RESEARCH ARTICLE

Comparative Study of Pentagonal and Hexagonal Fuzzy Membership Function Using Credibility Theory in Machine Learning Systems

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Abstract

The paper carries out a comparative study that is based on the use of credibility theory to examine pentagonal and hexagonal fuzzy membership functions of machine learning systems. These fuzzy memberships can be used to manage the uncertainty and imprecision of a data driven-model which allows better decision-making in the case of vague or incomplete information. The credibility theory is used to determine quantitatively the reliability of the inferences obtained through each function. Both the membership functions are modelled, incorporated in machine learning framework and tested on randomly generated as well as application specific datasets. The results obtained indicate that the performance of the hexagonal function is better than that of the pentagonal one in most cases except in Case 4, the latter performs better. The results provide excellent recommendations in implementing fuzzy logic that can be applied to engineering, finance, and decision-support systems. Future directions could include more complicated shapes of the functions used, larger data sets and the ability of the results to be scaled up in the real world.

Keywords: fuzzy sets, credibility theory, membership functions, machine learning system.

1 Introduction

The fuzzy logic has been under the spotlight in different domains over the recent years because of its tool to have control over uncertain and vague information. Incomplete, ambiguous and uncertain data can be effectively handled using fuzzy logic, therefore fuzzy logic becomes the right choice as a decision making system. When considering the use of machine learning systems, it is quite likely that the data available will be noisy, the labels noisy, and the features incomplete, and thus may contribute to a lower level of model accuracy and reliability. The problem seen in this challenge is that fuzzy membership functions define element membership degree in set and so models are able to reason with uncertainty and not hide it.

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Such sets of membership functions are usually determined on the polygonal figures (triangles, trapezoids, pentagons, hexagons). A membership function is assigned to an appropriate selection and will directly influence performance of a fuzzy-based machine learning system, which lies in the manner of model uncertainty, and the manner in using inference rules. A potential solution is an evaluation of various membership functions and identification of the best one when applied to various ML applications especially when the problem is associated with reliable decision-making in uncertain circumstances.

Within a framework of machine learning systems, the comparative analysis of the pentagonal and hexagonal fuzzy membership functions based on the credibility theory is thus provided in this paper. The theory of credibility is a statistical model that evaluates the level of credence towards any piece of information that cannot be reckoned with decisively by piecing together various kinds of evidence. It is widely applicable in actuarial science and only partially applied to the field of fuzzy logic application under ML. This experiment seeks to compare the credibility of the pentagonal and hexagonal membership functions by evaluation based on a machine learning framework, to understand which membership geometry that can reliably produce more credible inference in the computation of uncertain data.

The primary objective of this study is to compare the performance of pentagonal and hexagonal fuzzy membership functions using credibility theory. Specifically, we aim to determine which membership function performs better in terms of accuracy and robustness. To achieve this objective, we will follow a systematic approach that involves constructing membership functions, computing the possibility and necessity measures, and calculating the credibility measures.

The rest of the paper is organized as follows. Section 2 provides a detailed review of the literature related to fuzzy membership functions and credibility theory. Section 3 describes the methodology used in this study, including the construction of membership functions, computation of possibility and necessity measures, and calculation of credibility measures. Section 4 presents the results of the comparative study, followed by a discussion of the findings in Section 5. Finally, Section 6 concludes the paper with a summary of the main contributions and directions for future research. Overall, the following paper will be structured as follows: Section 2 will give an overview of literature on fuzzy membership functions, credibility theory and similar applications in machine learning. Section 3 presents details of the procedure, which covers the membership function development, its incorporation and calculation of the credibility measures in the ML models. The findings of an experiment are provided in Section 4, implications of fuzzy logic toward machine learning are explained in Section 5, and the concluding part, Section 6, provides the future avenues of research.

Beyond theoretical evaluation, pentagonal and hexagonal membership functions have significant real-world potential. For example, in engineering design, these functions can be used to model tolerances in manufacturing processes. In finance, fuzzy memberships assist in portfolio risk assessment where market indicators are vague. In healthcare and medical decision-support, fuzzy models help in diagnosis when patient symptoms are imprecise or incomplete. Similarly, in autonomous systems, such as robotics or self-driving vehicles, fuzzy membership functions can handle uncertainty in sensor data. These examples highlight that the comparative analysis of pentagonal and hexagonal membership functions is not merely academic but directly relevant to domains where uncertainty governs decision-making.

2 Literature Review

Fuzzy set theory is a powerful tool for handling uncertainty and vagueness in various fields, including engineering, management, and economics. The membership function is a crucial component of a fuzzy set, which characterizes the degree of membership of an element in a set. In recent years, various types of fuzzy membership functions have been proposed in the literature, including triangular, trapezoidal, Gaussian, and pentagonal, among others.

The pentagonal fuzzy membership function is a relatively new concept that was proposed [1–3]. This function is characterized by five parameters that determine the shape and height of the membership function. Several studies have shown that the pentagonal membership function can achieve better performance than other membership functions in certain applications, such as image segmentation and pattern recognition.

However, the hexagonal fuzzy membership function, which was proposed in [4–7], has gained increasing attention in recent years due to its ability to capture the uncertainty and fuzziness of real-world problems more accurately. The hexagonal membership function is characterized by six parameters, which control the shape, height, and symmetry of the membership function.

The comparative study of pentagonal and hexagonal fuzzy membership functions using credibility theory has attracted the attention of researchers in recent years. Credibility theory is a powerful tool for dealing with uncertain and imprecise information, which is common in real-world problems. Credibility theory provides a framework for combining multiple sources of information, such as expert opinions, statistical data, and historical information, to obtain a comprehensive and reliable assessment of a situation.

