RESEARCH ARTICLE

Preventive Maintenance and Competitive Strategies in IIoT-enable After-sales Markets: A Degradation Modeling and Game Theoretic Approach

Xiaojun Liang[1], **Li Yang**[2] and **Qingan Qiu**[3,*]

[1] School of Mathematics & Physics, Hulunbuir College, Hulun Buir 021008, China
[2] School of Reliability and Systems Engineering, Beihang University, Beijing 100083, China
[3] School of Management, Beijing Institute of Technology, Beijing 100081, China

Abstract

In the after-sales service market, understanding both the internal degradation of products and the external incentives within warranty period is crucial. Efforts into preventive maintenance can slow down the internal degradation, but these efforts are also influenced by external strategic services. Enabling an Industrial Internet of Things (IIoT) platform for preventive maintenance requires carefully considering the benefits instead of merely increasing efforts. This paper addresses these complexities by first proposing an additive degradation model to characterize the internal deterioration of products and the impact of efforts into preventive maintenance. It then introduces a sequential game model based on the IIoT platform, examining interactions between manufacturers and cooperative competitors under three competitive schemes: traditional competition, monopolistic competition, and shared competition. Equilibrium prices for new products and after-sales services are used to analyze external incentives. Utilizing these equilibrium prices, the paper derives profit and reliability functions of manufacturers and cooperative competitors under each competition scheme. Finally, this study combines the efforts into preventive maintenance and the internal degradation mechanism of products through equilibrium reliability functions.

Keywords: product warranty, after-sales service, degradation process, sequential games, Industrial Internet of Things (IIoT).

1 Introduction

1.1 Background

The after-sales service market for complex equipment industries—such as automotive, aviation maintenance, and medical equipment—is rapidly expanding and generates substantial revenue. In 2020, the automotive after-sales market in the United States reached approximately $153.5 billion, outpacing China at $77.996 billion, with Russia reporting $8 billion and Germany $22.3 billion, respectively [6]. Globally, the aviation maintenance and repair service market is forecasted to grow from $84 billion in 2022 to $133.7 billion by 2030, reflecting a compound annual growth

Submitted: 19 May 2025
Accepted: 07 July 2025
Published: 30 July 2025

Vol. 1, **No.** 1, 2025.
10.62762/TSSR.2025.782610

***Corresponding author:**
Qingan Qiu
qiu_qingan@bit.edu.cn

Citation
Liang, X., Yang, L., & Qiu, Q. (2025). Preventive Maintenance and Competitive Strategies in IIoT-enable After-sales Markets: A Degradation Modeling and Game Theoretic Approach. *ICCK Transactions on Systems Safety and Reliability*, 1(1), 21–42.

rate of 5.5% [20]. Similarly, the medical equipment maintenance market is projected to rise from $28.1 billion in 2021 to $49.2 billion by 2026, with an annual growth rate of 11.1% [17]. Product warranties, a significant aspect of manufacturers' after-sales services, represent a lucrative market segment. For example, in Japan, the extended warranty service generated sales of 1.56 trillion Japanese yen in 2021, expected to reach close to 1.8 trillion yen by 2030 [26]. This growth highlights the growing economic significance of warranty services on a global scale.

In the face of a vast market opportunity, numerous repair companies such as Easy Buy Industrial Products, SERVEONE, and Doctor of Engineering Mall are actively competing with manufacturers to secure their market share. They employ diverse strategies and techniques, including extending warranties, enhancing product reliability, reducing warranty costs in new product markets, and tackling challenges in the secondary market through methods such as obsolescence management, buybacks, trade-ins, leasing, or relicensing.

However, it's crucial to underscore that all these strategies hinge on one critical factor: product reliability, or more broadly, product quality. Product reliability depends not only on inherent quality but also on the level of effort into preventive maintenance (PM). Increasing efforts into PM typically result in improved product reliability and, correspondingly, fewer failures, although inherent product degradation trends are difficult to alter. This enhanced reliability serves as a catalyst for competitive strategies among savvy industry players. Yet, increasing efforts into PM can also lead to higher maintenance costs, potentially diminishing the effectiveness of these efforts. Therefore, achieving a balance between higher product reliability and lower associated costs is essential and requires careful consideration, especially as reliability is pivotal to various competitive strategies in this dynamic market landscape.

Additionally, the industrial Internet of Things (IIoT) platform represents one of the most promising and dynamic technology trends of our time, offering a wealth of opportunities that have yet to be fully realized. Take the Airbus's IIoT platform, Skywise, as an example. Skywise serves as a digital ecosystem hub connecting Airbus with its customers, showing how competition and cooperation can coexist. On the one hand, there exists a competitive relationship between the Airbus's IIoT platform and third parties in terms of the market share, service quality and efficiency, and innovation capability. For example, aiming to capture a larger market share, third-parties maintenance, repair, and overhaul (MRO) companies may leverage their specialized technical expertise and extensive industry experience to attract customers, potentially diverting clients from Airbus's after-sales services. Conversely, Airbus's IIoT platform, with its deep understanding of aircraft equipment and rich data resources, could develop more tailored after-sales services to retain customers and reduce reliance on third-party services. On the other hand, Airbus collaborates with third parties in terms of data sharing and integration, service complementarity, joint technology development, and ecosystem building. For example, Airbus and third parties can collaborate on developing advanced sensor technologies, data analysis algorithms, or security protocols. The Airbus-led "Factory of the Future" project involves partnerships with technology companies, research institutions, and suppliers. Through joint efforts, they explore the application of emerging technologies in aircraft manufacturing and after-sales services, driving innovation in IIoT technologies. Of course, Airbus can explore win-win business models with third parties, such as shared revenue or joint ventures.

Based upon the IIoT platform, three game processes are involved: when the manufacturer determines to establish the IIoT platform; whether the manufacturer should enable the platform to the cooperative competitor once it is established; and whether the cooperative competitor decides to access the platform once the manufacturer opens it to him.

This article will investigate the preventive maintenance and competitive strategies in product warranty and sequential games based on an IIoT platform.

1.2 Related literature

Understanding the internal degradation mechanisms of products is fundamental to shaping effective warranty policies, relying on detailed characterization of degradation processes. Typically, these processes involve continuous states, with prominent examples such as Wiener, gamma, and inverse Gaussian processes extensively studied in the literature on product warranties [11, 28]. Alternatively, for discrete state spaces, Markov processes provide a suitable framework [25].

The degradation process directly impacts product reliability and failure rates, especially as degradation reaches critical thresholds in soft failure scenarios. Research often examines product warranties through the lens of failure rates [12, 18], alongside analyses using reliability or distribution functions [3]. Product reliability influences age, usage patterns, and usage rates (the ratio of usage to age), crucial aspects in designing comprehensive warranty strategies [22]. While preventive maintenance (PM) may not alter the fundamental degradation process, rigorous PM practices can effectively slow it down. Evaluating the impact of PM typically involves stochastic variables [25], usage rates [18], failure rates [12], and copula-based failure models [23, 24]. Currently, literature on the influence of PM intensity on product degradation processes remains limited. This paper aims to fill this gap by proposing a comprehensive model. Peng et al. [16] have proposed an additive hazard model based on the baseline failure rate and deterioration process. Inspired by it, we propose the additive degradation model over time. Note that these are two completely different models, and the latter is more likely to obtain closed-form expressions when calculating the reliability of product.

Secondly, the external incentive mechanism of a product serves as the driving force for product warranty, while the equilibrium pricing and competitive strategies of efforts into PM have installed guardrails for the external incentive. In the fierce after-sales service market, operators will utilize all external resources and try their best efforts to obtain maximum profits. Some interesting topics about exogenous impacts, such as warranty length choice [5], planned obsolescence [21], buybacks [27], trade-in [4, 7–9], leasing [1], and relicensing [14], have been preliminarily studied. In addition, the rise of emerging technologies such as additive manufacturing [19], and IIoT platform [15] has also contributed to faster and more accurate product warranty. Equilibrium pricing in the after-sales service market is shaped by the interplay of market supply and demand, while the level of efforts into PM is constrained by associated maintenance costs. Understanding these dynamics is essential for analyzing various pricing strategies and their impact on profit functions for all participants. This becomes particularly critical in the era of Industrial Internet of Things (IIoT), where optimizing PM efforts is pivotal for maximizing profitability. In the competitive landscape of the after-sales service market, manufacturers often find themselves in warranty battles with cooperative competitors. The decision to adopt an IIoT platform initiates a strategic game where manufacturers must decide whether to open their platform to cooperative competitors and whether these competitors choose to participate. This strategic decision-making significantly influences profitability and competitive dynamics [10, 13]. Despite extensive research on the effects of IIoT platforms on preventive maintenance and pricing strategies, many studies overlook the internal degradation mechanisms of products. They often assume product reliability remains constant without exploring the dynamic nature of degradation processes. Addressing this gap requires research that integrates a deep understanding of product degradation mechanisms with strategic decision-making in IIoT-enabled after-sales service markets. Such studies can provide valuable insights into optimizing PM strategies and enhancing profitability amid competitive pressures.

1.3 Overview

Our work will fill this study gap between the internal degradation mechanism and external incentive mechanism through equilibrium pricing and competitive strategies of efforts into preventive maintenance.

First, according to the additive hazards model [16] and the preventive maintenance cost model [15], we first established an additive degradation model that characterizes the internal degradation mechanism of a product by considering the levels of baseline degradation, additive degradation, and efforts into preventive maintenance. Subsequently, the product's reliability formulation is built based upon a degradation threshold to analyze the response relationship between scale parameter, shape parameter, efforts into PM and reliability. Then we have analyzed how reliability varies across different competitive modes—including traditional competition, monopolistic competition, and shared competition between manufacturer and its competitor. Then, through analysing the sequence games between manufacturer and its competitor in IIoT-enabled after-sales markets, we have obtained the equilibrium prices of a new product and that of after-sales service, thereby obtaining the equilibrium profits and reliability of manufacturers and competitors respectively. These equilibrium reliability are related to the internal degradation mechanism within the product, and we can also achieve the optimal efforts into PM.

In summary, the contributions of this article are listed as follows.

(i) An additive degradation model is developed to characterize the internal degradation mechanism of a product and its sensitivity to the effort level into preventive maintenance.

(ii) Product's reliability formulation is built to characterize the response relationship between scale parameter, shape parameter, efforts into PM and the reliability.

(iii) We have analyzed how reliability varies across different competitive modes—including traditional competition, monopolistic competition, and shared competition—between manufacturer and its competitor by adopting a game-theoretic approach.

(iv) The effort level in preventive maintenance is connected to the internal degradation processes of products through equilibrium reliability functions, providing insights into how maintenance efforts influence product reliability and market dynamics.

The remaining part of this article is organized as follows. In Section 2, we describe the model setup, including product characteristics, manufacturer characteristics, and cooperative competitor characteristics, and present sequential games and demands for product and service. In Section 3, we discuss the sequential games under manufacturer dominating and cooperative competitor dominating under three competition schemes, respectively. Section 4 explores the competitive strategies of effort into preventive maintenance through numerical experiment. Finally, the conclusion and future work are presented in Section 5. All proofs are provided in Appendix, except for the proof process that is necessary to appear in the main text.

2 Model Description

2.1 Notations

cdf	cumulative distribution function
pdf	probability density function
$W(t)$	cumulative degradation process with additive hazards and PM implication
$W_0(t)$	baseline degradation process in a comfortable environment
$W(t)$	cumulative degradation process with use in a harsh environment
L	failure threshold of degradation level for a new product
ρ	level of effort into preventive maintenance
$\varrho_M(\varrho_C)$	level for manufacturer's (competitor's) PM efforts
$Ga(\alpha, \beta)$	Gamma distribution with shape parameter α and scale parameter β
α_i, β	shape parameter and scale parameter for $W_i(t), i = 0, 1$
$f_{W(t)}(x)$	pdf of stochastic process $W(t)$
T	first hitting time of a new product
$F_{W(t)}(x)$	cdf of stochastic process $W(t)$, i.e. $F_{W(t)}(x) = P\{W(t) \leq x\}$
R	reliability function of product
$R_M(R_M)$	reliability function of a product provided by manufacturer (competitor)
p_{M_0}	price of a new product
$p_M(p_C)$	price of manufacturer's (cooperative competitor's) after-sales service
ζ	associated repair cost induced by a product failure
K	cost factor related endogenous technology investment level
ε	endogenous technology investment level
εH	cost induced by the data acquisition and analysis process
c_M	cost of producing product
$\eta_M(\eta_C)$	cost coefficient of PM for the manufacturer (competitor)
θ	customer evaluation for a perfect product
φ	competitor's royalty fee to access the IIoT-based platform
δ	relative willingness to choose manufacturer's after-sales service

2.2 Product Characteristics

Consider a single type of product being sold by a manufacturer to a unit mass of consumers. Assume that the degradation process of the product follows a stochastic process with independent increments and is finitely durable in nature. Under this assumption, the Wiener process, Gamma process, and Inverse Gaussian process are three potential candidates for characterizing such a process. To concretize the characterization process and avoid complications, we preselect the Gamma process as a satisfactory anchor. Referring to the additive hazards model with respect to failure rate [16] and the preventive maintenance cost model [15], we propose the following additive degradation model.

