



Polynomial-Based Kudryashov Analytical Methods for a Generalized Sharma–Tasso–Olver Equation with Non-Integer Nonlinearity

Sarita Pippal^{1,*} and Vikramjeet²

¹Department of Mathematics, Panjab University, Chandigarh 160014, India

²Department of Computer Science and Applications, Panjab University, Chandigarh 160014, India

Abstract

We study travelling-wave solutions of the generalized Sharma-Tasso-Olver (GSTO) equation $u_t + \lambda(u^r)_x + \mu(u^s)_{xx} + \nu u_{xxx} = 0$, in which the exponents $r, s > 0$ need not be integers. The classical, modified and generalized Kudryashov methods are applied to the reduced ordinary differential equation. A balance argument shows that the convective term fixes the degree of the expansion through $N = 2/(r - 1)$, whereas the nonlinear diffusion term requires $N = 1/(s - 1)$; the two requirements are compatible only when $r = 2s - 1$, a condition satisfied by the classical STO equation ($r = 3, s = 2$). For the representative non-integer cases $(r, s) = (2, \frac{3}{2})$ and $(\frac{3}{2}, 2)$ we show that bounded solutions within the polynomial ansatz exist only in the dispersive limit $\mu = 0$, and we derive them in closed form: a sech^2 solitary wave for $r = 2$ and a sech^4 solitary wave with speed $\kappa = 4\nu L^2$ for $r = \frac{3}{2}$. The generalized Kudryashov method extends both results to hyperbolic, singular

periodic and rational families classified by the discriminant $\Delta = \mathcal{B}_1^2 - 4\mathcal{B}_0\mathcal{B}_2$, with $\kappa = 4\nu\Delta$ in the case $r = \frac{3}{2}$. Every solution reported here was checked by direct substitution into the governing equation.

Keywords: generalized Sharma-Tasso-Olver equation, Kudryashov method, non-integer nonlinearity, travelling-wave solutions, solitary waves.

1 Introduction

Coherent structures in dispersive media, such as solitary waves, kinks and periodic wave trains, arise when nonlinear steepening is balanced by dispersion or diffusion [1–3]. The Korteweg-de Vries (KdV) equation

$$u_t + 6uu_x + u_{xxx} = 0 \quad (1)$$

is the standard example: its quadratic nonlinearity uu_x is balanced by third-order dispersion u_{xxx} . When higher-order nonlinear effects matter, richer models are needed.

Citation

Pippal, S., & Vikramjeet (2026). Polynomial-Based Kudryashov Analytical Methods for a Generalized Sharma–Tasso–Olver Equation with Non-Integer Nonlinearity. *Journal of Numerical Simulations in Physics and Mathematics*, 3(1), 51–59.



© 2026 by the Authors. Published by Institute of Central Computation and Knowledge. This is an open access article under the CC BY license (<https://creativecommons.org/licenses/by/4.0/>).



Submitted: 16 April 2026
Revised: 23 April 2026
Accepted: 06 June 2026
Published: 18 October 2026

Vol. 3, No. 1, 2026.
doi:10.62762/JNSPM.2026.980251

*Corresponding author:
✉ Sarita Pippal
saritamath@pu.ac.in

The Sharma-Tasso-Olver (STO) equation [4, 5]

$$u_t + \alpha(u^3)_x + \frac{3}{2}\alpha(u^2)_{xx} + \alpha u_{xxx} = 0, \quad \alpha \neq 0, \quad (2)$$

combines cubic convection, quadratic nonlinear diffusion and linear dispersion. It is integrable and possesses infinitely many symmetries [5]. Exact solutions of (2) have been obtained by many methods: soliton fission and fusion through bilinear-type constructions [6], Lie symmetry reductions and conservation laws [7, 8], multi-kink solutions [9], tanh-type and related direct methods [10, 11], the Exp-function method [12], extended hyperbolic-function expansions [13], auxiliary-equation methods [14], the modified simple equation method [15, 16] and the improved G'/G -expansion [17]. Generalized STO equations with arbitrary coefficients [18], variable coefficients [19], conformable- or beta-type derivatives [20, 22] and further explicit solutions [21] have also been studied, together with numerical schemes [23, 24] and phase-plane analyses of STO and STO-Burgers equations [25, 26].

The Kudryashov method [27, 28] expands the solution as a polynomial in a function that satisfies a first-order Riccati equation; the expansion degree follows from a balance between the highest derivative and the leading nonlinearity. Its modified [29] and generalized [30] versions, and applications to high-order evolution equations [31] and the conformable Sharma-Tasso-Olver equation [32], show that the approach does not rely on integrability.

Most STO studies keep integer exponents. Here we consider

$$u_t + \lambda(u^r)_x + \mu(u^s)_{xx} + \nu u_{xxx} = 0, \quad (3)$$

with $\lambda, \mu, \nu \in \mathbb{R}$ and $r, s > 0$ not necessarily integers; equation (2) is recovered for $r = 3$, $s = 2$, $\lambda = \alpha$, $\mu = \frac{3}{2}\alpha$, $\nu = \alpha$. The contributions of this paper are as follows.

- We show that the degree conditions imposed by the convective and diffusive terms, $N = 2/(r - 1)$ and $N = 1/(s - 1)$, coincide only when $r = 2s - 1$, and we explain what happens otherwise.
- For $(r, s) = (2, \frac{3}{2})$ and $(\frac{3}{2}, 2)$ we prove that the polynomial ansatz yields bounded real solutions only for $\mu = 0$, and we give these solutions explicitly.
- Using the generalized Kudryashov method we obtain hyperbolic, periodic and rational families for both cases and state the sign conditions under which they are real.

All solutions were verified by substitution with a computer algebra system.

