

$$
B_{3\times3} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},
$$

$$
g(p(t)) = \begin{pmatrix} |p_1(t)+1| - |p_1(t)-1| \\ 0 \\ 0 \end{pmatrix},
$$

$$
p(t) = (p_1(t), p_2(t), p_3(t))^T. \qquad (12)
$$

With the initial values $p_1(0) = 0.2$, $p_2(0) = -0.5$ and $p_3(0) = 0.3$, the chaotic behaviors and state trajectories of system (11) without control are displayed in Figure 1.

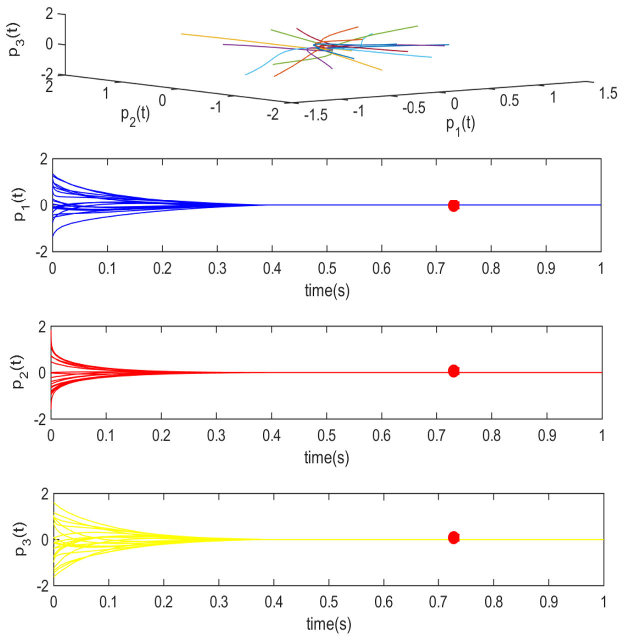

Figure 2. State trajectories of dynamical system (11) with controller (5).

Now, let $\gamma = 8$, $\eta = 1$, $q = 0.4999$ and $b = 4$. Since $g_1(\cdot)$ satisfies (2) with $\mathcal{M}_1 = 2$, and $g_2 = g_3 \equiv 0$ satisfy (2) with arbitrary $\mathcal{M}_2, \mathcal{M}_3 > 0$ (which do not affect $B^{+}\mathcal{M}$ because the second and third columns of B^{+} are zero), we take $\mathcal{M} = \mathrm{diag}\{2, \mathcal{M}_2, \mathcal{M}_3\}$. By solving condition (6) with MATLAB, one gets

$$
Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 1 \end{pmatrix}.
$$

Now, all conditions of Theorem 1 are satisfied; hence, the controlled system (11) with controller (5) is

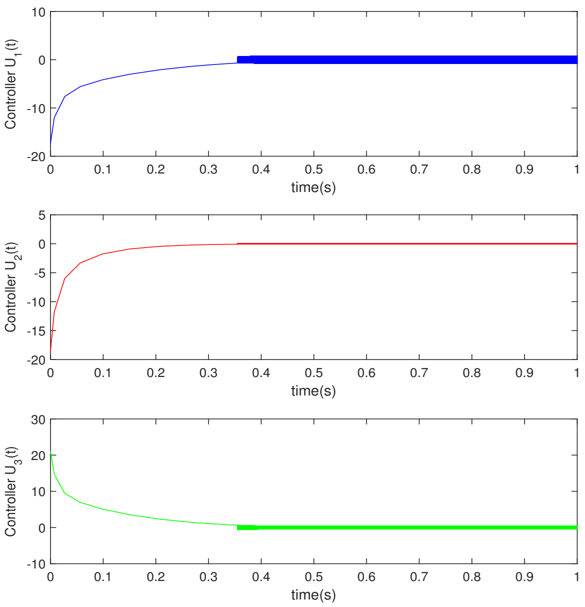

Figure 3. Input signals of controller (5).

fixed-time stable. Figure 2 shows the state trajectories of system (11) with controller (5), from which one can see that the states converge to zero within the settling-time upper bound $L = \frac{1}{2\eta q \ln b} \approx 0.72$ s. The corresponding control input signals generated by controller (5) are shown in Figure 3, which verifies the effectiveness of the designed controller.

5 Conclusions

By using feedback control with an exponential function and the fixed-time stability method, we have discussed the fixed-time stabilization of a class of general dynamical systems in this letter. Compared with previous work, our controller does not contain two power-index terms, and, when $0 < q < 1/2$, it is continuous and involves no sign function, which is more convenient for practical implementation. It should be noted that our method can be extended to investigate other dynamical systems, such as systems with discontinuous right-hand sides [8], complex-valued systems and memristive neural dynamical systems, which will be considered in our future work.

Data Availability Statement

Data will be made available on request.

Funding

This work was supported in part by the Science and Technology Research Program of the Hubei Provincial Department of Education under Grant B2024273, and the Doctoral Fund Program of Wuhan Business University under Grant 2024KB015.

Conflicts of Interest

The authors declare no conflicts of interest.

AI Use Statement

The authors declare that no generative AI was used in the preparation of this manuscript.

Ethical Approval and Consent to Participate

Not applicable.

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