Let the Gamma process, denoted by $\{W(t), t \geq 0\}$, represent the cumulative degradation over time t with the following additive degradation model:

$$W(t) = W_0(t) + \frac{W_1(t)}{1+\rho}, \tag{1}$$

where both $W_0(t)$ and $W_1(t)$ follow a stationary Gamma process with distinct shape parameters and the same scale parameter. Specifically, $W_0(t) \sim Ga(\alpha_0 t, \beta)$ and $W_1(t) \sim Ga(\alpha_1 t, \beta)$, in which $W_0(t) \sim Ga(\alpha_0 t, \beta)$ indicates $W_0(t)$ follows a Gamma process with shape parameter $\alpha_0 t$ and scale parameter β, while $W_1(t)$ follows a Gamma process with shape parameter $\alpha_1 t$ and scale parameter β. $W_0(t)$ may represent the baseline degradation process for the product in a comfortable operating environment or at a minimum usage rate, while $W_1(t)$ represents the additive degradation induced by the usage in a relatively harsh environment.

It is reasonable to divide the degradation process of a product into two parts: the baseline degradation process and the additive degradation process. On the one hand, the degradation process of a product is unavoidable in many situation, even in the most comfortable environment. Thus, the stochastic process $M_0(t)$ is used to characterize such a process. On the other hand, some after-sales services such as maintenance and repair of the product can only delay the degradation advancement rather than restore the product to a new state. This implies that the effect of the after-sales service is limited to partially correcting the degradation process. Therefore, $W_1(t)$ is dedicated to describe such an additive degradation process.

The same scale parameter is chosen in $W_0(t)$ and $W_1(t)$ since the characteristics of the product and the properties of the Gamma process are considered together. The scale parameter of the Gamma process determines the volatility of the process. The larger the scale parameter, the greater the volatility of the process, and vice versa. Considering the additive property of the Gamma distribution, we will choose the same volatility to characterize the baseline degradation process and the additive degradation process. In addition, implementing preventive maintenance (PM) is an affective way to decelerate the degradation process of products, especially for durable ones. The coefficient $\frac{1}{1+\rho}$ features the mitigation effect of PM efforts, where $\rho > 0$ represents the effort in PM. Note that an increase in the effort level corresponds to a reduction in the degree of degradation, and the resultant reduction in the probability of product failure. Thus, a product degradation model considering the effort in PM, as in Equation (1), has been established.

Several crucial results (with their detailed proofs found in Appendix A) are first presented to lay the foundation for further discussion.

Theorem 2.1 *Assume that the degradation process $W_0(t) \sim Ga(\alpha_0 t, \beta)$, $W_1(t) \sim Ga(\alpha_1 t, \beta)$, and ρ is the effort in PM, then*

$$\frac{W_1(t)}{1+\rho} \sim Ga(\alpha_1 t, (1+\rho)\beta). \tag{2}$$

The probability density function (pdf) of $W(t)$ may be expressed as

$$\begin{aligned} f_{W(t)}(x) =& \frac{(1+\rho)^{\alpha_1 t} \beta^{(\alpha_0+\alpha_1)t} x^{(\alpha_0+\alpha_1)t-1} e^{-\beta x}}{\Gamma(\alpha_0 t)\Gamma(\alpha_1 t)} \\ & \times \int_0^1 (1-u)^{\alpha_0 t-1} u^{\alpha_1 t-1} e^{-\rho\beta x u} du. \end{aligned} \tag{3}$$

Especially, when $\rho = 0$, the Equation (2) are reduced to $W_1(t) \sim Ga(\alpha_1 t, \beta)$, and consequently $W(t) = W_0(t) + W_1(t) \sim Ga((\alpha_0+\alpha_1)t, \beta)$.

After building the degradation model of the product, we will then determine the relationship between product failure and degradation level. The product is considered to be failed if its degradation level reaches a predetermined threshold L, i.e., $W(t) \geq L$. Thus, the failure time of each product is then characterized by the first hitting time (FHT), which is defined as

$$T = \inf\{t | W(t) \geq L\}. \tag{4}$$

In such a model, $T \leq t$ is equivalent to $W(t) \geq L$ due to the monotonicity of the Gamma process. Thus, the cumulative distribution function (cdf) of T may be readily expressed as

$$F_T(t) = P\{T \le t\} = P\{W(t) \ge L\} = 1 - F_{W(t)}(L), \quad (5)$$

where $F_{W(t)}(x) = P\{W(t) \le x\}$ is the cdf of $W(t)$. This further indicates that the reliability of the product, denoted by $R(t;\alpha_0,\alpha_1,\beta,\rho)$, may be expressed as

$$\begin{aligned} R(t;\alpha_0,\alpha_1,\beta,\rho) &= F_{W(t)}(L) = \int_0^L f_{W(t)}(x)dx \\ &= \frac{(1+\rho)^{\alpha_1 t}\beta^{(\alpha_0+\alpha_1)t}}{\Gamma(\alpha_0 t)\Gamma(\alpha_1 t)} \\ &\times \int_0^L x^{(\alpha_0+\alpha_1)t-1}e^{-\beta x}dx \\ &\times \int_0^1 (1-u)^{\alpha_0 t-1}u^{\alpha_1 t-1}e^{-\rho\beta x u}du \\ &= \frac{(1+\rho)^{\alpha_1 t}}{\Gamma(\alpha_0 t)\Gamma(\alpha_1 t)}\int_0^{\beta L} v^{(\alpha_0+\alpha_1)t-1}e^{-v}dv \\ &\times \int_0^1 (1-u)^{\alpha_0 t-1}u^{\alpha_1 t-1}e^{-\rho u v}du \end{aligned} \quad (6)$$

where α_0, α_1, β, and ρ are four related parameters, and the last equation has leveraged the integral transformation with $v = \beta x$. Especially, when the effort degree in PM decreases to minimum ($\rho = 0$), the above reliability (Equation (6)) reduces to

$$R(t;\alpha_0,\alpha_1,\beta,0) = \frac{1}{\Gamma((\alpha_0+\alpha_1)t)}\int_0^{\beta L} u^{(\alpha_0+\alpha_1)t-1}e^{-u}du. \quad (7)$$

In the following text, to simplify the presentation, we omit the constant parameters in the reliability expressions. For example, $R(t;\alpha_0,\alpha_1,\beta,\rho)$ is abbreviated as $R(t;\rho)$, which indicates that the reliability of the product at time t varies depending on ρ, while α_0, α_1, and β remain constant. Additionally, the subscript 'M' or 'C' indicates that the quantity is related to the manufacturer or its cooperative competitor through the rest of the article, respectively. Thus, the reliability $R(\rho_M)$ with the effort degree of manufacturer for PM can readily be written as R_M, and $R(\rho_C)$ with that of its cooperative competitor can be written as R_C.

2.3 Manufacturer Characteristics

A manufacturer produces aforementioned product with finite durability, with a cost of c_M, and sells it at a price of p_{M_0}. The product is subject to nontrivial reliability $R(t;\alpha_0,\alpha_1,\beta,\rho_M)$, where parameters such as α_0, α_1, and β feature the characteristics of the pre-sales product, while parameter ρ_M depicts the manufacturer's effort degree in PM for after-sales products. Indeed, the degradation process of the corresponding product without PM intervention, as stated in Theorem 2.1, satisfies $W(t) = W_0(t) + W_1(t) \sim Ga((\alpha_0+\alpha_1)t,\beta)$ with a mean of $\frac{(\alpha_0+\alpha_1)t}{\beta}$ and a variance of $\frac{(\alpha_0+\alpha_1)t}{\beta^2}$. In other words, the PM in our model only impacts the evolution of the additive degradation process $W_1(t)$ with coefficient $\frac{1}{1+\rho_M}$.

Assume that the manufacturer provides the after-sales service for customers at a price of p_M and then commits to maintain and repair the product within the service terms. Generally, the increase in PM efforts corresponds to the reduction in product failures. Similarly to recent studies [4, 10, 15], when the effort degree is enhanced from 0 to ρ_M, the reliability of the product will increase from $R(t;\alpha_0,\alpha_1,\beta,0)$ to $R(t;\alpha_0,\alpha_1,\beta,\rho_M)$, while the additive costs associated with PM will also increase to $\eta_M\rho_M^2$, where parameter η_M is the PM cost coefficient for the manufacturer, and a lower value of η_M indicates a higher PM efficiency. Since PM cannot eliminate all product failures, the maximum reliability of product is $R(t;\alpha_0,\alpha_1,\beta,1)$, less than 1. Thus, the manufacturer will undertake a repair cost ζ once a product fails, in which ζ might include the connected losses for customers induced by product failure. For example, the owner of a automobile may have to pay a fee to travel by other means, such as a car-hailing service, as the automobile breaks down.

In order to accurately improve the effectiveness of PM by monitoring the system's operation status in real time, many leading enterprises, such as Airbus and GE, have recently begun to establish Industrial Internet of Things (IIoT) platform, which leverages interconnected devices and sensors in industrial settings to collect and exchange data for monitoring, controlling, and optimizing PM processes. To take the impact of IIoT into account, employing the quadratic cost model adopted by some references [2, 15], assume that the manufacturer will undertake an investment $\tau + K\varepsilon^2$ to establish an IIoT platform, where τ is an initial installation cost for the necessary infrastructure and software, K represents a cost factor and $\varepsilon \in [0,1]$ denotes the endogenous technology investment level. Based upon such an IIoT platform, the cost coefficient of manufacturer's PM decreases from initial η_M to current $\frac{\eta_M}{(1+\varepsilon)}$, and the corresponding cost for PM becomes $\frac{\eta_M\rho_M^2}{1+\varepsilon}$. Simultaneously, an additional cost εH is incurred by the data acquisition and analysis process. Thus, a higher technology investment level ε indicates that the manufacturer can acquire and analyze an larger amount of operational data and hence more precise PM plan. Note that the manufacturer's marginal repair cost for each failure still keeps ζ since PM cost based upon IIoT platform does not decrease the repair cost of a failed product.

2.4 Cooperative Competitor Characteristics

When the manufacturer utilizes its own advantages to make participating in after-sales services an important part of revenue streams, enormous third parties, who specialize in providing independent maintenance, repair, and operations, also want to enter the after-sales service market and get a share of the pie. An independent third-party, referred to as cooperative competitor, also participates in after-sales service for customers at a price p_C after obtaining permission from the manufacturer. When the effort degree in PM remains at ρ_C, the degradation process runs according to $W(t) = W_0(t) + \frac{W_1(t)}{1+\rho_C}$, and resultant product reliability enhances from $R(\alpha_0, \alpha_1, \beta, 0)$ to $R(\alpha_0, \alpha_1, \beta, \rho_C)$, while the additive cost of associated PM will also increase to $\eta_C \rho_C^2$, wherein parameter η_C is the PM cost coefficient for the cooperative competitor, and a lower value of η_C indicates a higher PM efficiency. Correspondingly, ρ_C depicts the competitor's effort degree for after-sales products. Since PM cannot eliminate all product failures, assume that the cooperative competitor will bear a repair cost of ζ for each failure. Although the manufacturer and cooperative competitor may have different maintenance costs after product failures, we still assume that they are the same here, because our main focus is on the impact of effort degree about reliability, profit function, etc. This assumption simplifies the discussion of secondary issues. When the IIoT platform is shared, the cost coefficient for cooperative competitor will decrease from η_C to $\frac{\eta_C}{1+\varepsilon}$, and the resultant cost will vary from $\eta_C \rho_C^2$ to $\frac{\eta_C \rho_C^2}{1+\varepsilon}$. Although the cooperative competitor is not required to bear the cost of establishing an IIoT platform and the cost incurred by the data acquisition and analysis, he still needs to pay a royalty fee of φ to the manufacturer to access the platform.

2.5 Characteristics of Cooperation and Competition, and Sequential Games

Assume that there is still a unit volume of customers, who are heterogeneous in product valuation. The customer will acquire a net value of θ from product if it works well. Here we refer to such customer as type θ customer, where θ is a random variable and uniformly distributed on $[0, 1]$. Thus, with the support of new technologies, manufacturer can establish an IIoT-based platform and upgrade their PM level to compete with the cooperative competitor. Consequently, there have always been three alternative schemes between manufacturer and cooperative competitor: Traditional competition (TC) scheme, in which the manufacturer does not establish the IIoT-based platform, and both the manufacturer and cooperative competitor perform traditional PM; Monopolistic competition (MC) scheme, in which the manufacturer establishes the platform but the cooperative competitor does not access the platform; Shared competition (SC) scheme, in which the manufacturer establishes and opens the IIoT-based platform, and the competitor accesses it with a royalty fee φ.