2 Methods

With $u(x, t) = U(\xi)$, $\xi = x - \kappa t$, equation (3) becomes

$$-\kappa U' + \lambda(U^r)' + \mu(U^s)'' + \nu U''' = 0, \quad (4)$$

where the prime denotes $d/d\xi$. Expanded,

$$-\kappa U' + \lambda r U^{r-1} U' + \mu s(s-1) U^{s-2} (U')^2 + \mu s U^{s-1} U'' + \nu U''' = 0. \quad (5)$$

2.1 Classical and modified Kudryashov methods

The modified method seeks

$$U(\xi) = \sum_{k=0}^N A_k Y^k(\xi), \quad Y' = L(Y^2 - Y), \quad L \neq 0, \quad (6)$$

whose solution is

$$Y(\xi) = \frac{1}{1 + K e^{L\xi}} = \frac{1}{2} \left[1 - \tanh \frac{L(\xi - \xi_0)}{2} \right], \quad \xi_0 = \frac{\ln K}{L}, \quad (7)$$

for $K > 0$; $K < 0$ gives the singular coth branch, and $Y \equiv 0$, $Y \equiv 1$ are the equilibria. The classical method is the special case $L = 1$. Useful identities are

$$Y - Y^2 = \frac{1}{4} \operatorname{sech}^2 \frac{L(\xi - \xi_0)}{2}, \quad (8)$$

$$(Y - Y^2)' = -L(Y - Y^2)(1 - 2Y).$$

Lemma 1 (Degree balance). *Let Φ satisfy a Riccati equation with quadratic right-hand side and let U be a polynomial of degree N in Φ . Then $U^{(m)}$ has degree $N + m$ and $\frac{d^\ell}{d\xi^\ell}(U^\sigma)$ has degree $\sigma N + \ell$. Balancing the two gives*

$$N = \frac{m - \ell}{\sigma - 1}, \quad \sigma \neq 1. \quad (9)$$

Proof. Each differentiation raises the degree by one because Φ' is quadratic in Φ . Equating $N + m$ and $\sigma N + \ell$ gives (9). $\square$

This balance rule is standard in expansion methods; see [28, 33].

2.2 Generalized Kudryashov method

The generalized method [30] uses

$$U(\xi) = \mathcal{A}_0 + \sum_{k=1}^N \frac{\mathcal{A}_k}{(1 + Z(\xi))^k}, \quad Z' = \mathcal{B}_0 + \mathcal{B}_1 Z + \mathcal{B}_2 Z^2, \quad (10)$$

with $\mathcal{B}_2 \neq 0$ and $\Delta = \mathcal{B}_1^2 - 4\mathcal{B}_0\mathcal{B}_2$. The Riccati equation has the solutions

$$Z_h = -\frac{\mathcal{B}_1}{2\mathcal{B}_2} - \frac{\sqrt{\Delta}}{2\mathcal{B}_2} \tanh\left(\frac{\sqrt{\Delta}}{2}(\xi - \xi_0)\right), \quad \Delta > 0, \quad (11)$$

$$Z_p = -\frac{\mathcal{B}_1}{2\mathcal{B}_2} + \frac{\Omega}{2\mathcal{B}_2} \tan\left(\frac{\Omega}{2}(\xi - \xi_0)\right), \quad \Delta = -\Omega^2 < 0, \quad (12)$$

$$Z_r = -\frac{\mathcal{B}_1}{2\mathcal{B}_2} - \frac{1}{\mathcal{B}_2(\xi - \xi_0)}, \quad \Delta = 0, \quad (13)$$

and $\tanh \rightarrow \coth$ in (11), $\tan \rightarrow -\cot$ in (12) give further branches. Setting $G = 1/(1 + Z)$ one finds

$$\begin{aligned} G' &= -(\alpha_B G^2 + bG + \mathcal{B}_2), \\ \alpha_B &= \mathcal{B}_0 - \mathcal{B}_1 + \mathcal{B}_2, \quad b = \mathcal{B}_1 - 2\mathcal{B}_2, \end{aligned} \quad (14)$$

with $b^2 - 4\alpha_B\mathcal{B}_2 = \Delta$. Thus (10) is a polynomial in G , and Lemma 1 applies unchanged. We also use

$$R(\xi) = \frac{\mathcal{B}_0 + \mathcal{B}_1 Z + \mathcal{B}_2 Z^2}{(1 + Z)^2} = -G'(\xi). \quad (15)$$

For $\Delta > 0$ the function Z_h stays away from -1 , and the solutions below are bounded, if and only if $\alpha_B\mathcal{B}_2 > 0$.

3 Degree analysis for the GSTO equation

In (4), U''' has degree $N + 3$, the convective term $(U^r)'$ has degree $rN + 1$ and the diffusive term $(U^s)''$ has degree $sN + 2$. Lemma 1 gives

$$N_c = \frac{2}{r-1} \quad (r \neq 1), \quad N_d = \frac{1}{s-1} \quad (s \neq 1). \quad (16)$$

Proposition 1. *Let $r > 1$, $s \neq 1$ and $N = N_c$. Then*

$$(sN_c + 2) - (N_c + 3) = (s-1)(N_c - N_d) = \frac{2s-1-r}{r-1}. \quad (17)$$

Consequently:

- (i) if $r = 2s - 1$, all three terms of (4) enter the leading-order balance;
- (ii) if $r < 2s - 1$, the diffusive term has the highest degree, its leading coefficient is proportional to μ , and a nonconstant polynomial solution of degree N_c requires $\mu = 0$;
- (iii) if $r > 2s - 1$, the diffusive term has lower degree than U''' and does not enter the leading-order balance.