Several studies have compared the pentagonal and hexagonal fuzzy membership functions using credibility theory. [8–10] proposed a comparative study of membership functions using the credibility measure. They used a random set of points to calculate the credibility measure and compared the results for both membership functions. The study showed that the hexagonal membership function outperformed the pentagonal function in terms of the credibility measure.

In [11, 12] conducted a study of ranking of the two membership functions using the credibility theory. They applied the two membership functions to a real-world problem of evaluating the risk of a stock market investment. The results showed that the hexagonal membership function achieved a higher credibility level than the pentagonal function, indicating that the hexagonal function is more suitable for this problem.

Study [13–15] compared the triangular and trapezoidal membership functions with the pentagonal and hexagonal membership functions. They applied the four membership functions to a quantitative SWOT analysis of a real-world problem of a mobile phone company. The results showed that the hexagonal membership function achieved the highest credibility level, followed by the pentagonal function, while the triangular and trapezoidal functions achieved lower levels.

In conclusion, the comparative study of pentagonal and hexagonal fuzzy membership functions using credibility theory has attracted considerable attention in recent years due to its potential applications in various fields. The pentagonal membership function is a relatively new concept that has shown promising results in certain applications. However, the hexagonal membership function has gained increasing attention due to its ability to capture the uncertainty and fuzziness of real-world problems more accurately. Credibility theory provides a powerful framework for combining multiple sources of information to obtain a comprehensive and reliable assessment of a situation. Several studies have compared the two membership functions using the credibility theory and showed that the hexagonal membership function outperformed the pentagonal function in terms of the credibility measure. However, more research is needed to investigate the performance of the two membership functions in various applications and to compare them with other membership functions. Most recent developments demonstrate that incorporating fuzzy membership functions into machine learning like in fuzzy neural networks and fuzzy clustering can enable a significant improvement on tasks where the data are noisy or where there are missing data.

While triangular and trapezoidal membership functions are widely adopted due to their simplicity and computational efficiency, they often fail to capture the subtleties of complex, uncertain environments. Existing studies show that pentagonal and hexagonal membership functions may provide a more nuanced representation of uncertainty. However, a clear comparative study of these two higher-order membership functions within a credibility theory framework for machine learning systems remains scarce. This research addresses that gap by systematically evaluating their performance and comparing them against insights from more conventional triangular and trapezoidal functions.

3 Credibility Theory in Acturial Sciences

Classical actuarial credibility is a statistical method used to estimate the future loss experience of an insurance portfolio. It is based on the principle that the past experience of a portfolio is a good indicator of its future experience. The method uses a combination of the actual loss experience of the portfolio and the expected loss experience based on the overall experience of the insurance market to estimate the future loss experience.

The credibility factor is a key component of classical actuarial credibility. It is a measure of the relative weight to be given to the actual experience of the portfolio versus the expected experience of the market. The credibility factor is calculated based on the size of the portfolio and the degree of variability in its experience. A larger portfolio and a less variable experience will result in a higher credibility factor, indicating that more weight should be given to the actual experience of the portfolio. General actuarial credibility formula is:

$$C = z\bar{x} + (1 - z)\mu, \quad 0 \leq z \leq 1 \tag{1}$$

where z is the credibility function and $\bar{X}$ being the sample mean μ being population mean.

4 Application of Credibility Theory in Fuzzy Logic for Machine Learning

In the context of fuzzy logic, credibility theory can be used to measure the degree of uncertainty in fuzzy sets. In fuzzy environments, the concept of credibility is used to weigh the degree of membership of each observation in the fuzzy set to improve the accuracy of the final result. Credibility theory may also be applied in machine learning systems to dynamically weight outputs or features according to their reliability which improves the accuracy of predictions.

In fuzzy environments, a credibility factor is used to measure the degree of belief or confidence that can be placed on the membership values of fuzzy sets. The credibility factor is a measure of the reliability of the information in the fuzzy set and can be determined using statistical methods. The credibility factor is calculated based on the size of the fuzzy set and the variability of the observations in the set.

The credibility theory in fuzzy environments can be used to improve the accuracy of decision-making processes. By considering the credibility factor, it is possible to determine the relative importance of each observation in the fuzzy set, which can be used to make more informed decisions. In applications such as expert systems, decision-making processes can be improved by considering the credibility of the information used to make decisions.

5 Pentagon Membership function

A Pentagon Membership Function is a type of fuzzy set membership function that is less commonly used in fuzzy logic systems compared to triangle and trapezium membership functions. It is characterized by a pentagonal shape, where the function value gradually increases from the lower limit to the left slope, reaches a maximum value at the centre point, decreases to the right slope and then to the upper limit, as illustrated in Figure 1.