Thus, a sequential game is built as follows:

(i) Based upon the investment amount of $\tau + K\varepsilon^2$, the manufacturer determines whether to establish the IIoT-based platform and the sales price p_{M_0}.

(ii) The manufacturer determines whether to open the platform to the cooperative competitor when the platform is established.

(iii) The cooperative competitor determines whether to access the platform when the manufacturer opens the platform to cooperative competitor.

Thus, the manufacturer and cooperative competitor cooperate and compete in the after-sales service market. Specifically, under the traditional competition scheme, the manufacturer and cooperative competitor provide the traditional after-sales service with the effort degrees of ρ_M and ρ_C, and then claim the after-sales fees p_M and p_C, respectively. Under the monopolistic competition scheme, the manufacturer provides the improved after-sales service based on the IIoT platform while its competitor still provides the traditional one, although their effort degree in PM and service prices remain unchanged. Note that this is a competitive scheme dominated by the manufacturer. Under the shared competition scheme, both the manufacturer and cooperative competitor provide the improved after-sales services based on the IIoT platform altogether with effort degrees of ρ_M and ρ_C, respectively.

(iv) Customers decide whether to purchase the product, and choose the corresponding after-sales service from the manufacturer or cooperative competitor.

2.6 Demands for Product and Service

When choosing product and after-sales services, rational consumers only pay attention to the prices of product and services and do not pay attention to whether the platform has established, whether the platform has opened to the cooperative competitor, or any other questions like these that are irrelevant to consumers. However, most customers might have underlying preference towards the after-sales services provided by either manufacturer or cooperative competitor. Consequently, assume that $\delta \in (0,1)$ represents the relative willingness to choose the manufacturer's after-sales service, and the resultant $1-\delta$ represents that of the cooperative competitor. Under these assumptions, the gross utility surplus received by customer of type θ is divided into two categories as follows:

(i)When customers buy the product and choose the manufacturer's service, the gross utility is

$$u_M = \delta\theta R_M - (1-R_M)\zeta - p_{M_0} - p_M. \quad (8)$$

(ii)When customers buy the product and choose the cooperative competitor's service, the gross utility is

$$u_C = (1-\delta)\theta R_C - (1-R_C)\zeta - p_{M_0} - p_C, \quad (9)$$

where $\delta \in [0,1]$ is the relative willingness to buy the product and manufacturer's after-sales service compared with the cooperative competitor.

Furthermore, assume that the volume of the after-sales service market is same as that of sales market. Then the related demands could be obtained in the following theorem.

Theorem 2.2 *(i)When $\frac{R_M}{R_C} > \frac{1}{\delta} - 1$, the total product demand is*

$$Q = \begin{cases} 1 - \dfrac{(1-R_C)\zeta + p_{M_0} + p_C}{(1-\delta)R_C}, & \delta \le \mathfrak{L}; \\ 1, & \delta > \mathfrak{L}. \end{cases} \quad (10)$$

The demand for manufacturer's after-sales service is

$$Q_M = \begin{cases} 1 - \dfrac{(R_C-R_M)\zeta + p_M - p_C}{\delta R_M - (1-\delta)R_C}, & \delta \le \mathfrak{L}; \\ 1 - \dfrac{(1-R_M)\zeta + p_{M_0} + p_M}{\delta R_M}, & \delta > \mathfrak{L}. \end{cases} \quad (11)$$

The demand for competitor's after-sales service is

$$Q_C = \begin{cases} \dfrac{(R_C-R_M)\zeta + p_M - p_C}{\delta R_M - (1-\delta)R_C} - \dfrac{(1-R_C)\zeta + p_{M_0} + p_C}{(1-\delta)R_C}, \delta \le \mathfrak{L}; \\ \dfrac{(1-R_M)\zeta + p_{M_0} + p_M}{\delta R_M}, \delta > \mathfrak{L}. \end{cases} \quad (12)$$

(ii) When $\frac{R_M}{R_C} < \frac{1}{\delta} - 1$, the demand for product is

$$Q = \begin{cases} 1, & \delta \le \mathfrak{L}; \\ 1 - \dfrac{(1-R_M)\zeta + p_{M_0} + p_M}{\delta R_M}, & \delta > \mathfrak{L}. \end{cases} \quad (13)$$

The demand for manufacturer's service is

$$Q_M = \begin{cases} \dfrac{(1-R_C)\zeta + p_{M_0} + p_C}{(1-\delta)R_C}, \delta \le \mathfrak{L}; \\ \dfrac{(R_C-R_M)\zeta + p_M - p_C}{\delta R_M - (1-\delta)R_C} - \dfrac{(1-R_M)\zeta + p_{M_0} + p_M}{\delta R_M}, \delta > \mathfrak{L}. \end{cases} \quad (14)$$

The demand for competitor's service is

$$Q_C = \begin{cases} 1 - \dfrac{(1-R_C)\zeta + p_{M_0} + p_C}{(1-\delta)R_C}, & \delta \le \mathfrak{L}; \\ 1 - \dfrac{(R_C-R_M)\zeta + p_M - p_C}{\delta R_M - (1-\delta)R_C}, & \delta > \mathfrak{L}. \end{cases} \quad (15)$$

where

$$\mathfrak{L} = R_C\left[(1-R_M)\zeta + p_{M_0} + p_M\right] / \{R_C\left[(1-R_M)\zeta + p_{M_0} + p_M\right] + R_M\left[(1-R_C)\zeta + p_{M_0} + p_C\right]\}$$

Firstly, the managerial implications of some symbols and conditions in the Theorem 2.2 need further explanations. If the product of reliability and purchasing willingness is used to characterize the level of after-sales service, then condition $\frac{R_M}{R_C} > \frac{1}{\delta} - 1$ implies that the manufacturer is dominant in service level since the condition $\frac{R_M}{R_C} > \frac{1}{\delta} - 1$ is equivalent to $\delta R_M > (1-\delta)R_C$. So we refer to such case as *manufacturer dominating*. Correspondingly, the condition $\frac{R_M}{R_C} < \frac{1}{\delta} - 1$ implies that the cooperative competitor is dominant, and we then refer to it as *competitor dominance*. Additionally, we can obtain $(1-R_M)\zeta + p_{M_0} + p_M > (1-R_C)\zeta + p_{M_0} + p_C$ according to both conditions $\frac{R_M}{R_C} > \frac{1}{\delta} - 1$ and $\delta \le \mathfrak{L}$. It indicates that the condition $\delta < \mathfrak{L}$ means the "negative" gross utility of manufacturer is greater than that of the cooperative competitor, since $(1 - R_M)\zeta + p_{M_0} + p_M$ and $(1 - R_C)\zeta + p_{M_0} + p_C$ are just the negative part of Equations (8) and (9), respectively. Corresponding to the case of $\frac{R_M}{R_C} < \frac{1}{\delta} - 1$, similarly, we can obtain $(1-R_M)\zeta+p_{M_0}+p_M < (1-R_C)\zeta+p_{M_0}+p_C$ according to both conditions $\frac{R_M}{R_C} < \frac{1}{\delta} - 1$ and $\delta > \mathfrak{L}$. In such case, the "negative" gross utility of cooperative competitor is greater than that of the manufacturer.

After identifying the total product demand, we need to distinguish two types of market coverage cases: the partially covered market ($Q_M + Q_C < 1$) and the fully covered market ($Q_M + Q_C = 1$). According to the results of Theorem 2.2, under the condition of $\frac{R_M}{R_C} > \frac{1}{\delta} - 1$, the market coverage is full when $\delta > \mathfrak{L}$, and partial when $\delta \leq \mathfrak{L}$. Under the condition of $\frac{R_M}{R_C} < \frac{1}{\delta} - 1$, the market coverage is full when $\delta \leq \mathfrak{L}$, and partial when $\delta > \mathfrak{L}$. In a partially covered market, there are still people who have not purchased any products, which indicates that these people are potential customers for purchasing products and their after-sales service. But in a fully covered market, everyone either buys the product and manufacturer's service or buys the product and the competitor's service, in other words, the market is fully occupied. Thus, we mainly focus on the partially covered market in the following discussions.

3 Sequential Games Based on IIoT Platform

In this section, we will consider the equilibrium profits of the manufacturer and cooperative competitor under three schemes, including the traditional competition, monopolistic competition, and shared competition.

3.1 Sequential Game under Manufacturer Dominating and Partially Covered Market

We first consider the case of manufacturer dominating and partially covered market, i.e., $\frac{R_M}{R_C} > \frac{1}{\delta} - 1$ (or equivalently $\delta R_M > (1 - \delta)R_C$) and $\delta < \mathfrak{L}$. In this case, sequential games should be unfolded in three different scenarios, including the traditional competition, monopolistic competition, and shared competition schemes as follows.

Lemma 3.1 *Under the traditional competition scheme, if the manufacturer is dominant and the service market is partially covered, the equilibrium profits of the manufacturer and cooperative competitor, respectively, are*

$$\pi_M^{TC} = \frac{[(1-\delta)R_C - (1-R_C)\zeta - p_C^{*1} - c_M]^2}{4(1-\delta)R_C} + \frac{[\delta R_M - (1-\delta)R_C - (1+R_C-2R_M)\zeta + p_C^{*1} - \eta_M \rho_M^2]^2}{4[\delta R_M - (1-\delta)R_C]}, \tag{16}$$

and

$$\pi_C^{TC} = \left\{(1-\delta)R_C\left[2(1-R_M)\zeta + c_M + \eta_M\rho_M^2\right] - \delta R_M\left[2(1-R_C)\zeta + c_M + \eta_C\rho_C^2\right]\right\}^2 / \left(8\delta(1-\delta)R_C R_M\left[\delta R_M - (1-\delta)R_C\right]\right) \tag{17}$$

where $p_C^{*1} = \frac{(1-\delta)R_C}{2\delta R_M}[2(1-R_M)\zeta + c_M + \eta_M\rho_M^2] + \frac{\eta_C\rho_C^2 - c_M}{2}$. *Furthermore, the equilibrium reliability functions for the manufacturer and the cooperative competitor, respectively, yields as follows:*

$$R_M^{TC} = R(t;\alpha_0,\alpha_1,\beta,0)\left(\frac{\eta_M(\rho_M^*)^2}{2\zeta + c_M} + 1\right). \tag{18}$$

and

$$R_C^{TC} = R(t;\alpha_0,\alpha_1,\beta,0) + \frac{\eta_C(\rho_C^*)^2}{\zeta - \frac{1-\delta}{2\delta R_M}[2\zeta - 2(\delta+\zeta)R_M + c_M + \eta_M\rho_M^2]}. \tag{19}$$

where $R(t;\alpha_0,\alpha_1,\beta,0)$ *is the initial condition defined in Equation 7, and* ρ_M^* *and* ρ_M^* *are the equilibrium effort degree in PM for the manufacturer and cooperative competitor, respectively.*

Proof. Under the traditional competition scheme, since the manufacturer does not establish the IIoT-based platform, and both the manufacturer and competitor perform regular PM, then the manufacturer's profit function is

$$\begin{aligned}\pi_M =& \left[1 - \frac{(1-R_C)\zeta + p_{M_0} + p_C}{(1-\delta)R_C}\right](p_{M_0} - c_M) \\ &+ \left[1 - \frac{(R_C - R_M)\zeta + p_M - p_C}{\delta R_M - (1-\delta)R_C}\right] \\ &\times [p_M - (1-R_M)\zeta - \eta_M\rho_M^2],\end{aligned}$$

and the competitor's profit function is

$$\begin{aligned}\pi_C =& \left[\frac{(R_C - R_M)\zeta + p_M - p_C}{\delta R_M - (1-\delta)R_C} - \frac{(1-R_C)\zeta + p_{M_0} + p_C}{(1-\delta)R_C}\right] \\ &\times [p_C - (1-R_C)\zeta - \eta_C\rho_C^2].\end{aligned}$$

We first solve the decision of the manufacturer and competitor on their equilibrium prices. By taking the derivatives with respect to p_{M_0}, p_M, and p_C, respectively, we obtain the results as follows:

$$\frac{\partial \pi_M}{\partial p_M} = \frac{\delta R_M - (1-\delta)R_C + (1-R_C)\zeta - 2p_M + p_C + \eta_M \rho_M^2}{\delta R_M - (1-\delta)R_C},$$
$$\frac{\partial \pi_M}{\partial p_{M_0}} = \frac{(1-\delta)R_C - (1-R_C)\zeta - 2p_{M_0} - p_C + c_M}{(1-\delta)R_C},$$
$$\frac{\partial \pi_C}{\partial p_C} = \frac{\frac{1}{2}(1+R_C-2R_M)\zeta - \frac{3}{2}p_C + p_M + \frac{\eta_C\rho_C^2}{2}}{\delta R_M - (1-\delta)R_C} - \frac{\frac{1}{2}(1-R_C)\zeta + \frac{3}{2}p_C + p_{M_0} - \frac{1}{2}\eta_C\rho_C^2}{(1-\delta)R_C}.$$

Let $\frac{\partial \pi_M}{\partial p_M} = \frac{\partial \pi_M}{\partial p_{M_0}} = \frac{\partial \pi_C}{\partial p_C} = 0$, we derive the equilibrium prices as

$$p_M^* = \frac{1}{2}[\delta R_M - (1-\delta)R_C + (1-R_C)\zeta + p_C + \eta_M\rho_M^2],$$
$$p_{M_0}^* = \frac{1}{2}[(1-\delta)R_C - (1-R_C)\zeta - p_C + c_M],$$
$$p_C^{*1} = \frac{(1-\delta)R_C}{2\delta R_M}[2(1-R_M)\zeta + c_M + \eta_M\rho_M^2] + \frac{\eta_C\rho_C^2 - c_M}{2},$$

where p_M^* represents the equilibrium price of p_M, and similar to $p_{M_0}^*$ and p_C^{*1}.