Proof. The first equality in (17) follows from $N_c + 3 = rN_c + 1$ and $N_d = 1/(s-1)$; the second from $N_c - N_d = (2s-1-r)/((r-1)(s-1))$. Since $r > 1$, the sign of the gap is the sign of $2s-1-r$. $\square$

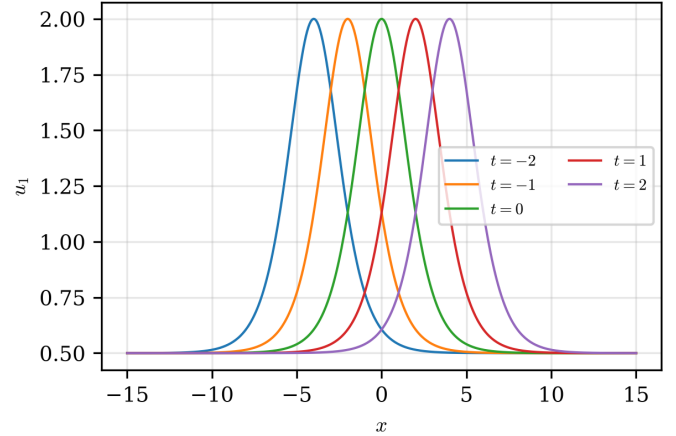
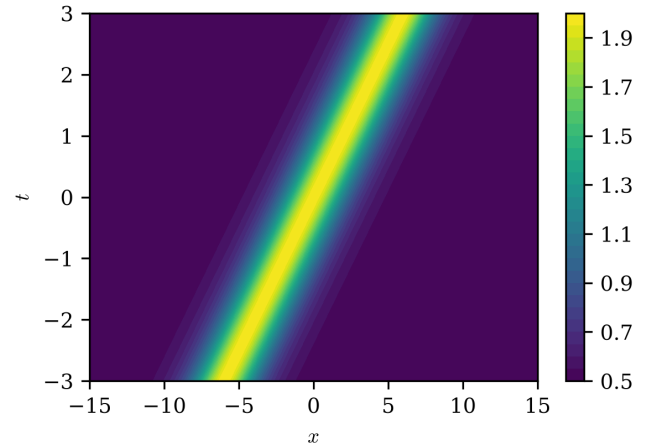
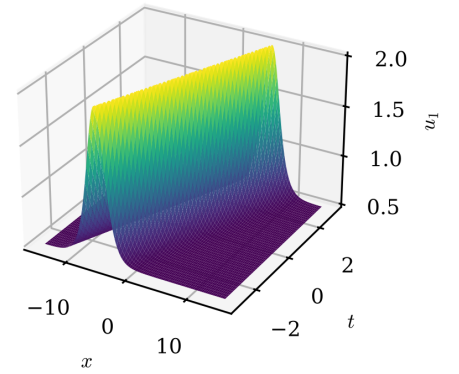


Figure 1. Solitary wave u_1 of Eq. (20) for $r = 2$, $\mu = 0$, $\kappa = 2$, $\lambda = \nu = L = 1$, $\xi_0 = 0$: surface (top), contour plot (middle) and profiles at $t = -2, \dots, 2$ (bottom).

The condition must be stated in terms of r and s , not as $N_c > N_d$: for $0 < s < 1$ one has $N_d < 0 < N_c$, yet by (17) the diffusive term is subdominant. For the classical STO equation, $r = 3$ and $s = 2$ give $N_c = N_d = 1$. For $(r, s) = (2, \frac{3}{2})$ case (i) applies ($N = 2$); for $(r, s) = (\frac{3}{2}, 2)$ case (ii) applies, since $r = \frac{3}{2} < 3 = 2s - 1$ ($N_c = 4$, and the diffusive term has degree 10 against 7).

Non-integer powers are handled by writing $U = V^2$ with $V \geq 0$, so that $U^{3/2} = V^3$ and $U^{1/2} = V$ are polynomials in Y (or G). The condition $V \geq 0$ is essential: if V changes sign, $V^3 \neq (V^2)^{3/2}$ and the substitution no longer solves (3).

4 Solutions by the classical and modified methods

4.1 Case $r = 2, s = \frac{3}{2}$

Here $N = 2$. Writing $U = V^2$ with $V = c_0 + c_1 Y$ and substituting into (4) gives a consistent algebraic system whose nontrivial solution is

$$c_0 = -\frac{c_1}{2}, \quad \kappa = \frac{c_1(3L\mu + 4c_1\lambda)}{12}, \quad \nu = -\frac{c_1(3L\mu + c_1\lambda)}{6L^2} \quad (18)$$

Then $V = c_1(Y - \frac{1}{2}) = -\frac{c_1}{2} \tanh \frac{L(\xi - \xi_0)}{2}$ changes sign at $\xi = \xi_0$, so $U = \frac{c_1^2}{4} \tanh^2 \frac{L(\xi - \xi_0)}{2}$ satisfies (3) only on the half-line where $V \geq 0$. A globally valid nonconstant solution therefore requires $\mu = 0$. In that case (4) with $U = A_0 + A_1 Y + A_2 Y^2$ gives

$$A_2 = -\frac{6\nu L^2}{\lambda}, \quad A_1 = \frac{6\nu L^2}{\lambda}, \quad A_0 = \frac{\kappa - \nu L^2}{2\lambda}, \quad (19)$$

with κ free, and hence, by (8),

$$\begin{aligned} u_1(x, t) &= \frac{\kappa - \nu L^2}{2\lambda} + \frac{6\nu L^2}{\lambda} (Y - Y^2) \\ &= \frac{\kappa - \nu L^2}{2\lambda} + \frac{3\nu L^2}{2\lambda} \operatorname{sech}^2 \frac{L(x - \kappa t - \xi_0)}{2}. \end{aligned} \quad (20)$$

The classical method ($L = 1$) and the modified method yield the same family; only the width parameter L differs. Figure 1 shows a bright solitary wave of amplitude $3\nu L^2/(2\lambda)$ on the background $(\kappa - \nu L^2)/(2\lambda)$ moving to the right with speed κ .