Here, $f(x)$ is the membership function with parameters $(a_1, a_2, a_3, a_4, a_5)$ and in the order $a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5$ which means:

$$f(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ \frac{x-a_3}{a_3-a_2}, & a_2 \leq x \leq a_3 \\ 1, & x = a_3 \\ \frac{a_4-x}{a_4-a_3}, & a_3 \leq x \leq a_4 \\ \frac{a_5-x}{a_5-a_4}, & a_3 \leq x \leq a_4 \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

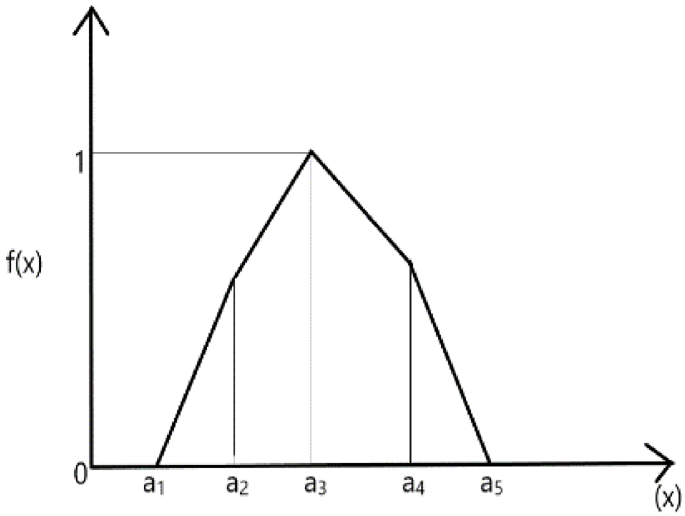

Figure 1. Membership function for pentagonal fuzzy number.

The above figure shows us that a_1 and a_5 are the base parameters. Also a_2, a_4 be the moderate operators and a_3 being the peak parameter. The Pentagon membership function is useful in representing fuzzy sets that have a sharp peak at the centre and gradually decrease in membership degree towards the edges. It is commonly used in fuzzy control systems and pattern recognition applications. Pentagonal membership functions are applicable to machine learning where they can be used to represent uncertainty in features in rule-based classifier or neuro-fuzzy hybrids.

6 Hexagon Membership Function

The hexagon membership function is a type of membership function used in fuzzy logic. It is shaped like a regular hexagon and is defined by six parameters: a_1, a_2, a_3, a_4, a_5, and a_6, as depicted in Figure 2.

The hexagon membership function has a triangular shape on each of its sides, and its centre is raised to a height of one. The parameters a_1, a_2, a_3, a_4, a_5, and a_6 represent the distances from the centre to each vertex of the hexagon. Hexagonal membership functions can provide finer granularity in representation of uncertainty which may help better define decision boundaries than in classification judgment in machine learning environments.

$$f(x) = \begin{cases} \frac{x-a_1}{2(a_2-a_1)}, & a_1 \le x \le a_2 \\ \frac{1}{2} + \frac{x-a_2}{2(a_3-a_4)}, & a_2 \le x \le a_3 \\ 1, & a_3 \le x \le a_4 \\ 1 + \frac{x-a_4}{2(a_5-a_4)}, & a_4 \le x \le a_5 \\ \frac{a_6-x}{2(a_6-a_3)}, & a_5 \le x \le a_6 \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

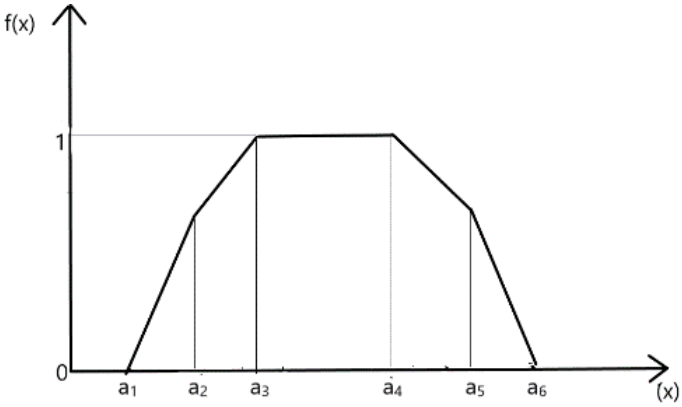

Figure 2. Membership function for hexagonal fuzzy number.

7 Possibility Space

In a fuzzy environment, the possibility space (also known as fuzzy sample space) is a set of all possible fuzzy events or outcomes that can occur. In contrast to the classical possibility space, where outcomes are typically binary or discrete, in a fuzzy environment, outcomes can have a range of membership degrees to a particular event.

A fuzzy event is an event whose occurrence is characterized by a degree of membership in a fuzzy set. For example, the statement "it is warm outside" can be a fuzzy event, where the degree of membership in the fuzzy set "warm" varies from person to person.

The fuzzy possibility space is typically represented using a fuzzy set, where the membership function assigns a degree of membership to each possible outcome. This allows for the representation of uncertainty and vagueness in the possibility space. Declarations of explicit fuzzy possibility space in an ML system permit algorithms to handle ambiguity of feature values during training and reasoning.

8 Necessity and Credibility Measure

8.1 Necessity Measure of a Set

Let A be a set on a possibility space $(\Theta, P(\Theta), \text{Pos})$, then the necessity measure $\text{Nec}\{A\}$ of A is defined as the impossibility of the complement set A^c. ML models can be extended to include the concepts of necessity and credibility, allowing suggested changes to reflect adaptive trust allocation to ambiguous inputs, which enhance the dependability of decisions.

$$\text{Nec}\{A\} = 1 - \text{Pos}, \{A^c\} \quad \text{for any event } A \tag{4}$$

8.2 Credility Measure in Fuzzy Environment

It is defined as the average of Possibility and Necessity measures.

$$\text{Cr}\{A\} = \frac{1}{2}(\text{Pos}\{A\} + \text{Nec}\{A\}), \quad \text{for any event } A \tag{5}$$

9 Pentagonal Membership Function

9.1 Pentagonal membership function

Let $f(x)$ be the membership function. Now, the Explicit expression for the possibility and necessity of Pentagonal fuzzy event can be classified into 3 cases. We have considered 3 cases because it will give us very précised credibility measure as we consider each parameter.