Substituting the sale and service prices of p_M^*, $p_{M_0}^*$ and p_C^*, the profit functions of the manufacturer and competitor become

$$\pi_M^{TC} = \frac{[(1-\delta)R_C - (1-R_C)\zeta - p_C^{*1} - c_M]^2}{4(1-\delta)R_C} + \frac{[\delta R_M - (1-\delta)R_C - (1+R_C-2R_M)\zeta + p_C^{*1} - \eta_M\rho_M^2]^2}{4[\delta R_M - (1-\delta)R_C]},$$

$$\pi_C^{TC} = \Big\{(1-\delta)R_C\left[2(1-R_M)\zeta + c_M + \eta_M\rho_M^2\right] - \delta R_M\left[2(1-R_C)\zeta + c_M + \eta_C\rho_C^2\right]\Big\}^2 \Big/ \left(8\delta(1-\delta)R_C R_M\left[\delta R_M - (1-\delta)R_C\right]\right)$$

Taking the further derivatives with respect to the level of PM efforts of the manufacturer, ρ_M, we can obtain

$$\frac{\partial p_{M_0}}{\partial \rho_M} = -\frac{(1-\delta)R_C}{4\delta}\frac{2\eta_M\rho_M R_M - (2\zeta + c_M + \eta_M\rho_M^2)R_M'}{R_M^2}.$$

Thus, we can obtain the reliability function with equilibrium level of PM efforts of the manufacturer as

$$\ln R_M = \int_0^{\rho_M^*} \frac{2\eta_M u}{2\zeta + c_M + \eta_M u^2} du,$$

or equivalently,

$$R_M^{TC} = R(t;\alpha_0,\alpha_1,\beta,0)\left(\frac{\eta_M(\rho_M^*)^2}{2\zeta + c_M} + 1\right).$$

where $R(t;\alpha_0,\alpha_1,\beta,0)$ is the initial condition defined in Equation 7.

Similarly, taking the derivative with respect to ρ_C, we can obtain

$$\frac{\partial p_{M_0}}{\partial \rho_C} = \frac{1}{2}\Big\{(1-\delta)R_C' + \zeta R_C' - \frac{(1-\delta)R_C'}{2\delta R_M}\left[2(1-R_M)\zeta + c_M + \eta_M\rho_M^2\right] - \eta_C\rho_C\Big\}$$

Let $\frac{\partial p_{M_0}}{\partial \rho_C} = 0$, we can obtain the reliability function with equilibrium level of PM efforts of the competitor as

$$R_C^{TC} = R(t;\alpha_0,\alpha_1,\beta,0) + \frac{\eta_C(\rho_C^*)^2}{\zeta - \frac{1-\delta}{2\delta R_M}[2\zeta - 2(\delta+\zeta)R_M + c_M + \eta_M\rho_M^2]}.$$

Lemma 3.2 *Under the monopolistic competition scheme, if the manufacturer is dominant and the service market is partially covered, the equilibrium profits of the manufacturer and cooperative competitor, respectively, are*

$$\pi_M^{MC} = \frac{\left[(1-\delta)R_C - (1-R_C)\zeta - {p_C^*}^2 - c_M\right]^2}{4(1-\delta)R_C} + \frac{\left[\delta R_M - (1-\delta)R_C - (1+R_C-2R_M)\zeta + {p_C^*}^2 - \frac{\eta_M\rho_M^2}{1+\varepsilon} - \varepsilon H\right]^2}{4\left[\delta R_M - (1-\delta)R_C\right]} - K\varepsilon^2$$

and

$$\pi_C^{MC} = \Big\{(1-\delta)R_C\left[2(1-R_M)\zeta + c_M + \frac{\eta_M\rho_M^2}{1+\varepsilon} + \varepsilon H\right] - \delta R_M\left[2(1-R_C)\zeta + \eta_C\rho_C^2 + c_M\right]\Big\}^2 \Big/ \Big(8\delta(1-\delta)R_C R_M[\delta R_M - (1-\delta)R_C]\Big) \tag{20}$$

where $p_C^{*2} = \frac{(1-\delta)R_C}{2\delta R_M}[2(1-R_M)\zeta + c_M + \frac{\eta_M\rho_M^2}{1+\varepsilon} + \varepsilon H] + \frac{\eta_C\rho_C^2 - c_M}{2}$. *Furthermore, the equilibrium reliability functions for the manufacturer and the cooperative competitor, respectively, can be obtained as*

$$R_M^{MC} = R(t;\alpha_0,\alpha_1,\beta,0)\left(\frac{\eta_M{\rho_M^*}^2}{2\zeta + c_M} + 1\right), \tag{21}$$

and

$$R_C^{MC} = R(t;\alpha_0,\alpha_1,\beta,0) \tag{22}$$
$$+ \frac{\eta_C\rho_C^2}{\zeta - \frac{1-\delta}{2\delta R_M}[2\zeta - 2(\delta+\zeta)R_M + c_M + \eta_M\rho_M^2]}. \tag{23}$$

Comparing the results of Lemma 3.1 and Lemma 3.2, it can be seen that, except for the profit function of the producer being very similar, all other quantities, including the profit function of the cooperative competitor, π_C, and the equilibrium reliability of the manufacturer and cooperative competitor, R_C and R_M, have the same formula. This means that the the IIoT platform introduced by manufacturers, under the monopolistic competition scheme, has not caused an impact on the profits and reliability for the cooperative competitor. However, the question that the following theorem aims to solve is what impact exclusive ownership of an IIoT platform will have on itself.

Theorem 3.1 *Under the conditions of manufacturer dominating and partially covered market, i.e.,* $\frac{R_M}{R_C} > \frac{1}{\delta} - 1$ *and* $\delta < \mathfrak{L}$*, a necessary condition for the manufacturer to establish an IIoT is* $\frac{\eta_M \rho_M^2}{1+\varepsilon} > H$*, and then the optimal endogenous investment level is subject to*

$$\varepsilon + 1 = \sqrt{\frac{\eta_M \rho_M^2}{H}}. \tag{24}$$

Meanwhile, if $\frac{\eta_M \rho_M^2}{1+\varepsilon} > H$*, the profits of the cooperative competitor become lower.*

Proof. By comparing Equations (16) and (20), it is easy to find that if $\frac{\eta_M \rho_M^2}{1+\varepsilon} + \varepsilon H > \eta_M \rho_M^2$, the manufacturer's profit will decrease. In fact, $\frac{\eta_M \rho_M^2}{1+\varepsilon} + \varepsilon H > \eta_M \rho_M^2$ means that the introduction of the IIoT platform will result in repair and construction costs exceeding the original repair costs. Consequently, $\frac{\eta_M \rho_M^2}{1+\varepsilon} + \varepsilon H < \eta_M \rho_M^2$, or equivalently $\frac{\eta_M \rho_M^2}{1+\varepsilon} > H$, becomes a necessary condition to establish an IIoT platform. Additionally, $f_1(\varepsilon) = K\varepsilon^2$ is an upward-opening quadratic function with a minimum value of 0, while $f_2(\varepsilon) = \frac{\eta_M \rho_M^2}{1+\varepsilon} + \varepsilon H$ is equivalent to a Hook function with minimum point $(\sqrt{\frac{\eta_M \rho_M^2}{H}}, 2\sqrt{H \eta_M \rho_M^2} - H)$ when considering ε as the dependent variable. Thus, the optimal endogenous investment level is subject to $\varepsilon + 1 = \sqrt{\frac{\eta_M \rho_M^2}{H}}$.

Meanwhile, by comparing Equations (17) and (20), it is easy to find that $\pi_C^{MC} < \pi_C^{TC}$ yields when $\frac{\eta_M \rho_M^2}{1+\varepsilon} > H$. This implies the cooperative competitor will gain less profit.

Note that under condition of $\frac{\eta_M \rho_M^2}{1+\varepsilon} > H$, the equilibrium price of the cooperative operator's after-sales service has been deceased. In fact, we can, from the results of Lemma 3.1 and 3.2, obtain

$$\begin{aligned} p_C^{*2} - p_C^{*2} &= \frac{(1-\delta)R_C}{2\delta R_M}(\varepsilon H - \frac{\varepsilon \eta_M \rho_M^2}{1+\varepsilon}) \\ &= \frac{\varepsilon(1-\delta)R_C}{2\delta R_M}(H - \frac{\eta_M \rho_M^2}{1+\varepsilon}) < 0. \end{aligned}$$

It is not surprising that when the manufacturer introduces the IIoT platform and incurs construction costs, the cooperative competitor can enhance competitiveness by lowering price of after-sales service.

Additionally, only from the perspective of production profit, it may not be profitable for the manufacturer to introduce the IIoT platform only for his own benefit when $\frac{\eta_M \rho_M^2}{1+\varepsilon} + \varepsilon H > \eta_M \rho_M^2$ or $K\varepsilon^E$ is great enough, and at the same time, the equilibrium benefits of the cooperative competitor will also decrease. This also fully demonstrates the harm of non-cooperative competition to the manufacturer's own benefits, and it, in turn, also motivates the cooperative competitor to adopt repair models based on the IIoT platform.

Lemma 3.3 *Under the shared competition scheme, if the manufacturer is dominant and the service market is partially covered, the equilibrium profits of the manufacturer and cooperative competitor, respectively, yield as*

$$\begin{aligned} \pi_M^{SC} =& \frac{[(1-\delta)R_C - (1-R_C)\zeta - p_C^{*3} - c_M]^2}{4(1-\delta)R_C} \\ &+ \frac{[\delta R_M - (1-\delta)R_C - (1+R_C-2R_C)\zeta + p_C^{*3} - \frac{\eta_M \rho_M^2}{1+\varepsilon} - \varepsilon H]^2}{4[\delta R_M - (1-\delta)R_C]} \\ &- K\varepsilon^2 + \varphi, \end{aligned}$$

and

$$\begin{aligned} \pi_C^{SC} = &\Big\{(1-\delta)R_C\Big[2(1-R_M)\zeta + c_M + \frac{\eta_M \rho_M^2}{1+\varepsilon} + \varepsilon H\Big] \\ &- \delta R_M\Big[2(1-R_C)\zeta + c_M + \frac{\eta_C \rho_C^2}{1+\varepsilon}\Big]\Big\}^2 / \\ &\Big(8\delta(1-\delta)R_C R_M[\delta R_M - (1-\delta)R_C]\Big) - \varphi \end{aligned} \tag{25}$$

where $p_C^{*3} = \frac{(1-\delta)R_C}{2\delta R_M}[2(1-R_M)\zeta + c_M + \frac{\eta_M \rho_M^2}{1+\varepsilon} + \varepsilon H] + \frac{\eta_C \rho_C^2}{2(1+\varepsilon)} - \frac{c_M}{2}$*. Further, the equilibrium reliability functions for the manufacturer and cooperative competitor, respectively, can be obtained as*

$$R_M^{SC} = R(t, \alpha_0, \alpha_1, \beta, 0)[\frac{(\rho_M^*)^2 \eta_M}{(1+\varepsilon)(2\zeta + c_M + \varepsilon H)} + 1], \tag{26}$$

and

$$R_C^{SC} = R(t;\alpha_0,\alpha_1,\beta,0) + \frac{\rho_C^2\eta_C}{2(1+\varepsilon)[\zeta - \frac{1-\delta}{2\delta R_M}(2\zeta - 2(\delta+\zeta)R_M + c_M + \frac{\eta_M\rho_M^2}{1+\varepsilon} + \varepsilon H)]}, \quad (27)$$

where $R_M^{'}$ represents the derivative of R_M and ρ_M^* is the optimal effort degree in PM.