4.2 Case $r = \frac{3}{2}, s = 2$

By Proposition 1 we must take $\mu = 0$. Then $N = 4$ and we set $U = V^2$, $V = c_0 + c_1 Y + c_2 Y^2$, so that $U^{3/2} = V^3$ when $V \geq 0$. The algebraic system has two nonconstant solutions. The first is

$$c_0 = 0, \quad c_1 = -c_2 = \frac{20\nu L^2}{\lambda}, \quad \kappa = 4\nu L^2, \quad (21)$$

which in terms of (6) means $A_0 = A_1 = 0$, $A_2 = A_4 = 400\nu^2 L^4/\lambda^2$, $A_3 = -800\nu^2 L^4/\lambda^2$. Using (8),

$$u_2(x, t) = \frac{25\nu^2 L^4}{\lambda^2} \operatorname{sech}^4 \frac{L(x - 4\nu L^2 t - \xi_0)}{2}, \quad \frac{\nu}{\lambda} > 0. \quad (22)$$

The condition $\nu/\lambda > 0$ ensures $V \geq 0$. The second solution, $c_0 = -10\nu L^2/(3\lambda)$ with $\kappa = -4\nu L^2$, gives a

V that changes sign and is therefore rejected. Unlike u_1 , the speed of u_2 is fixed by the width, and its amplitude scales with ν^2/λ^2 . Figure 2 shows the narrower sech⁴ pulse that results.

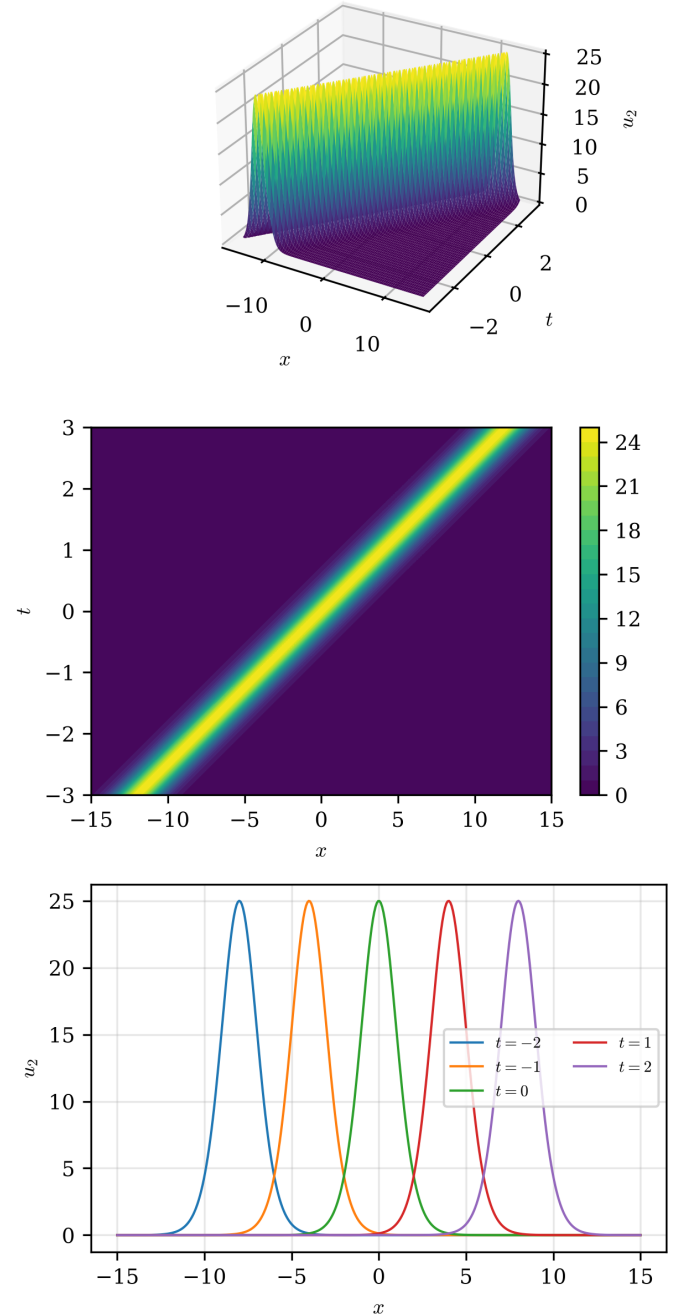


Figure 2. Solitary wave u_2 of Eq. (22) for $r = \frac{3}{2}$, $\mu = 0$, $\lambda = \nu = L = 1$ (hence $\kappa = 4$), $\xi_0 = 0$: surface, contour plot and profiles at $t = -2, \dots, 2$.

Remark 1. Lemma 1 gives a necessary degree; it does not guarantee that the algebraic system is solvable. The original choice of extending $N = 1$ to $N = 4$ for $r = \frac{3}{2}$ is justified only after the diffusion term is removed, because then $N_c = 4$ is the only balance left.