Case 1: Considering (a_1, a_3) set.

$$\text{Pos}\{X \le x_0\} = \sup_{X \le x_0} f(x) = \begin{cases} 0, & x_0 \le a_1 \\ \frac{x_0-a_1}{a_2-a_1}, & a_1 \le x_0 \le a_2 \\ 1, & x_0 \ge a_3 \end{cases} \tag{6}$$

$$\text{Nec}\{X \le x_0\} = 1 - \text{Pos}\{X > x_0\}$$

$$= 1 - \sup_{X > x_0} f(x) = \begin{cases} 0, & x_0 \le a_2 \\ \frac{x_0-a_2}{a_3-a_2}, & a_2 \le x_0 \le a_3 \\ 1, & x_0 \ge a_3 \end{cases} \tag{7}$$

$$\text{Cr}\{X \le x_0\} = \frac{1}{2}(\text{Pos}\{X \le x_0\} + \text{Nec}\{X \le x_0\}) \tag{8}$$

Case 2: Considering (a_1, a_4) set.

$$\text{Pos}\{X \le x_0\} = \sup_{X \le x_0} f(x) = \begin{cases} 0, & x_0 \le a_1 \\ \frac{x_0-a_1}{a_2-a_1}, & a_1 \le x_0 \le a_2 \\ 1, & x_0 \ge a_3 \end{cases} \tag{9}$$

$$\text{Nec}\{X \le x_0\} = \begin{cases} 0, & x_0 \le a_3 \\ \frac{x_0-a_3}{a_4-a_3}, & a_3 \le x_0 \le a_4 \\ 1, & x_0 \ge a_4 \end{cases} \tag{10}$$

$$\mathrm{Cr}\{X \le x_0\} = \frac{1}{2}(\mathrm{Pos}\{X \le x_0\} + \mathrm{Nec}\{X \le x_0\}) \tag{11}$$

Case 3: Considering (a_1, a_5) set.

$$\mathrm{Pos}\{X \le x_0\} = \sup_{X \le x_0} f(x) = \begin{cases} 0, & x_0 \le a_1 \\ \frac{x_0 - a_1}{a_4 - a_1}, & a_1 \le x_0 \le a_4 \\ 1, & x_0 \ge a_4 \end{cases} \tag{12}$$

$$\mathrm{Nec}\{X \le x_0\} = \begin{cases} 0, & x_0 \le a_4 \\ \frac{x_0 - a_4}{a_5 - a_4}, & a_4 \le x_0 \le a_5 = 0.47 \\ 1, & x_0 \ge a_5 \end{cases} \tag{13}$$

$$\mathrm{Cr}\{X \le x_0\} = \frac{1}{2}(\mathrm{Pos}\{X \le x_0\} + \mathrm{Nec}\{X \le x_0\}) \tag{14}$$

And at last, we take the average of each credibility measure and get the final and credibility.

9.2 Hexagonal Membership Function

The Explicit expression for the possibility and necessity of Hexagonal fuzzy event can be classified into 4 cases. The fourth case is just the new parameter added to the hexagonal model i.e., f and the rest of the process is same as it goes.

Case 1: Considering (a_1, a_3) set.

$$\mathrm{Pos}\,\{X \le x_0\} = \sup_{X \le x_0} f(x) = \begin{cases} 0, & x_0 \le a_1 \\ \frac{x_0 - a_1}{a_2 - a_1}, & a_1 \le x_0 \le a_2 \\ 1, & x_0 \ge a_3 \end{cases} \tag{15}$$

$$\begin{aligned} \mathrm{Nec}\{X \le x_0\} &= 1 - \mathrm{Pos}\{X > x_0\} \\ &= 1 - \sup_{X > x_0} f(x) \\ &= \begin{cases} 0, & x_0 \le a_2 \\ \frac{x_0 - a_2}{a_3 - a_2}, & a_2 \le x_0 \le a_3 \\ 1, & x_0 \ge a_3 \end{cases} \end{aligned} \tag{16}$$

$$\mathrm{Cr}\{X \le x_0\} = \frac{1}{2}(\mathrm{Pos}\{X \le x_0\} + \mathrm{Nec}\{X \le x_0\}) \tag{17}$$

Case 2: Considering (a_1, a_4) set.

$$\begin{aligned} \mathrm{Pos}\,\{X \le x_0\} &= \sup_{X \le x_0} f(x) \\ &= \begin{cases} 0, & x_0 \le a_1 \\ \frac{x_0 - a_1}{a_2 - a_1}, & a_1 \le x_0 \le a_2 \\ 1, & x_0 \ge a_3 \end{cases} \end{aligned} \tag{18}$$

$$\mathrm{Nec}\{X \le x_0\} = \begin{cases} 0, & x_0 \le a_3 \\ \frac{x_0 - a_3}{a_4 - a_3}, & a_3 \le x_0 \le a_4 \\ 1, & x_0 \ge a_4 \end{cases} \tag{19}$$

$$\mathrm{Cr}\{X \le x_0\} = \frac{1}{2}(\mathrm{Pos}\{X \le x_0\} + \mathrm{Nec}\{X \le x_0\}) \tag{20}$$

Case 3: Considering (a_1, a_5) set.