Theorem 3.2 *Under the condition of the manufacturer dominating and partially covered market, i.e., $\frac{R_M}{R_C} > \frac{1}{\delta} - 1$ and $\delta < \mathfrak{L}$, a necessary condition for the cooperative competitor to access the IIoT platform is $\frac{\varepsilon\eta_C\rho_C^2}{1+\varepsilon} \leq \varphi$, and the indifference point satisfies*

$$\varphi = \Big((1-\delta)R_C\left[2(1-R_M)\zeta + c_M + \frac{\eta_M\rho_M^2}{1+\varepsilon} + \varepsilon H\right] - \delta R_M\left[2(1-R_C)\zeta + c_M + \frac{(2+\varepsilon)\eta_C\rho_C^2}{1+\varepsilon}\right]\Big) \Big/ \Big(4\delta(1-\delta)R_C R_M[\delta R_M - (1-\delta)R_C]\Big) \times \frac{\varepsilon\eta_C\rho_C^2}{1+\varepsilon}$$

Meanwhile, manufacturers will benefit from an additional royalty fee for accessing the IIoT platforms to the cooperative competitor.

Proof. The cooperative competitor might determine to access the IIoT platform when the manufacturer opens it to cooperative competitor if accessing the platform is profitable, that is, $\frac{\eta_C\rho_C^2}{1+\varepsilon} - \eta_C\rho_C^2 \leq \varphi$. Thus, the necessary condition for the cooperative competitor to access the IIoT platform is $\frac{\varepsilon\eta_C\rho_C^2}{1+\varepsilon} \leq \varphi$.

Additionally, substituting Equations (20) and (21) into $\pi_C^{SC} - \pi_C^{MC} = 0$, we can obtain the indifference point

$$\varphi = \Big((1-\delta)R_C\left[2(1-R_M)\zeta + c_M + \frac{\eta_M\rho_M^2}{1+\varepsilon} + \varepsilon H\right] - \delta R_M\left[2(1-R_C)\zeta + c_M + \frac{(2+\varepsilon)\eta_C\rho_C^2}{1+\varepsilon}\right]\Big) \Big/ \Big(4\delta(1-\delta)R_C R_M[\delta R_M - (1-\delta)R_C]\Big) \times \left(\varepsilon\eta_C\rho_C^2/(1+\varepsilon)\right)$$

Meanwhile, comparing the Equations (20) and (25), we can readily find that the manuscript will benefit φ from the royalty fee for accessing the IIoT platforms to the cooperative competitor.

As expected, Theorem 3.1 demonstrates the win-win effect of cooperation in the after-sales service market.

3.2 Sequential Game under Cooperative Competitor Dominance and Partially Covered Market

Here we will then consider the sequential game under the condition of $\frac{R_M}{R_C} < \frac{1}{\delta} - 1$ and $\delta > \mathfrak{L}$ based upon the discussion in Section 2.6. In such case, the sequential games should also be unfolded in three different scenarios, including the traditional competition, monopolistic competition, and shared competition schemes.

Lemma 3.4 *Under the traditional competition scheme, if the cooperative competitor is dominant and the service market is partially covered, the equilibrium profits of the cooperative competitor and manufacturer, respectively, are*

$$\pi_C^{TC} = \Big(p_M^* + (1-\delta)R_C - \delta R_M - (1+R_M-2R_C)\zeta - \eta_C\rho_C^2\Big)^2 \Big/ \Big(4\left[(1-\delta)R_C - \delta R_M\right]\Big) \quad (28)$$

and

$$\pi_M^{TC} = \frac{[2p_M^* + c_M - \delta R_M + \eta_M\rho_M^2]^2}{4\delta R_M} + [\frac{1}{2} - \frac{p_M^* + c_M + (1-R_M)\zeta}{\delta R_M} - \frac{p_M^* - (1+R_M-2R_C)\zeta - \eta_C\rho_C^2}{2[(1-\delta)R_C - \delta R_M]}] \times [p_M^* - (1-R_M)\zeta - \eta_M\rho_M^2]. \quad (29)$$

where $p_M^* = \frac{1}{3}[\delta R_M - (1-\delta)R_C + (3-2R_C-R_M)\zeta + 2\eta_M\rho_M^2 + \eta_C\rho_C^2]$. Further, the equilibrium reliability functions for the cooperative competitor and manufacturer, respectively, can be obtained as

$$R_M = R(t;\alpha_0,\alpha_1,\beta,0) + \frac{\eta_M\rho^2}{2|\zeta-\delta|}(\zeta \neq \delta). \quad (30)$$

and

$$R_C = R(t;\alpha_0,\alpha_1,\beta,0) + \frac{2\eta_C\rho_C^2}{\zeta+\delta-1}. \quad (31)$$

Lemma 3.5 *Under the monopolistic competition scheme, if the cooperative competitor is dominant and the service market is partially covered, the equilibrium profits of the cooperative competitor and manufacturer, respectively, are*

$$\pi_C^{MC} = \Big(p_M^* + (1-\delta)R_C - \delta R_M - (1+R_M-2R_C)\zeta - \eta_C\rho_C^2\Big)^2 \Big/ \Big(4\left[(1-\delta)R_C - \delta R_M\right]\Big) \quad (32)$$

and

$$\pi_M^{MC} = \frac{[2p_M^* + c_M - \delta R_M + \frac{\eta_M \rho_M^2}{1+\varepsilon} + \varepsilon H]^2}{4\delta R_M} + [\frac{1}{2} - \frac{p_M^* + c_M + (1-R_M)\zeta}{\delta R_M} - \frac{p_M^* - (1+R_M-2R_C)\zeta - \eta_C \rho_C^2}{2[(1-\delta)R_C - \delta R_M]}] \times [p_M^* - (1-R_M)\zeta - \frac{\eta_M \rho_M^2}{1+\varepsilon} - \varepsilon H]. \quad (33)$$

where $p_M^* = \frac{1}{3}[\delta R_M - (1-\delta)R_C + (3-2R_C-R_M)\zeta + \eta_C \rho_C^2 + \frac{2\eta_M \rho_M^2}{1+\varepsilon} + 2\varepsilon H]$. *Further, the equilibrium reliability functions for the cooperative competitor and manufacturer, respectively, can be obtained as*

$$R_M = \frac{2\eta_M \rho_M^2}{1+\varepsilon} + R(t;\alpha_0,\alpha_1,\beta,0). \quad (34)$$

and

$$R_C = \frac{2\eta_C \rho_C^2}{|\zeta - (1-\delta)|} + R(t;\alpha_0,\alpha_1,\beta,0). \quad (35)$$

Similar to Theorem 3.1, we will derive the following conclusion without proof.

Theorem 3.3 *Under the conditions of the cooperative competitor dominance and partially covered market, i.e.,* $\frac{R_M}{R_C} < \frac{1}{\delta} - 1$ *and* $\delta > \mathfrak{L}$*, a necessary condition for the manufacturer to establish an IIoT is* $\frac{\eta_M \rho_M^2}{1+\varepsilon} > H$*, and then the optimal endogenous investment level is subject to*

$$\varepsilon + 1 = \sqrt{\frac{\eta_M \rho_M^2}{H}}. \quad (36)$$

Meanwhile, if $\frac{\eta_M \rho_M^2}{1+\varepsilon} > H$*, the profits of the cooperative competitor become lower.*

Lemma 3.6 *Under the shared competition scheme, if the cooperative competitor is dominant and the service market is partially covered, the equilibrium profits of the cooperative competitor and manufacturer, respectively, are*

$$\pi_C^{SC} = \Big(p_M^* + (1-\delta)R_C - \delta R_M - (1+R_M-2R_C)\zeta - \frac{\eta_C \rho_C^2}{1+\varepsilon}\Big)^2 \Big/ \Big(4\left[(1-\delta)R_C - \delta R_M\right]\Big),$$

$$\pi_M^{SC} = \frac{[2p_M^* + c_M - \delta R_M + \frac{\eta_M \rho_M^2}{1+\varepsilon} + \varepsilon H]^2}{4\delta R_M} + [\frac{1}{2} - \frac{p_M^* + c_M + (1-R_M)\zeta}{\delta R_M} - \frac{p_M^* - (1+R_M-2R_C)\zeta - \frac{\eta_C \rho_C^2}{1+\varepsilon}}{2[(1-\delta)R_C - \delta R_M]}] \times [p_M^* - (1-R_M)\zeta - \frac{\eta_M \rho_M^2}{1+\varepsilon} - \varepsilon H].$$

where $p_M^* = \frac{1}{3}[\delta R_M - (1-\delta)R_C + (3-2R_C-R_M)\zeta + \frac{\eta_C \rho_C^2}{1+\varepsilon} + \frac{2\eta_M \rho_M^2}{1+\varepsilon} + 2\varepsilon H]$. *Further, the equilibrium reliability functions for the cooperative competitor and manufacturer, respectively, can be obtained as*

$$R_M = \frac{2\eta_M \rho_M^2}{|\zeta - \delta|(1+\varepsilon)} + R(t;\alpha_0,\alpha_1,\beta,0), \quad (37)$$

and

$$R_C = \frac{2\eta_C \rho_C^2}{|\zeta - 1 + \delta|(1+\varepsilon)} + R(t;\alpha_0,\alpha_1,\beta,0). \quad (38)$$

Similar to Theorem 3.2, we will derive the following theorem without proof.

Theorem 3.4 *Under the condition of the cooperative competitor dominance and partially covered market, i.e.,* $\frac{R_M}{R_C} < \frac{1}{\delta} - 1$ *and* $\delta > \mathfrak{L}$*, a necessary condition for the cooperative competitor to access the IIoT platform is* $\frac{\varepsilon \eta_C \rho_C^2}{1+\varepsilon} \le \varphi$*, and the indifference point satisfies*

$$\varphi = \Big((1-\delta)R_C\left[2(1-R_M)\zeta + c_M + \frac{\eta_M \rho_M^2}{1+\varepsilon} + \varepsilon H\right] - \delta R_M\left[2(1-R_C)\zeta + c_M + \frac{(2+\varepsilon)\eta_C \rho_C^2}{1+\varepsilon}\right]\Big) \Big/ (4\delta(1-\delta)R_C R_M[\delta R_M - (1-\delta)R_C]) \times \big(\varepsilon \eta_C \rho_C^2/(1+\varepsilon)\big)$$

Meanwhile, manufacturers will benefit from an additional royalty fee for accessing the IIoT platforms to the cooperative competitor.

4 Trade-off of Effort Degree for Preventive Maintenance

First, let us examine the parameters in Equation (6), including t, α_0, α_1, β, and ρ. Considering both Equations (1) and (4), assume that the following assumption holds:

$$E[W(t)] = \frac{\alpha_0 t}{\beta} + \frac{\alpha_1 t}{(1+\rho)\beta} \le L, \tag{39}$$

where $E[W(t)]$ is the mean function of $W(t)$. Equation (39) implies that the Mean Time to First Failure (MTTFF) of the product, denoted by $E[T]$, satisfies

$$E[T] = \sup\{t | \frac{\alpha_0 t}{\beta} + \frac{\alpha_1 t}{(1+\rho)\beta} \le L\} = \frac{\beta L}{\alpha_0 + \frac{\alpha_1}{1+\rho}}. \tag{40}$$

Based on the above fact, we simply assign the value of $\frac{\beta L}{\alpha_0+\frac{\alpha_1}{1+\rho}}$ to the parameter t. In other words, we will straightforwardly consider the reliability at time $\frac{\beta L}{\alpha_0+\frac{\alpha_1}{1+\rho}}$. Additionally, for exogenous parameter, L, without loss of generality, assume that $L = 1$. Furthermore, assume that $W_1(t) \sim Ga(\frac{nt}{2}, \frac{1}{2})$ since the second additive term of Equation (1) represents the increment of the degradation or depreciation process. It implies that $W_1(t)$ follows a Chi-square process with n degrees of freedom. Thus, taking $\alpha_0 = 1$, we can reduce the product reliability of form Equation (6) as

$$R(\frac{\beta}{\alpha_0 + \frac{\alpha_1}{1+\rho}}; \alpha_0, \alpha_1, \beta, \rho) = R(\frac{1}{2 + \frac{n}{1+\rho}}; 1, \frac{n}{2}, \frac{1}{2}, \rho)$$
$$= \frac{(1+\rho)^{\frac{n}{2(2+\frac{n}{1+\rho})}}}{\Gamma(\frac{1}{2+\frac{n}{1+\rho}})\Gamma(\frac{n}{2(2+\frac{n}{1+\rho})})} \int_0^{\frac{1}{2}} v^{\frac{2+n}{2(2+\frac{n}{1+\rho})}-1} e^{-v} dv$$
$$\int_0^1 (1-u)^{\frac{1}{2+\frac{n}{1+\rho}}-1} u^{\frac{n}{2(2+\frac{n}{1+\rho})}-1} e^{-\rho u v} du. \tag{41}$$

Next we will examine the reliability under sequential games.