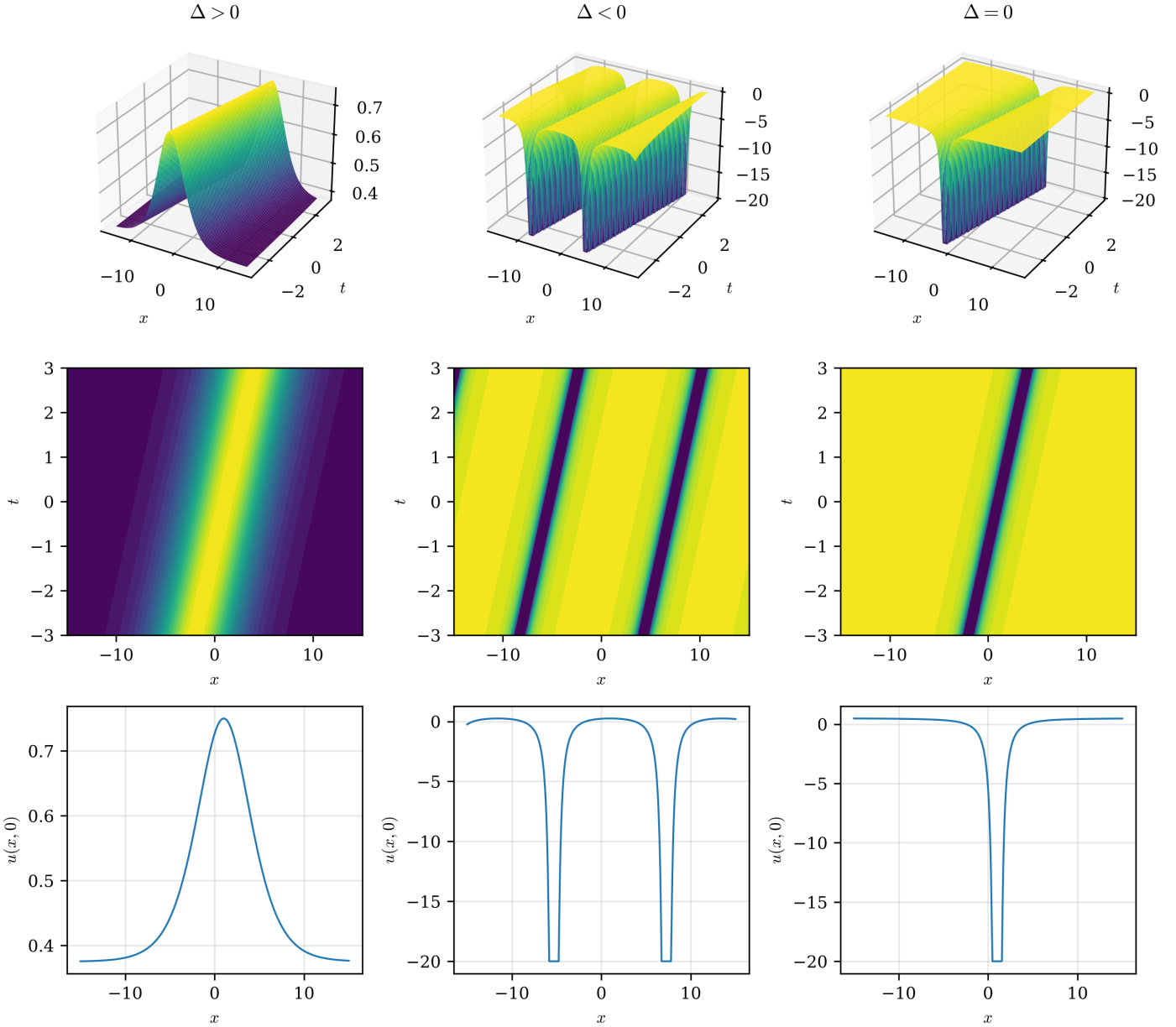


Figure 3. Generalized Kudryashov solutions for $r = 2, \mu = 0, \lambda = \nu = \kappa = 1, \xi_0 = 0$. Columns: u_3 with $(\mathcal{B}_0, \mathcal{B}_1, \mathcal{B}_2) = (-\frac{1}{16}, 0, 1), \Delta = \frac{1}{4}$; u_4 with $(\frac{1}{16}, 0, 1), \Delta = -\frac{1}{4}$; u_5 with $(0, 0, 1), \Delta = 0$. Rows: surface, contour plot, profile at $t = 0$. Values below -20 are truncated near the poles.

5 Solutions by the generalized Kudryashov method

We keep $\mu = 0$ for the reasons given above; α_B is defined in (14).

5.1 Case $r = 2$

With $N = 2$ in (10), the coefficients of powers of G vanish for

$$\begin{aligned} \mathcal{A}_2 &= -\frac{6\nu\alpha_B^2}{\lambda}, & \mathcal{A}_1 &= -\frac{6\nu\alpha_B(\mathcal{B}_1 - 2\mathcal{B}_2)}{\lambda}, \\ \mathcal{A}_0 &= \frac{\kappa - \nu(\mathcal{B}_1^2 - 12\mathcal{B}_1\mathcal{B}_2 + 8\mathcal{B}_0\mathcal{B}_2 + 12\mathcal{B}_2^2)}{2\lambda}, \end{aligned} \quad (23)$$

with κ free. Using (15), the solution simplifies to

$$U(\xi) = \frac{\kappa - \nu\Delta}{2\lambda} - \frac{6\nu\alpha_B}{\lambda}R(\xi), \quad \xi = x - \kappa t. \quad (24)$$

Inserting (11)-(13) gives three families:

$$u_3 = \frac{\kappa - \nu\Delta}{2\lambda} - \frac{6\nu\alpha_B}{\lambda}R(Z_h), \quad \Delta > 0, \quad (25)$$

$$u_4 = \frac{\kappa + \nu\Omega^2}{2\lambda} - \frac{6\nu\alpha_B}{\lambda}R(Z_p), \quad \Delta = -\Omega^2, \quad (26)$$

$$u_5 = \frac{\kappa}{2\lambda} - \frac{6\nu\alpha_B}{\lambda}R(Z_r), \quad \Delta = 0, \quad (27)$$

where $R(Z)$ denotes (15) evaluated at the indicated Z . For $\Delta > 0$ and $\alpha_B\mathcal{B}_2 > 0$, u_3 is a bounded solitary