$$\mathrm{Pos}\,\{X \le x_0\} = \sup_{X \le x_0} f(x) = \begin{cases} 0, & x_0 \le a_1 \\ \frac{x_0 - a_1}{a_2 - a_1}, & a_1 \le x_0 \le a_2 \\ 1, & x_0 \ge a_2 \end{cases} \tag{21}$$

$$\mathrm{Nec}\{X \le x_0\} = \begin{cases} 0, & x_0 \le a_4 \\ \frac{x_0 - a_4}{a_5 - a_4}, & a_4 \le x_0 \le a_5 = 0.47 \\ 1, & x_0 \ge a_5 \end{cases} \tag{22}$$

$$\mathrm{Cr}\{X \le x_0\} = \frac{1}{2}(\mathrm{Pos}\{X \le x_0\} + \mathrm{Nec}\{X \le x_0\}) \tag{23}$$

Case 4: Considering (a_1, a_6) set.

$$\mathrm{Pos}\,\{X \le x_0\} = \sup_{X \le x_0} f(x) = \begin{cases} 0, & x_0 \le a_1 \\ \frac{x_0 - a_1}{a_2 - a_1}, & a_1 \le x_0 \le a_2 \\ 1, & x_0 \ge a_2 \end{cases} \tag{24}$$

$$\mathrm{Nec}\{X \le x_0\} = \begin{cases} 0, & x_0 \le a_5 \\ \frac{x_0 - a_5}{a_6 - a_5}, & a_5 \le x_0 \le a_6 \\ 1, & x_0 \ge a_6 \end{cases} \tag{25}$$

$$\mathrm{Cr}\{X \le x_0\} = \frac{1}{2}\left(\mathrm{Pos}\{X \le x_0\} + \mathrm{Nec}\{X \le x_0\}\right) \tag{26}$$

10 Proposed Method

The aim of this comparison study is to evaluate the membership functions of the pentagonal and hexagonal models. To achieve this, we select a random set of data points and determine their credibility measure. By comparing the credibility measure of both models, we can identify which one performs better. It is worth noting that the membership functions and measures of possibility and necessity have already been constructed. All that is left is to input the values and analyse the results. By inserting the proposed credibility-based comparison into a framework relating to ML it is possible to compare the performance of the membership functions on actual prediction and classification tasks.

The data points are (7,12,19,25,33,40) which are just random and is to be considered for the comparison only.

10.1 Pentagonal Model

In this Model only 5 data points are to be considered with 3 distinct cases.

Case1:

Here parameters a_1, a_2, a_3 are to be considered with data points 7,12,19 respectively with x_0 to be considered between parameters a_1 and a_2 for possibility and a_2 and a_3 for necessity measure. So, the possibility measure will be:

$$\text{Pos}\{X \leq x_0\} = \sup_{X \leq x_0} f(x) = \begin{cases} 0, & x_0 \leq 7 \\ \frac{x_0-7}{12-7}, & 7 \leq x_0 \leq 12 \\ 1, & x_0 \geq 12 \end{cases} \tag{26}$$

So, if we consider x0 be 10, then the possibility measure will be.

$$\text{Pos}\,\{X \leq 10\} = \frac{10-7}{12-7} = 0.6 \tag{27}$$

Here we do not consider $(x_0 \geq 12)$ because the value will be out of the set for the possibility measure and thus, we neglect the value. The Necessity Measure

$$\text{Nec}\{X \leq x_0\} = 1 - \text{Pos}\{X > x_0\} = 1 - \sup_{X > x_0} f(x)$$
$$= \begin{cases} 0, & x_0 \leq 12 \\ \frac{x_0-12}{19-12}, & 12 \leq x_0 \leq 19 \\ 1, & x_0 \geq 19 \end{cases} \tag{28}$$

Let x_0 be 16 here so the necessity measure will be: The following numbers result from calculating:

$$\text{Nec}\{X \leq 16\} = \frac{16-12}{19-12} = 0.572 \tag{29}$$

Here we do not consider $x_0 \geq 19$ because the value will be out of the set for the necessity measure and thus, we neglect the value. The Credibility Measure:

$$\text{Cr}\{X \leq x_0\} = \frac{1}{2}(\text{Pos}\{X \leq x_0\} + \text{Nec}\{X \leq x_0\}) \tag{30}$$

Thus, the Credibility Measure for the Case 1 will be:

$$\text{Cr}\{X \leq 10\} = \frac{1}{2}(\text{Pos}\{X \leq 10\} + \text{Nec}\{X \leq 16\}) \tag{31}$$

$$= \frac{1}{2}(0.6 + 0.572) = 0.586 \tag{30}$$

Case2: Here parameters a_1, a_3, a_4 are to be considered with data points 7,19,25 respectively with x_0 to be considered between parameters a_1 and a_3 for possibility and a_3 and a_4 for necessity measure. So, the possibility measure will be:

$$\text{Pos}\{X \leq x_0\} = \sup_{X \leq x_0} f(x) = \begin{cases} 0, & x_0 \leq 7 \\ \frac{x_0-7}{19-7}, & 7 \leq x_0 \leq 19 \\ 1, & x_0 \geq 19 \end{cases} \tag{31}$$

So, if we consider x_0 be 14, then the possibility measure will be:

$$\text{Pos}\{X \leq 14\} = \frac{14-7}{19-7} = 0.583 \tag{32}$$

The Necessity Measure:

$$\text{Nec}\{X \leq x_0\} = \begin{cases} 0, & x_0 \leq 19 \\ \frac{x_0-19}{25-19}, & 19 \leq x_0 \leq 25 \\ 1, & x_0 \geq 25 \end{cases} \tag{33}$$