4.1 Manufacturer Dominating and Partially Covered Market

First of all, we will consider the case of manufacturer dominating and partially covered market. The parameter spaces (such as $\delta \in (0,1)$, $\theta \in (0,1)$, $\zeta \in (0,\infty)$, $\eta \in (0,1)$) denoted by the above closed expression in Section 3 are non-empty, and all of the equilibrium outcomes are valid within these parameter spaces. Examining the impact of each parameter on equilibrium reliability one by one would be tedious and not an ideal program. Our purpose here is to study the impact of effort degree for PM on reliability under both equilibrium and non-equilibrium conditions. Therefore, using numerical experiments instead of discussing each parameter is a simple and feasible method. Assigning $\zeta = 0.15, \theta = 0.8, \delta = 0.4, \eta_C = \eta_M = 0.0015, c_M = 0.2$, and $n = 3$, the related equilibrium reliability functions with respect to effort degree ρ under the traditional competition scheme are depicted in Figure 1 (a). From the graph, it can be seen that the product reliability $R(\rho)$, which only considers the effort degree for PM and is denoted by the blue dashed line, is monotonically increasing, while the manufacturer's equilibrium reliability R_M or the competitor's equilibrium reliability R_C is monotonically decreasing. No surprisingly, such curve trend just reflects the trade-off between the product reliability and its costs since the higher effort degree will incur more costs in this model. Figure 1 (b) depicts the graph of the equilibrium pricing $p^*_{M_0}$, p^*_M, and p^*_C, where both p^*_M, and p^*_C increase with increasing effort degree, but $p^*_{M_0}$ decreases with increasing effort degree. It is interesting that Figure 1 (c) shows that under the monopolistic competition scheme, the manufacturer, as the first participant in the sequential game, gains more profits, but the total amount of profits is very small, although the difference between the two is significant. This is also an incentive for manufacturers to establish IIoT-based platform.

Further assigning $\varepsilon = 0.02, \tau = 0.0001, K = 0.5, H = 0.2$, and $\varphi = 0$, under the monopolistic competition scheme, the related reliability functions, optimal prices, and profit functions are drawn in Figure 2. Although Figure 2 (a) presents seemingly repetitive curves, Figures 2 (b) and (c) have depicted the curves of prices and profit functions, full of fascinating insights. They show that a higher effort degree does not necessarily mean a decrease in optimal pricing, nor does it mean an increase in profits for both manufacturer and the cooperative competitor. Especially the profits of cooperative competitor actually decrease with the increase of effort degree. This also means that the manufacturer has an advantage in profits after establishing the IIoT platform, and it also gives cooperative competitor the motivation to join the IIoT platform.

Under the shared competition scheme, the related reliability functions, optimal prices, and profit functions are depicted in Figure 3. From Figure 3 (b), we can obseve that under the shared competition scheme, the price of a new product decreases with

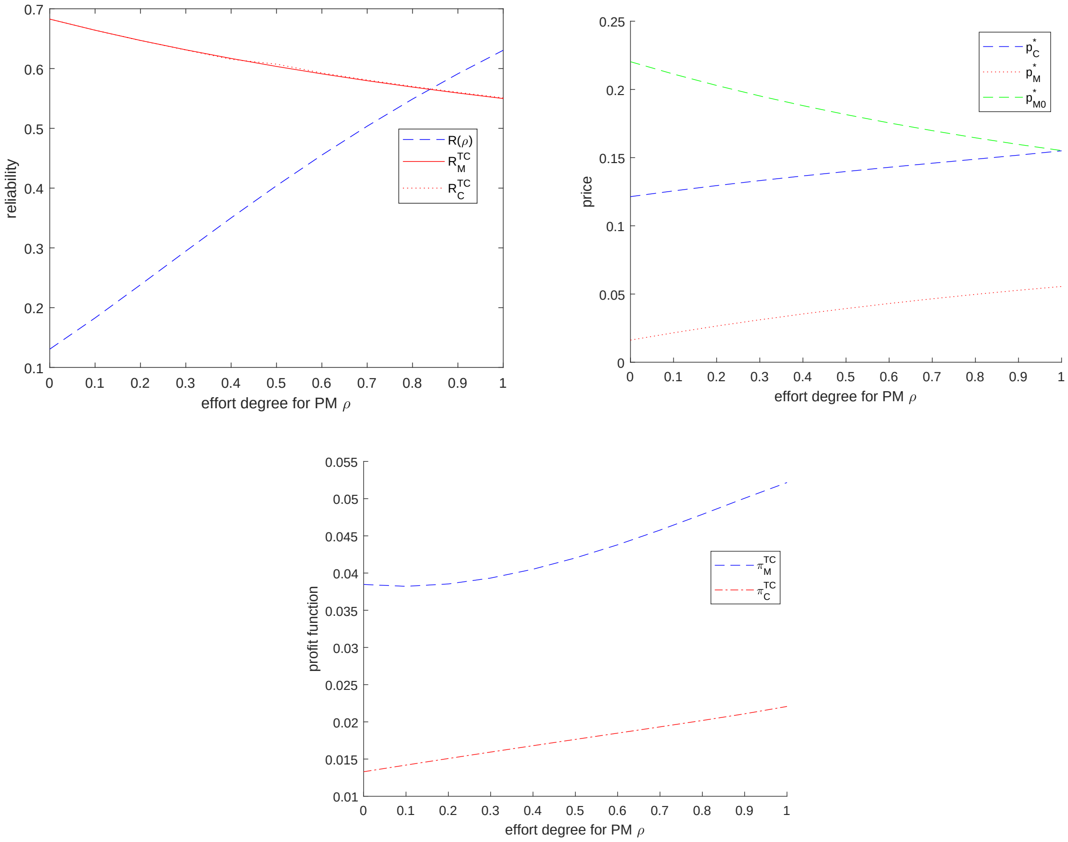

Figure 1. The optimal quantity of TC scheme under manufacturer dominating and $\delta < \mathfrak{L}$.

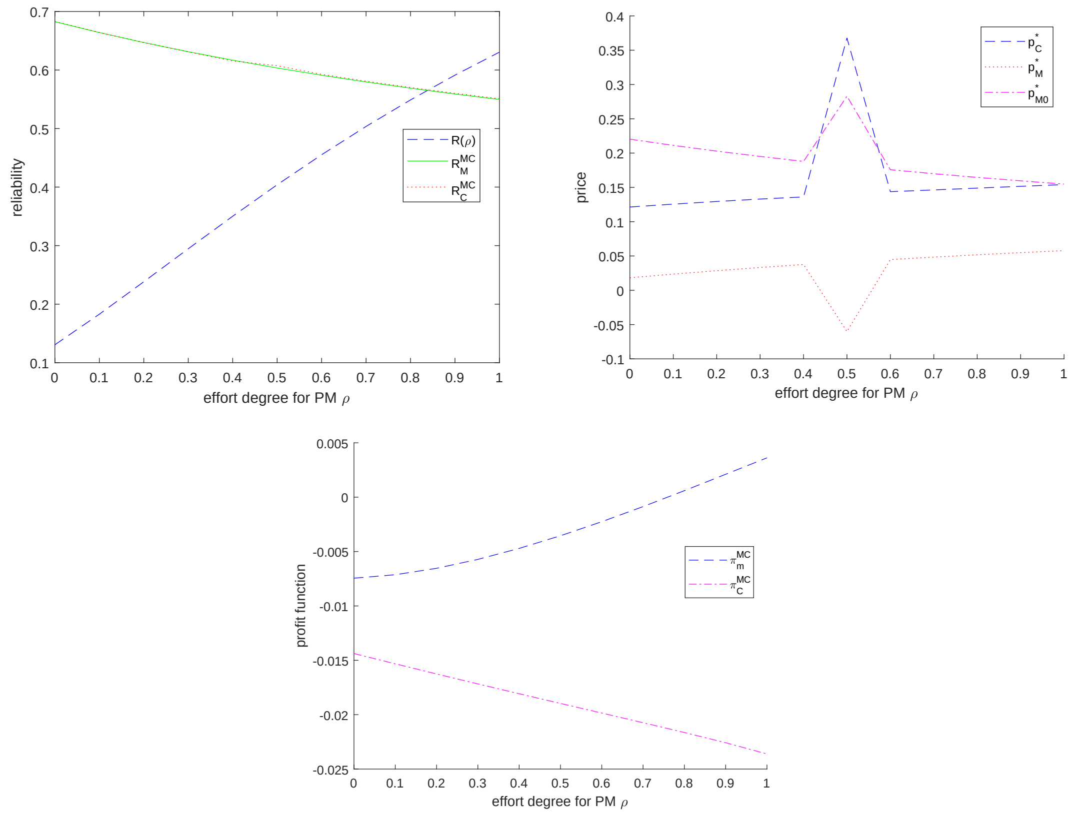

Figure 2. The optimal quantity of MC scheme under manufacturer dominating and $\delta < \mathfrak{L}$.

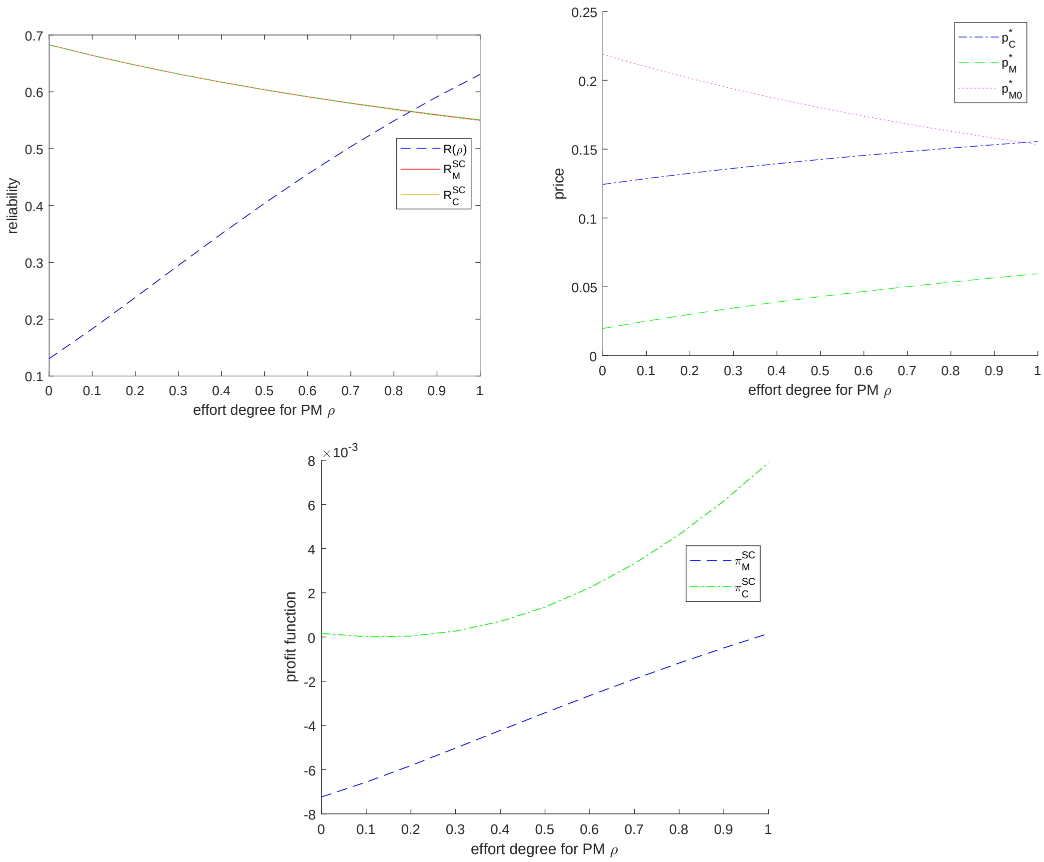

Figure 3. The optimal quantity of SC scheme under manufacturer dominating and $\delta < \mathfrak{L}$.

increasing effort degree, while the price of after-sales service increases with higher effort degree. This can be interpreted as high-quality after-sales service being associated with a higher service price. As shown in Figure 3 (c), although the cooperative competitor must pay a royalty fee φ, both the manufacturer and cooperative competitor can enhance their profits as they increase their effort degree.