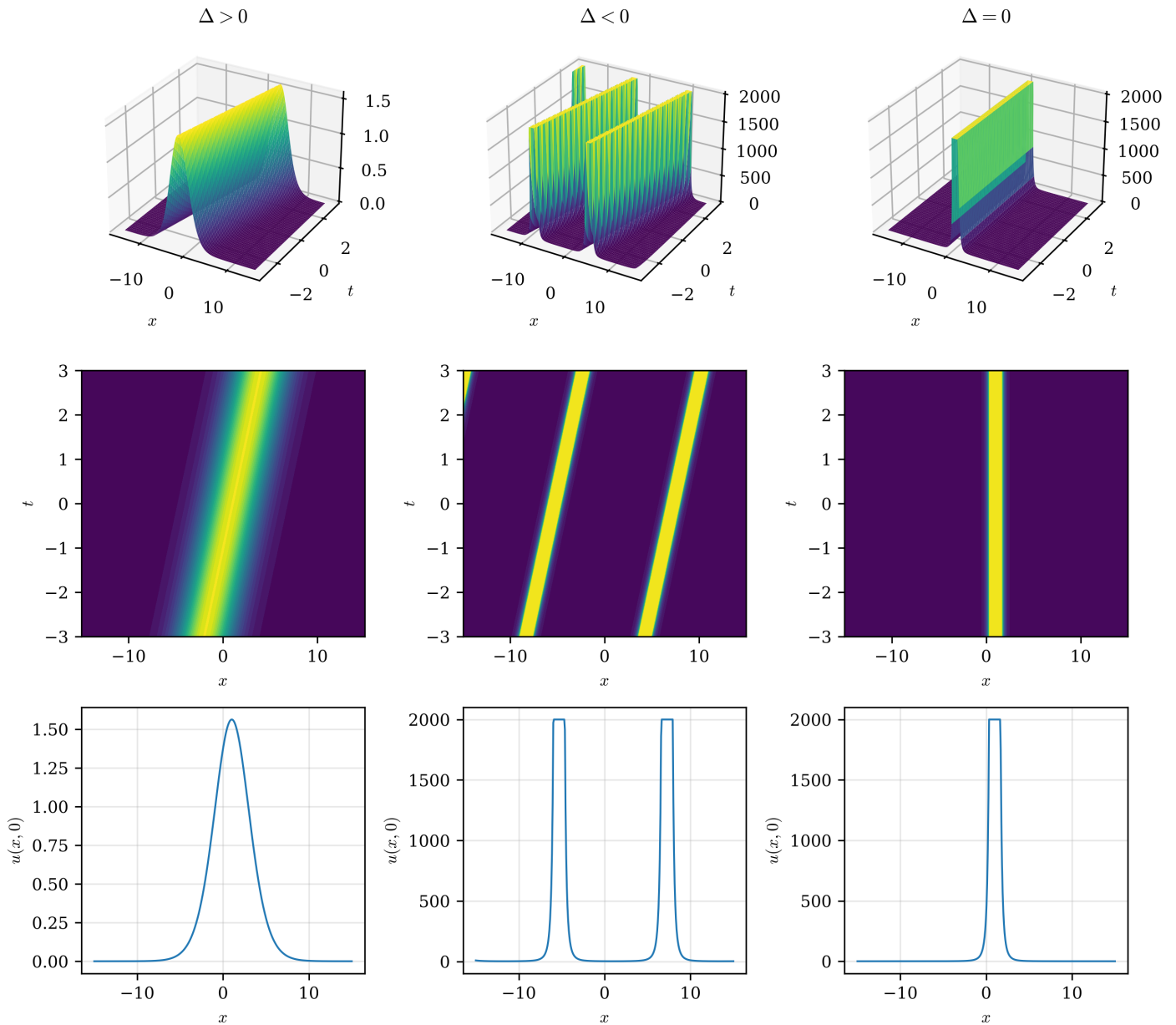


Figure 4. Generalized Kudryashov solutions for $r = \frac{3}{2}$, $\mu = 0$, $\lambda = 1$, $\xi_0 = 0$, with the same $(\mathcal{B}_0, \mathcal{B}_1, \mathcal{B}_2)$ as in Figure 3. Columns: u_6 ($\nu = 1, \kappa = 1$); u_7 ($\nu = -1, \kappa = 1$); u_8 ($\nu = -1, \kappa = 0$). Rows: surface, contour plot, profile at $t = 0$. Values above 2000 are truncated near the poles.

wave and coincides with u_1 after a shift of ξ and the identification $L^2 = \Delta$. The periodic family u_4 and the rational family u_5 are singular where $Z = -1$.

5.2 Case $r = \frac{3}{2}$

With $U = V^2$ and V quadratic in G , we obtain

$$U(\xi) = \frac{400\nu^2\alpha_B^2}{\lambda^2}R^2(\xi), \quad \kappa = 4\nu\Delta, \quad (28)$$

which is valid wherever $V = -20\nu\alpha_B R/\lambda \geq 0$. The sign of R is that of $-\mathcal{B}_2$ for Z_h and that of $\mathcal{B}_2$ for Z_p and Z_r , and $\alpha_B\mathcal{B}_2 > 0$ in the bounded hyperbolic case

and always for $\Delta \leq 0$. Hence

$$u_6 = \frac{400\nu^2\alpha_B^2}{\lambda^2}R^2(Z_h), \quad \kappa = 4\nu\Delta, \quad \frac{\nu}{\lambda} > 0, \quad (29)$$

$$u_7 = \frac{400\nu^2\alpha_B^2}{\lambda^2}R^2(Z_p), \quad \kappa = -4\nu\Omega^2, \quad \frac{\nu}{\lambda} < 0, \quad (30)$$

$$u_8 = \frac{400\nu^2\alpha_B^2}{\lambda^2}R^2(Z_r), \quad \kappa = 0, \quad \frac{\nu}{\lambda} < 0. \quad (31)$$

For $\Delta > 0$, u_6 reduces to the sech^4 pulse (22) with $L^2 = \Delta$. For $\Delta = 0$ the wave is stationary.

Figures 3 and 4 compare the three families. For $\Delta > 0$ both cases give a smooth pulse travelling along $x - \kappa t =$

const. For $\Delta < 0$ the profiles are periodic with period $2\pi/\Omega$ and blow up where $Z_p = -1$; for $\Delta = 0$ there is a single algebraic singularity. The $r = \frac{3}{2}$ solutions are squares of the $r = 2$ profile shape, which makes them narrower and non-negative.