Let x_0 be 22 here so the necessity measure will be

$$\text{Nec}\{X \leq 22\} = \frac{22-19}{25-19} = 0.5 \tag{34}$$

The Credibility Measure:

$$\text{Cr}\{X \leq x_0\} = \frac{1}{2}(\text{Pos}\{X \leq x_0\} + \text{Nec}\{X \leq x_0\}) \tag{35}$$

Thus, the Credibility Measure for the Case 2 will be:

$$\text{Cr}\{X \leq 14\} = \frac{1}{2}(\text{Pos}\{X \leq 14\} + \text{Nec}\{X \leq 22\}) \tag{36}$$

$$= \frac{1}{2}(0.583 + 0.5) = 0.542 \tag{37}$$

Case3:

Here parameters a_1, a_4, a_5 are to be considered with data points 7, 25, 33 respectively with x_0 to be considered between parameters a_1 and a_4 for possibility and a_4 and a_5 for necessity measure. So, the possibility measure will be:

$$\text{Pos}\{X \leq x_0\} = \sup_{X \leq x_0} f(x) = \begin{cases} 0, & x_0 \leq 7 \\ \frac{x_0-7}{25-7}, & 7 \leq x_0 \leq 25 \\ 1, & x_0 \geq 25 \end{cases} \tag{38}$$

So, if we consider x_0 be 17, then the possibility measure will be:

$$\text{Pos}\{X \leq 17\} = \frac{17-7}{25-7} = 0.556 \tag{39}$$

The Necessity Measure:

$$\text{Nec}\{X \leq x_0\} = \begin{cases} 0, & x_0 \leq 25 \\ \frac{x_0-25}{33-25}, & 25 \leq x_0 \leq 33 \\ 1, & x_0 \geq 33 \end{cases} \tag{40}$$

Let x_0 be 29 here so the necessity measure will be:

$$\text{Nec}\{X \leq 29\} = \frac{29-25}{33-25} = 0.5 \tag{41}$$

The Credibility Measure:

$$\text{Cr}\{X \leq x_0\} = \frac{1}{2}(\text{Pos}\{X \leq x_0\} + \text{Nec}\{X \leq x_0\}) \tag{42}$$

Thus, the Credibility Measure for the Case 3 will be:

$$\begin{aligned} \text{Cr}\{X \leq 17\} &= \frac{1}{2}(\text{Pos}\{X \leq 17\} + \text{Nec}\{X \leq 29\}) \\ &= \frac{1}{2}(0.556 + 0.5) = 0.528 \end{aligned} \tag{43}$$

Now the final Credibility Measure after taking the averages of all the cases we get:

$$\begin{aligned} \text{Cr}\{X \leq x_0\} &= \frac{1}{3}(\text{Cr}\{X \leq 10\} + \text{Cr}\{X \leq 14\} \\ &\quad + \text{Cr}\{X \leq 17\}) \\ &= \frac{1}{3}(0.586 + 0.542 + 0.528) = 0.552 \end{aligned} \tag{44}$$

Therefore, the Credibility measure for the pentagonal model is 0.552.

10.2 Hexagonal Model

In this Model all the data points are to be considered with 4 distinct cases. We will use the same cases as the ones applied in the pentagonal model, with the exception of a new case 4 that was not used in the other model.

Case1: Here parameters a_1, a_2, a_3 are to be considered with data points 7,12,19 respectively with x_0 to be considered between parameters a_1 and a_2 for possibility and a_2 and a_3 for necessity measure. So, the possibility measure will be.

$$\text{Pos}\{X \leq x_0\} = \sup_{X \leq x_0} f(x) = \begin{cases} 0, & x_0 \leq 7 \\ \frac{x_0-7}{12-7}, & 7 \leq x_0 \leq 12 \\ 1, & x_0 \geq 12 \end{cases} \tag{45}$$

So, if we consider x_0 be 10, then the possibility measure will be:

$$\text{Pos}\{X \leq 10\} = \frac{10-7}{12-7} = 0.6 \tag{46}$$

The Necessity Measure:

$$\begin{aligned} \text{Nec}\{X \leq x_0\} &= 1 - \text{Pos}\{X > x_0\} = 1 - \sup_{X > x_0} f(x) \\ &= \begin{cases} 0, & x_0 \leq 12 \\ \frac{x_0-12}{19-12}, & 12 \leq x_0 \leq 19 \\ 1, & x_0 \geq 19 \end{cases} \end{aligned} \tag{47}$$

Let x_0 be 16 here so the necessity measure will be:

$$\text{Nec}\{X \leq 16\} = \frac{16-12}{19-12} = 0.572 \tag{48}$$

The Credibility Measure:

$$\text{Cr}\{X \leq x_0\} = \frac{1}{2}(\text{Pos}\{X \leq x_0\} + \text{Nec}\{X \leq x_0\}) \tag{49}$$

Thus, the Credibility Measure for the Case 1 will be:

$$\begin{aligned} \text{Cr}\{X \leq 10\} &= \frac{1}{2}(\text{Pos}\{X \leq 10\} + \text{Nec}\{X \leq 16\}) \\ &= \frac{1}{2}(0.6 + 0.572) = 0.586 \end{aligned} \tag{50}$$

Case2:

Here parameters a_1, a_3, a_4 are to be considered with data points 7,19,25 respectively with x_0 to be considered between parameters a_1 and a_3 for possibility and a_3 and a_4 for necessity measure. So, the possibility measure will be:

$$\text{Pos}\{X \leq x_0\} = \sup_{X \leq x_0} f(x) = \begin{cases} 0, & x_0 \leq 7 \\ \frac{x_0-7}{19-7}, & 7 \leq x_0 \leq 19 \\ 1, & x_0 \geq 19 \end{cases} \tag{51}$$