4.2 Cooperative Competitor Dominance and Partially Covered Market

Then we will consider the case of cooperative competitor dominance and partially covered market. Assigning $\zeta = 0.15, \theta = 0.8, \delta = 0.4, \eta_C = \eta_M = 0.0015, c_M = 0.2$, $n = 3$, $\varepsilon = 0.02, \tau = 0.0001, K = 0.5, H = 0.2$, and $\varphi = 0$, the related equilibrium reliability functions with respect to effort degree ρ under the traditional competition scheme are depicted in Figures 3 - 6. These images have an explanation similar to Figures 1 - 3, which will not be repeated here.

5 Conclusion and Future Work

In the fierce competition of the after-sales service market, it is necessary to consider not only the internal degradation mechanism of a product but also external incentive model for product warranty. In other words, the purpose of this article is to establish the connection between the degradation process of products and sequential games through the effort degree for preventive maintenance. First of all, the additive degradation model $W(t) = W_0(t) + \frac{W_1(t)}{1+\rho}$ is used to characterize the intrinsic degradation by $W_0(t)$ and the impact of effort level for PM on overall degradation level by $\frac{W_1(t)}{1+\rho}$. The greatest advantage of this model is that it can easily obtain the density function (Equation 3) and reliability function of the product degradation process (Equations 6, 7). Manufacturers and competitors face two parallel problems simultaneously. From the consumer market perspective, the primary concern is the demand for products and services. For manufacturers considering an IIoT platform, several strategic decisions affect their operational strategy: firstly, whether to invest in establishing the IIoT platform; secondly, whether to open it to cooperative competitors once established; and thirdly, whether these competitors opt to access it. Setting up an IIoT platform requires an initial investment from the manufacturer, potentially boosting their maintenance

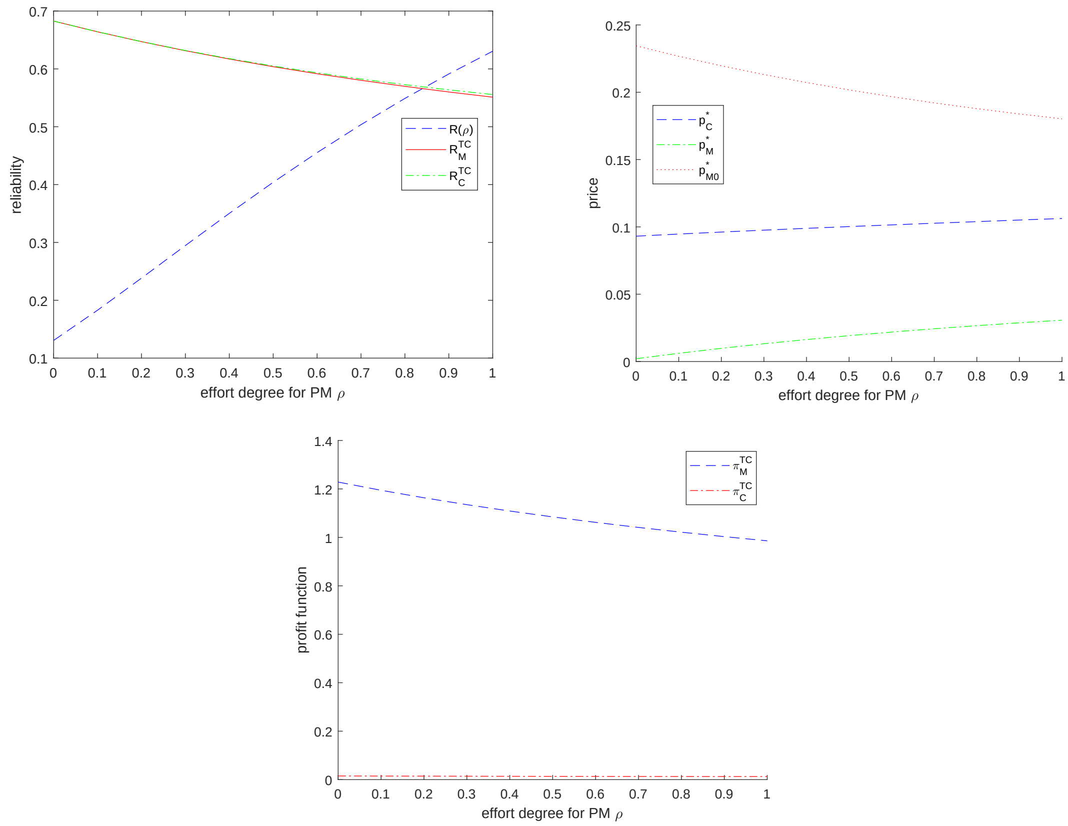

Figure 4. The optimal quantity of TC scheme under competitor dominance and $\delta > \mathfrak{L}$.

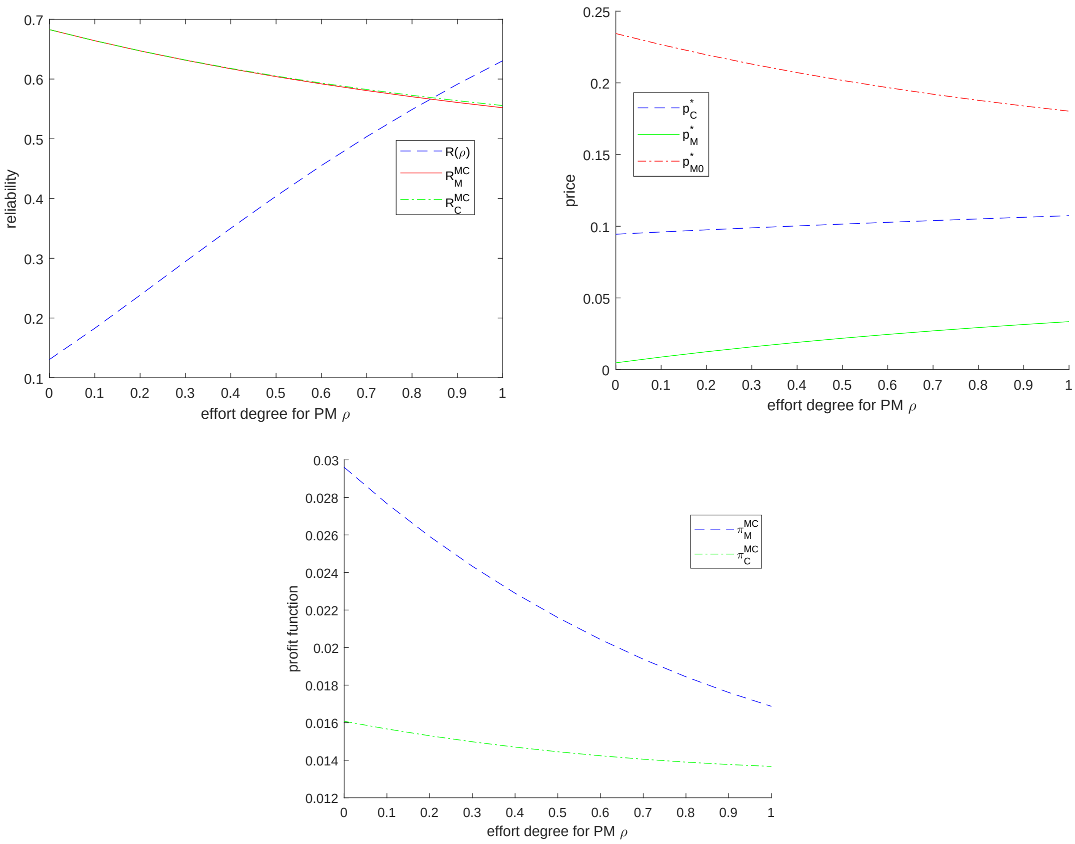

Figure 5. The optimal quantity of MC scheme under competitor dominance and $\delta > \mathfrak{L}$.

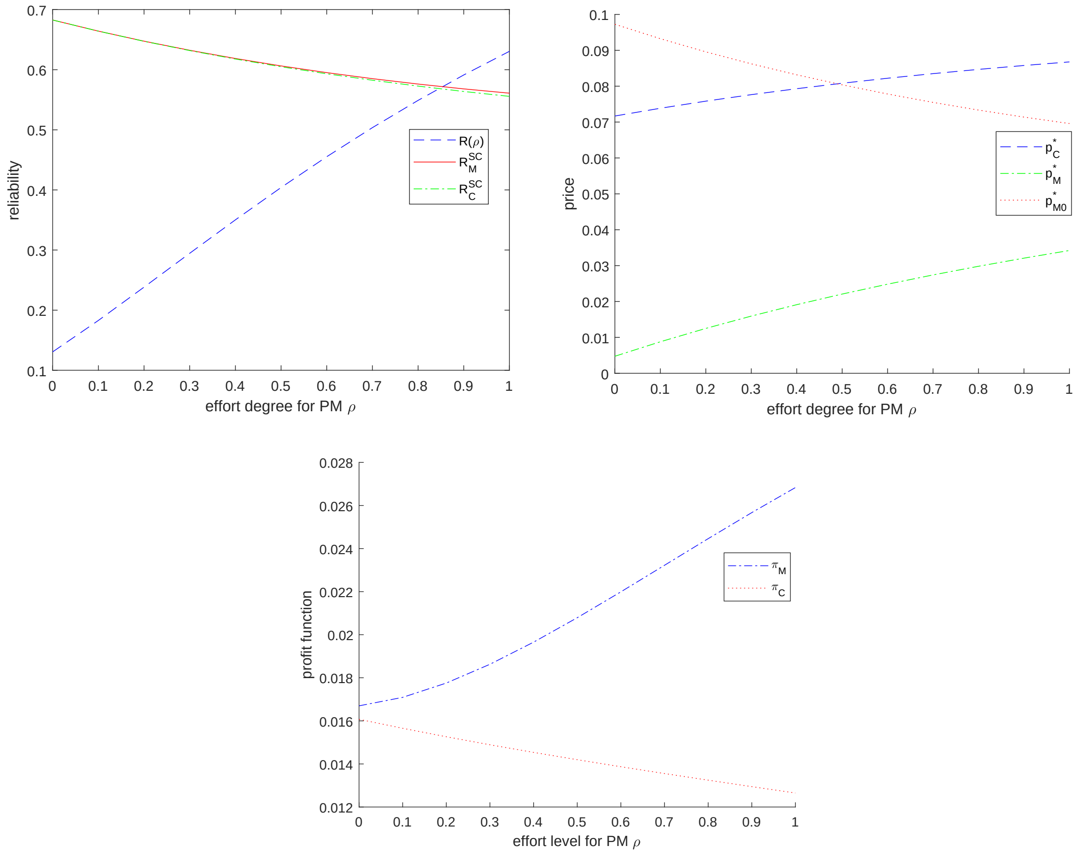

Figure 6. The optimal quantity of SC scheme under competitor dominance and $\delta > \mathfrak{L}$.

capabilities and increasing profitability. Opening the platform to competitors involves charging a royalty fee while intensifying market competition. Consequently, a sequential game unfolds between the manufacturer and cooperative competitors across three scenarios: traditional competition (TC), unicorn competition (UC), and shared competition (SC). Equilibrium pricing, encompassing new product prices and after-sales service costs, determines external incentives. These prices help derive profit and reliability functions for both the manufacturer and cooperative competitors under each competition scheme. Moreover, the effort allocated to preventive maintenance aligns with internal product degradation mechanisms, as reflected in equilibrium reliability functions for both parties. This approach establishes a connection between product degradation processes and sequential game theory, driven by the level of effort in preventive maintenance.

There are still some shortcomings in the article. The range of values for parameters ρ (effort degree for PM) and ε (investment level), and their impact on product reliability and degradation process are still interesting issues that need to be considered in future research.

Appendix A

The proof of Theorem 2.1

First of all, we assume that $\{W_1(t), t \geq 0\}$ (similar to the case of $\{W_0(t), t \geq 0\}$) follow positive stationary Gamma process, which possesses the following properties:

(i) $W_1(0) = 0$;

(ii) $W_1(t_2) - W_1(t_1) \sim Ga(\alpha_1(t_2 - t_1), \beta)$;

(iii) $\{W_1(t), t \geq 0\}$ is monotonically increasing and has independent increments, which indicates that $W_1(t_3) - W_1(t_2)$ and $W_1(t_2) - W_1(t_1)$ are independent of each other for any $0 \leq t_1 \leq t_2 \leq t_3$ within its field.

Thus, the probability density function (pdf) of $W_1(t_2) - W_1(t_1)$ can be characterized as

$$f_{W_1(t_2)-W_1(t_1)}(x) = \frac{\beta^{\alpha_1(t_2-t_1)}}{\Gamma(\alpha_1(t_2 - t_1))} x^{\alpha_1(t_2-t_1)-1} e^{-\beta x}.$$

Especially, if taking $t_1 = 0$, $f_{W_1(t_2)-W_0(t_1)}(x)$ reduces to $f_{W_1(t)}(x) = \frac{\beta^{\alpha_1 t}}{\Gamma(\alpha_1 t)} x^{\alpha_1 t-1} e^{-\beta x}$.