6 Discussion

For the classical STO equation, $r = 3$ and $s = 2$ give $N = 1$ and the three terms of (4) balance simultaneously; this is the setting of the known kink and multi-kink solutions [9, 10]. In the generalized equation the same simultaneous balance requires $r = 2s - 1$. The two non-integer cases examined here show two different outcomes. For $(r, s) = (2, \frac{3}{2})$ the degrees match, but the nonconstant solution of the form $U = (c_0 + c_1 Y)^2$ with $\mu \neq 0$ changes sign, so it is valid on a half-line only. For $(r, s) = (\frac{3}{2}, 2)$ the diffusion term dominates and must vanish. In both cases the bounded solutions are those of the dispersive limit $\mu = 0$, i.e. of a KdV equation with general power nonlinearity, for which $\text{sech}^{2/(r-1)}$ pulses are expected; u_1 and u_2 are the cases $r = 2$ and $r = \frac{3}{2}$.

The degree $N = 2/(r - 1)$ grows as $r \rightarrow 1^+$, and the pulse sech^N becomes narrower relative to its width parameter. Finding nontrivial solutions with $\mu \neq 0$ and $r \neq 2s - 1$ will require ansätze that are not polynomial in a Riccati variable, for example power-type ansätze $U = V^\gamma$ with γ adapted to both exponents.

7 Conclusion

We applied the classical, modified and generalized Kudryashov methods to the GSTO equation (3) with non-integer exponents. The main findings are:

1. the convective and diffusive terms require $N = 2/(r - 1)$ and $N = 1/(s - 1)$ respectively, and a full balance needs $r = 2s - 1$;
2. for $(r, s) = (2, \frac{3}{2})$ and $(\frac{3}{2}, 2)$ bounded solutions within the polynomial ansatz exist only for $\mu = 0$; they are the sech^2 wave (20) and the sech^4 wave (22) with $\kappa = 4\nu L^2$;
3. the generalized method yields hyperbolic, singular periodic and rational families, whose type is set by Δ and whose reality depends on the sign of ν/λ .

Future work will address ansätze suited to $\mu \neq 0$ with $r \neq 2s - 1$, numerical simulation of the GSTO equation, and the stability of the solitary waves found here.

Data Availability Statement

No new data were generated. The scripts used for symbolic verification and for the figures are available from the corresponding author upon request.

Funding

This work was supported without any funding.

Conflicts of Interest

The authors declare no conflicts of interest.

AI Use Statement

The authors declare that ChatGPT-5 was used for language editing and translation of the manuscript. The authors have carefully reviewed, revised, and verified the AI-assisted output and take full responsibility for the content of the manuscript.

Ethical Approval and Consent to Participate

Not applicable.

References

- [1] Drazin, P. G., & Johnson, R. S. (1989). *Solitons: An Introduction*. Cambridge University Press. <https://physic.s.puchd.ac.in/scholars/talks/Amit-Goyal-solitons.pdf>
- [2] Ablowitz, M. J., & Clarkson, P. A. (1991). *Solitons, Nonlinear Evolution Equations and Inverse Scattering*. Cambridge University Press. <https://assets.cambridge.org/97805213/87309/sample/9780521387309ws.pdf>
- [3] Whitham, G. B. (1974). *Linear and Nonlinear Waves*. John Wiley & Sons. https://eclass.uoa.gr/modules/document/file.php/PHYS289/%CE%92%CE%B9%CE%B2%CE%BB%CE%AF%CE%B1/Whitham_Linear_and_Nonlinear_Waves.pdf
- [4] Sharma, A. S., & Tasso, H. (1977). *Connection between wave envelope and explicit solution of a nonlinear dispersive wave equation* (No. IPP-6/158). Max-Planck-Institut fuer Plasmaphysik, Garching/Muenchen (Germany, FR).
- [5] Olver, P. J. (1977). Evolution equations possessing infinitely many symmetries. *Journal of Mathematical Physics*, 18(6), 1212-1215. [CrossRef]
- [6] Wang, S., Tang, X. Y., & Lou, S. Y. (2004). Soliton fission and fusion: Burgers equation and Sharma-Tasso-Olver equation. *Chaos, Solitons & Fractals*, 21(1), 231-239. [CrossRef]
- [7] Lian, Z. J., & Lou, S. Y. (2005). Symmetries and exact solutions of the Sharma-Tasso-Olver equation. *Nonlinear Analysis: Theory, Methods & Applications*, 63(5-7), e1167-e1177. [CrossRef]
- [8] Johnpillai, A. G., & Khalique, C. M. (2014). On the solutions and conservation laws for the