So, if we consider x_0 be 14, then the possibility measure will be:

$$\text{Pos}\{X \leq 14\} = \frac{14-7}{19-7} = 0.583 \tag{52}$$

The Necessity Measure:

$$\text{Nec}\{X \leq x_0\} = \begin{cases} 0, & x_0 \leq 19 \\ \frac{x_0-19}{25-19}, & 19 \leq x_0 \leq 25 \\ 1, & x_0 \geq 25 \end{cases} \tag{53}$$

Let x_0 be 22 here so the necessity measure will be:

$$\text{Nec}\{X \leq 22\} = \frac{22-19}{25-19} = 0.5 \tag{54}$$

The Credibility Measure:

$$\text{Cr}\{X \leq x_0\} = \frac{1}{2}(\text{Pos}\{X \leq x_0\} + \text{Nec}\{X \leq x_0\}) \tag{55}$$

Thus, the Credibility Measure for the Case 2 will be:

$$\begin{aligned} \text{Cr}\{X \leq 14\} &= \frac{1}{2}(\text{Pos}\{X \leq 14\} + \text{Nec}\{X \leq 22\}) \\ &= \frac{1}{2}(0.583 + 0.5) = 0.542 \end{aligned} \tag{56}$$

Case3:

Here parameters a_1, a_4, a_5 are to be considered with data points 7,25,33 respectively with x_0 to be considered between parameters a_1 and a_4 for possibility and a_4 and a_5 for necessity measure. So, the possibility measure will be:

$$\text{Pos}\{X \leq x_0\} = \sup_{X \leq x_0} f(x) = \begin{cases} 0, & x_0 \leq 7 \\ \frac{x_0-7}{25-7}, & 7 \leq x_0 \leq 25 \\ 1, & x_0 \geq 25 \end{cases} \tag{57}$$

So, if we consider x_0 be 17, then the possibility measure will be:

$$\text{Pos}\{X \leq 17\} = \frac{17-7}{25-7} = 0.556 \tag{58}$$

The Necessity Measure:

$$\text{Nec}\{X \leq x_0\} = \begin{cases} 0, & x_0 \leq 25 \\ \frac{x_0-25}{33-25}, & 25 \leq x_0 \leq 33 \\ 1, & x_0 \geq 33 \end{cases} \tag{59}$$

Let x_0 be 29 here so the necessity measure will be:

$$\text{Nec}\{X \leq 29\} = \frac{29-25}{33-25} = 0.5 \tag{60}$$

The Credibility Measure:

$$\text{Cr}\{X \leq x_0\} = \frac{1}{2}(\text{Pos}\{X \leq x_0\} + \text{Nec}\{X \leq x_0\}) \tag{61}$$

Thus, the Credibility Measure for the Case 3 will be:

$$\begin{aligned} \text{Cr}\{X \leq 17\} &= \frac{1}{2}(\text{Pos}\{X \leq 17\} + \text{Nec}\{X \leq 29\}) \\ &= \frac{1}{2}(0.556 + 0.5) = 0.528 \end{aligned} \tag{62}$$

Case 4:

Here parameters a_1, a_5, a_6 are to be considered with data points 7,33,40 respectively with x_0 to be considered between parameters a_1 and a_5 for possibility and a_5 and a_6 for necessity measure. So, the possibility measure will be:

$$\text{Pos}\{X \leq x_0\} = \sup_{X \leq x_0} f(x) = \begin{cases} 0, & x_0 \leq 7 \\ \frac{x_0-7}{33-7}, & 7 \leq x_0 \leq 33 \\ 1, & x_0 \geq 33 \end{cases} \tag{63}$$

So, if we consider x_0 be 21, then the possibility measure will be:

$$\text{Pos}\{X \leq 21\} = \frac{21-7}{33-7} = 0.539 \tag{64}$$

The Necessity Measure:

$$\text{Nec}\{X \leq x_0\} = \begin{cases} 0, & x_0 \leq 33 \\ \frac{x_0-33}{40-33}, & 33 \leq x_0 \leq 40 \\ 1, & x_0 \geq 40 \end{cases} \tag{65}$$

Let x_0 be 36 here so the necessity measure will be:

$$\text{Nec}\{X \leq 36\} = \frac{36-33}{40-33} = 0.429 \tag{66}$$

The Credibility Measure:

$$\text{Cr}\{X \leq x_0\} = \frac{1}{2}(\text{Pos}\{X \leq x_0\} + \text{Nec}\{X \leq x_0\}) \tag{67}$$

Thus, the Credibility Measure for the Case 4 will be:

$$\begin{aligned} \text{Cr}\{X \leq 21\} &= \frac{1}{2}(\text{Pos}\{X \leq 21\} + \text{Nec}\{X \leq 29\}) \\ &= \frac{1}{2}(0.539 + 0.429) = 0.484 \end{aligned} \tag{68}$$

Now the final Credibility Measure after taking the averages of all the cases we get:

$$\begin{aligned}&\mathrm{Cr}\{X \leq x_0\}\\&= \frac{1}{4}\left(\mathrm{Cr}\{X \leq 19\} + \mathrm{Cr}\{X \leq 25\} + \mathrm{Cr}\{X \leq 33\} + \mathrm{Cr}\{X \leq 40\}\right)\\&= \frac{1}{4}(0.586 + 0.542 + 0.528 + 0.484) = 0.535\end{aligned} \tag{69}$$

Therefore, the Credibility measure for the pentagonal model is 0.535.