- Sharma-Tasso-Olver equation. *ScienceAsia*, 40(6), 451-455. https://scienceasia.org/2014.40.n6/scias40_451.pdf
- [9] Chen, A. (2010). Multi-kink solutions and soliton fission and fusion of Sharma-Tasso-Olver equation. *Physics Letters A*, 374(23), 2340-2345. [CrossRef]
- [10] Wazwaz, A. M. (2007). New solitons and kinks solutions to the Sharma-Tasso-Olver equation. *Applied Mathematics and Computation*, 188(2), 1205-1213. [CrossRef]
- [11] Uğurlu, Y., & Kaya, D. (2007). Analytic method for solitary solutions of some partial differential equations. *Physics Letters A*, 370(3-4), 251-259. [CrossRef]
- [12] Erbaş, B., & Yusufoglu, E. (2009). Exp-function method for constructing exact solutions of Sharma-Tasso-Olver equation. *Chaos, Solitons & Fractals*, 41(5), 2326-2330. [CrossRef]
- [13] Shang, Y., Qin, J., Huang, Y., & Yuan, W. (2008). Abundant exact and explicit solitary wave and periodic wave solutions to the Sharma-Tasso-Olver equation. *Applied Mathematics and Computation*, 202(2), 532-538. [CrossRef]
- [14] Pan, J. T., & Chen, W. Z. (2009). A new auxiliary equation method and its application to the Sharma-Tasso-Olver model. *Physics Letters A*, 373(35), 3118-3121. [CrossRef]
- [15] Jawad, A. J. A. M., Petković, M. D., & Biswas, A. (2010). Modified simple equation method for nonlinear evolution equations. *Applied Mathematics and Computation*, 217(2), 869-877. [CrossRef]
- [16] Zayed, E. M. E. (2011). A note on the modified simple equation method applied to Sharma-Tasso-Olver equation. *Applied Mathematics and Computation*, 218(7), 3962-3964. [CrossRef]
- [17] He, Y., Li, S., & Long, Y. (2013). Exact solutions to the Sharma-Tasso-Olver equation by using improved G'/G -expansion method. *Journal of Applied Mathematics*, 2013(1), 247234. [CrossRef]
- [18] Salas, A. H. (2011). Exact solutions to a generalized Sharma-Tasso-Olver equation. *Applied Mathematical Sciences*, 5(46), 2289-2295. <https://www.m-hikari.com/ams/ams-2011/ams-45-48-2011/salasAMS45-48-2011-2.pdf>
- [19] Sheikh, M. A. N., Taher, M. A., Hossain, M. M., & Akter, S. (2023). Variable coefficient exact solution of Sharma-Tasso-Olver model by enhanced modified simple equation method. *Partial Differential Equations in Applied Mathematics*, 7, 100527. [CrossRef]
- [20] Pleumpreedaporn, C., Moore, E. J., Sirisubtawee, S., Khansai, N., & Pleumpreedaporn, S. (2024). Exact solutions for the Sharma-Tasso-Olver equation via the Sardar subequation method with a comparison between Atangana space-time beta-derivatives and classical derivatives. *Mathematics*, 12(14), 2155. [CrossRef]
- [21] Abdou, M. A., Ouahid, L., Owyed, S., Abdel-Baset, A. M., Akinlar, M. A., & Chu, Y.-M. (2020). Explicit solutions to the Sharma-Tasso-Olver equation. *AIMS Mathematics*, 5(6), 7272. [CrossRef]
- [22] Butt, A. R., Zaka, J., Akgül, A., & El Din, S. M. (2023). New structures for exact solution of nonlinear fractional Sharma-Tasso-Olver equation by conformable fractional derivative. *Results in Physics*, 50, 106541. [CrossRef]
- [23] Feng, Y., Dai, H., & Wei, X. (2023). Numerical solutions to the Sharma-Tasso-Olver equation using Lattice Boltzmann method. *International Journal for Numerical Methods in Fluids*, 95(9), 1546-1564. [CrossRef]
- [24] Zebari, L., Pirdawood, M. A., & Sadeeq, M. I. (2024). Efficient Numerical Solution of the Sharma-Tasso-Olver Equation Using Radial Basis Function-Pseudo Spectral Method with Comparative Analysis. *Wasit Journal for Pure sciences*, 3(2), 1-7. [CrossRef]
- [25] Zhou, Y., & Zhuang, J. (2022). Dynamics and exact traveling wave solutions of the Sharma-Tasso-Olver-Burgers equation. *Symmetry*, 14(7), 1468. [CrossRef]
- [26] Li, J. (2019). Geometric properties and exact travelling wave solutions for the generalized Burger-Fisher equation and the Sharma-Tasso-Olver equation. *Journal of Nonlinear Modeling and Analysis*, 1(1), 1-10. [CrossRef]
- [27] Kudryashov, N. A. (1988). Exact soliton solutions of the generalized evolution equation of wave dynamics. *Journal of applied mathematics and mechanics*, 52(3), 361-365. https://primat.mephi.ru/~kudryashov/articles/eng/88d_PMM.pdf
- [28] Kudryashov, N. A. (2005). Simplest equation method to look for exact solutions of nonlinear differential equations. *Chaos, Solitons & Fractals*, 24(5), 1217-1231. [CrossRef]
- [29] Kudryashov, N. A. (2012). One method for finding exact solutions of nonlinear differential equations. *Communications in Nonlinear Science and Numerical Simulation*, 17(6), 2248-2253. [CrossRef]
- [30] Kaplan, M., Bekir, A., & Akbulut, A. (2016). A generalized Kudryashov method to some nonlinear evolution equations in mathematical physics. *Nonlinear Dynamics*, 85(4), 2843-2850. [CrossRef]
- [31] Ryabov, P. N., Sinelshchikov, D. I., & Kochanov, M. B. (2011). Application of the Kudryashov method for finding exact solutions of the high order nonlinear evolution equations. *Applied Mathematics and Computation*, 218(7), 3965-3972. [CrossRef]
- [32] Sirisubtawee, S., Koonprasert, S., & Sungnol, S. (2020). New exact solutions of the conformable space-time Sharma-Tasso-Olver equation using two reliable methods. *Symmetry*, 12(4), 644. [CrossRef]
- [33] Malfliet, W. (1992). Solitary wave solutions of nonlinear wave equations. *American Journal of Physics*, 60(7), 650-654. [CrossRef]



Sarita Pippal is an Assistant Professor in the Department of Mathematics, Panjab University, Chandigarh, India, a position she has held since 2014; before that she taught at Ramjas College, University of Delhi. Her research interests include heat and mass transfer, fluid dynamics, fractional calculus and the mathematical modelling of natural and mixed convection flows. She has published in international journals, presented at national and international conferences, given invited talks and supervised doctoral research. (Email: saritamath@pu.ac.in)



Vikramjeet is a Ph.D. candidate in the Department of Computer Science and Applications, Panjab University, Chandigarh, India. Her research interests are natural language processing and machine learning, with an interdisciplinary focus on linguistics, cognitive science and artificial intelligence. She works on data visualization (Matplotlib) and programs in C, C++ and Java.