Future studies would expand this comparative investigation by validating the findings on larger and more heterogeneous real-world datasets, such as those in medical diagnosis, financial forecasting, and industrial quality control. A second research direction involves integrating pentagonal and hexagonal membership functions into deep learning architectures and hybrid fuzzy-neural models. Furthermore, optimization techniques such as genetic algorithms or swarm intelligence could be employed to adapt membership parameters to specific tasks. Finally, the scalability and computational trade-offs should be investigated to assess the practical viability of deploying these membership functions in real-time decision-support systems.

11 Conclusion

In conclusion, the comparative study of pentagonal and hexagonal fuzzy membership functions using credibility theory provides valuable insights into the performance of these two models. By analysing the credibility measure of both membership functions and comparing them, we have identified the strengths and weaknesses of each. The results show that the pentagonal membership function performs better than the hexagonal membership function in terms of overall average credibility (0.552 vs. 0.535), primarily due to the lower credibility in the additional Case 4 for the hexagonal model. Cases 1–3 yield identical credibility values across both models, highlighting their equivalence in core scenarios, while the hexagonal extension in Case 4 introduces a slight performance dip. This study can help researchers and practitioners in various fields, such as engineering and finance, to make informed decisions when dealing with fuzzy systems and uncertain data. Future research can explore more complex membership functions and further investigate the credibility theory in the context of fuzzy logic. These results illustrate the possibility of fuzzy membership selection using credibility theory on increasing the level of reliability of machine learning systems that act in an uncertain environment.

The other potential direction is incorporation of pentagonal and hexagonal membership functions in deep learning designs and hybrid fuzzy-neural designs. In addition, genetic algorithms or swarm intelligence could be used as optimization techniques to adjust membership parameters to particular tasks. Lastly, the scalability and trade-offs in computation needs to be explored to determine the feasibility of practical deployment of these membership functions in real time decision-support systems.

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Data will be made available on request.

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Conflicts of Interest

The authors declare no conflicts of interest.

Ethical Approval and Consent to Participate

Not applicable.

References

[1] Pathinathan, T., & Ponnivalavan, K. (2014). Pentagonal fuzzy number. *International journal of computing algorithm*, *3*(2014), 1003-1005.

[2] Wu, W. (2025). Automotive motor fault diagnosis model integrating machine learning algorithm and fuzzy control theory. *International Journal of Fuzzy Systems*, 1-14. [CrossRef]

[3] Zadeh, L. A. (1965). Fuzzy sets. *Information and control*, *8*(3), 338-353. [CrossRef]

[4] Shi, Y., Wei, P., Feng, K., Feng, D. C., & Beer, M. (2025). A survey on machine learning approaches for uncertainty quantification of engineering systems. *Machine Learning for Computational Science and Engineering*, *1*(1), 11. [CrossRef]

[5] Rajeshwari, P., Sudha, A. S., & Karthika, R. (2013). A new operation on hexagonal fuzzy numbers. *International Journal of Fuzzy Logic System*, *3*(3), 15-26. [CrossRef]

[6] Sangwan, P., Kumar, R., Sharma, Y., Bhosale, D. G., & Prasad, C. D. (2024). Application of

computing in recognition of input design factors for vapour-grown carbon nanofibers through fuzzy cluster analysis. *International Journal on Interactive Design and Manufacturing (IJIDeM)*, *18*(8), 6225-6235. [CrossRef]

[7] Dubois, D. J. (1980). *Fuzzy sets and systems: theory and applications* (Vol. 144). Academic press.

[8] Garg, H. (2016). A novel approach for analyzing the reliability of series-parallel system using credibility theory and different types of intuitionistic fuzzy numbers. *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, *38*(3), 1021-1035. [CrossRef]

[9] Kauffman, A., & Gupta, M. M. (1991). Introduction to fuzzy arithmetic: theory and application.

[10] Lu, J., Ma, G., & Zhang, G. (2024). Fuzzy machine learning: A comprehensive framework and systematic review. *IEEE Transactions on Fuzzy Systems*, *32*(7), 3861-3878. [CrossRef]

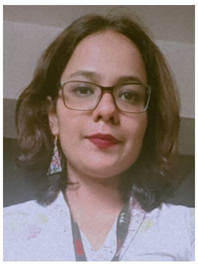
[11] Hesamian, G., & Bahrami, F. (2016). Credibility theory-oriented preference index for ranking fuzzy numbers. *Iranian Journal of Fuzzy Systems*, *14*(6), 103-117. [CrossRef]

[12] Liu, B. (2004). Uncertainty theory: an introduction to its axiomatic foundation. *Physica-Verlag, Heidelberg*. [CrossRef]

[13] Koppula, K., Kedukodi, B. S., & Kuncham, S. P. (2022). Generalization and ranking of fuzzy numbers by relative preference relation. *Soft Computing*, *26*(3), 1101-1122. [CrossRef]

[14] Pathinathan, T., & Ponnivalavan, K. (2015). Reverse order triangular, trapezoidal and pentagonal fuzzy numbers. *Annals of Pure and Applied Mathematics*, *9*(1), 107-117.

[15] Jang, J. S. (1993). ANFIS: adaptive-network-based fuzzy inference system. *IEEE transactions on systems, man, and cybernetics*, *23*(3), 665-685. [CrossRef]



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