Since the cumulative distribution function (cdf) of $kW_1(t)$ is $P\{kW_1(t) \leq x\} = P\{W_1(t) \leq \frac{x}{k}\}$ $(k > 0)$, the pdf of $kW_1(t)$ becomes

$$f_{kW_1(t)}(x) = \frac{d}{dx}P\{kW_1(t) \leq x\} = \frac{(\frac{\beta}{k})^{\alpha_1 t}}{\Gamma(\alpha_1 t)}x^{\alpha_1 t-1}e^{-\frac{\beta}{k}x}.$$

It indicates $kW_1(t) \sim Ga(\alpha_1 t, \frac{\beta}{k})$. Likewise, $\frac{W_1(t)}{1+\rho} \sim Ga(\alpha_1 t, (1+\rho)\beta)$. This means Equation (2) holds.

Secondly, let $W(t) = W_0(t) + \frac{W_1(t)}{1+\rho}$. The non-negative properties of $W_0(t)$ and $W_1(t)$ imply $W(t)$ is non-negative. Then, if $x < 0$, the pdf of $W(t)$ becomes $f_{W(t)}(x) = 0$. If $x > 0$, the corresponding pdf has, by employing the convolution formula of pdf, derived as follows:

$$\begin{aligned}
&f_{W(t)}(x) = \int_0^x f_{W_0(t)}(x-y)f_{\frac{W_1(t)}{1+\rho}}(y)dy \\
&= \int_0^x \frac{\beta^{\alpha_0 t}}{\Gamma(\alpha_0 t)}(x-y)^{\alpha_0 t-1}e^{-\beta(x-y)}\frac{((1+\rho)\beta)^{\alpha_1 t}}{\Gamma(\alpha_1 t)}y^{\alpha_1 t-1}e^{-(1+\rho)\beta y}dy \\
&= \frac{(1+\rho)^{\alpha_1 t}\beta^{(\alpha_0+\alpha_1)t}}{\Gamma(\alpha_0 t)\Gamma(\alpha_1 t)}\int_0^x (x-y)^{\alpha_0 t-1}y^{\alpha_1 t-1}e^{-\beta x-\rho\beta y}dy \\
&= \frac{(1+\rho)^{\alpha_1 t}\beta^{(\alpha_0+\alpha_1)t}e^{-\beta x}x^{(\alpha_0+\alpha_1)t-1}}{\Gamma(\alpha_0 t)\Gamma(\alpha_1 t)} \\
&\quad \int_0^1 (1-u)^{\alpha_0 t-1}u^{\alpha_1 t-1}e^{-\rho\beta x u}du.
\end{aligned}$$

This result means Equation (3) holds.

The proof of Theorem 2.2

Proof. Scenario A. Assume $\frac{R_M}{R_C} > \frac{1}{\delta} - 1$.

When customers choose to buy the product and the manufacturer's after-sales service, then they should derive the higher non-negative surplus from the option, that is

$$\begin{cases} u_M \geq u_C, \\ u_M \geq 0. \end{cases}$$

Solving the inequality, we derive the following result

$$\theta \geq \max\{\frac{(R_C-R_M)\zeta+p_M-p_C}{\delta R_M-(1-\delta)R_C}, \frac{(1-R_M)\zeta+p_{M_0}+p_M}{\delta R_M}\}.$$

When customers choose to buy the product and the competitor's after-sales service, the following inequality should hold

$$\begin{cases} u_C \geq u_M, \\ u_C \geq 0. \end{cases}$$

Solving the inequality, we derive the following two connected inequalities

$$\frac{(1-R_C)\zeta+p_{M_0}+p_C}{(1-\delta)R_C} \leq \theta \leq \frac{(R_C-R_M)\zeta+p_M-p_C}{\delta R_M-(1-\delta)R_C}.$$

Given both $\frac{(R_C-R_M)\zeta+p_M-p_C}{\delta R_M-(1-\delta)R_C} \geq \frac{(1-R_M)\zeta+p_{M_0}+p_M}{\delta R_M}$ and $\frac{(R_C-R_M)\zeta+p_M-p_C}{\delta R_M-(1-\delta)R_C} \geq \frac{(1-R_C)\zeta+p_{M_0}+p_C}{(1-\delta)R_C}$, we can derive the same result as follows.

$$\begin{aligned}
&\delta \leq R_C\left[(1-R_M)\zeta+p_{M_0}+p_M\right] / \\
&\quad \left(R_C\left[(1-R_M)\zeta+p_{M_0}+p_M\right]+R_M\left[(1-R_C)\zeta+p_{M_0}+p_C\right]\right)
\end{aligned}$$

Letting $\mathfrak{L} = \frac{R_C[(1-R_M)\zeta+p_{M_0}+p_M]}{R_C[(1-R_M)\zeta+p_{M_0}+p_M]+R_M[(1-R_C)\zeta+p_{M_0}+p_C]}$, then consumers of type

$$\theta \in [\frac{(R_C-R_M)\zeta+p_M-p_C}{\delta R_M-(1-\delta)R_C}, 1]$$

will choose (i) when $\delta \leq \mathfrak{L}$, while the consumers of type

$$\theta \in \left[\frac{(1-R_C)\zeta+p_{M_0}+p_C}{(1-\delta)R_C}, \frac{(R_C-R_M)\zeta+p_M-p_C}{\delta R_M-(1-\delta)R_C}\right]$$

will choose (ii) when $\delta \leq \mathfrak{L}$ (see Figure 1 (a)).

In Figure 7 (a), $\theta_1 = \frac{(R_C-R_M)\zeta+p_M-p_C}{\delta R_M-(1-\delta)R_C}$ is the indifferent point between choosing the manufacturer's after-sales service and the competitor's one, and $\theta_2 = \frac{(R_C-R_M)\zeta+p_M-p_C}{\delta R_M-(1-\delta)R_C}$ is the indifferent point between choosing the competitor's after-sales service and doing nothing.

Similarly, when $\delta > \mathfrak{L}$, the consumers of type

$$\theta \in [\frac{(1-R_M)\zeta+p_{M_0}+p_M}{\delta R_M}, 1]$$

will choose (i) and nobody choose (ii) (see Figure 7 (b)).

In Figure 7 (b), $\theta_1 = \frac{(1-R_M)\zeta+p_{M_0}+p_M}{\delta R_M}$ is the indifferent point between choosing the manufacturer's service and the competitor's service.

Thus, the product demand, manufacturer's service demand and manufacturer's service demand are readily obtained as the first part of Theorem 2.2.

Scenario B. Assume $\frac{R_M}{R_C} < \frac{1}{\delta} - 1$.

Similar to Scenario A, we can obtain two indifferent points below: $\theta_1 = \frac{(R_C-R_M)\zeta+p_M-p_C}{\delta R_M-(1-\delta)R_C}$, which is the indifferent point between choosing the manufacturer's after-sales service and the competitor's service, and $\theta_2 = \frac{(1-R_M)\zeta+p_{M_0}+p_M}{\delta R_M}$, which is the different point between choosing the manufacturer's after-sales service and doing nothing. Thus, the second part of Theorem 2.2 holds.

The proof of Lemma 3.2

Figure 7. The consumers' options when $\frac{R_M}{R_C} \geq \frac{1}{\delta} - 1$.

Proof. Under the monopolistic competition scheme, the manufacturer establishes the platform and improves its PM but the competitor still performs the regular PM, in other words, the 'unicorn competition' scheme remains between manufacturer and competitor. Thus, the manufacturer's profit function is

$$\begin{aligned}\pi_M =& [1 - \frac{(1-R_C)\zeta + p_{M_0} + p_C}{(1-\delta)R_C}](p_{M_0} - c_M)\\ &+ [1 - \frac{(R_C - R_M)\zeta + p_M - p_C}{\delta R_M - (1-\delta)R_C}]\\ &\times [p_M - (1-R_M)\zeta - \frac{\eta_M \rho_M^2}{1+\varepsilon} - \varepsilon H] - K\varepsilon^2,\end{aligned}$$

and the competitor's profit function yields

$$\begin{aligned}\pi_C =& [\frac{(R_C - R_M)\zeta + p_M - p_C}{\delta R_M - (1-\delta)R_C}\\ &- \frac{(1-R_C)\zeta + p_{M_0} + p_C}{(1-\delta)R_C}]\\ &\times [p_C - (1-R_C)\zeta - \eta_C \rho_C^2].\end{aligned}$$

Similar to Lemma 3.1, we can obtain the following equilibrium prices:

$$\begin{aligned}p_M^* =& \frac{1}{2}[\delta R_M - (1-\delta)R_C + (1-R_C)\zeta + p_C + \frac{\eta_M \rho_M^2}{1+\varepsilon} + \varepsilon H],\\ p_{M_0}^* =& \frac{1}{2}[(1-\delta)R_C - (1-R_C)\zeta - p_C + c_M],\\ p_C^{*2} =& \frac{(1-\delta)R_C}{2\delta R_M}[2(1-R_M)\zeta + c_M + \frac{\eta_M \rho_M^2}{1+\varepsilon} + \varepsilon H]\\ &+ \frac{\eta_C \rho_C^2 - c_M}{2}.\end{aligned}$$

where p_M^*, $p_{M_0}^*$, and p_C^{*2} represent the equilibrium prices of p_M, p_{M_0}, and p_C, respectively.

Substituting these equilibrium prices into the profit functions of the manufacturer and cooperative competitor, we can obtain Equations 20 and 20.

Furthermore, taking the derivative of function of p_{M_0} with respect to ρ_M and $\frac{\partial p_{M_0}}{\partial \rho_M} = 0$, we can obtain the results as Equations 21 and 22.

The proof of Lemma 3.3

Proof. Under SC scheme, the manufacturer establishes and opens the IIoT-based platform, and the competitor accesses it. Consequently, both the manufacturer and competitor improve PM, and thus the manufacturer's profit function is

$$\begin{aligned}\pi_M =& [1 - \frac{(1-R_C)\zeta + p_{M_0} + p_C}{(1-\delta)R_C}](p_{M_0} - c_M)\\ &+ [1 - \frac{(R_C - R_M)\zeta + p_M - p_C}{\delta R_M - (1-\delta)R_C}]\\ &[p_M - (1-R_M)\zeta - \frac{\eta_M \rho_M^2}{1+\varepsilon} - \varepsilon H] - K\varepsilon^2 + \varphi.\end{aligned} \tag{42}$$

The competitor's profit function yields

$$\begin{aligned}\pi_C =& [\frac{(R_C - R_M)\zeta + p_M - p_C}{\delta R_M - (1-\delta)R_C} - \frac{(1-R_C)\zeta + p_{M_0} + p_C}{(1-\delta)R_C}]\\ &[p_C - (1-R_C)\zeta - \frac{\eta_C \rho_C^2}{1+\varepsilon}] - \varphi.\end{aligned} \tag{43}$$

Similar to the operation of Lemma 3.2, we can readily obtain the equilibrium prices below

$$\begin{aligned}p_M^* =& \frac{1}{2}[\delta R_M - (1-\delta)R_C + (1-R_C)\zeta + \frac{\eta_M \rho_M^2}{1+\varepsilon} + \varepsilon H + p_C],\\ p_{M_0}^* =& \frac{1}{2}[(1-\delta)R_C - (1-R_C)\zeta - p_C + c_M],\\ p_C^{*3} =& \frac{(1-\delta)R_C}{2\delta R_M}[2(1-R_M)\zeta + c_M + \frac{\eta_M \rho_M^2}{1+\varepsilon} + \varepsilon H]\\ &+ \frac{\eta_C \rho_C^2}{2(1+\varepsilon)} - \frac{c_M}{2}.\end{aligned}$$

Substituting the sales prices of p_M^*, $p_{M_0}^*$, and p_C^*, the profits of the manufacturer and cooperative competitor reduce to Equations 25 and 21.

Furthermore, taking the derivative of function of p_{M_0} with respect to ρ_M and $\frac{\partial p_{M_0}}{\partial \rho_M} = 0$, we can obtain the reliability function with equilibrium level of PM efforts of the manufacturer as

$$\ln R_M = \int_0^{\rho_M^*} \frac{2x\eta_M}{x^2\eta_M + (1+\varepsilon)(2\zeta + c_M + \varepsilon H)} dx,$$

or equivalently,

$$R_M^{SC} = R(t, \alpha_0, \alpha_1, \beta, 0)[\frac{(\rho_M^*)^2 \eta_M}{(1+\varepsilon)(2\zeta + c_M + \varepsilon H)} + 1], \tag{44}$$

where $R_M^{'}$ represents the derivative of R_M and ρ_M^* is the optimal effort degree in PM.

Similarly, Equations 26 and 27 hold.

The proofs of Lemmas 3.4- 3.6

Proof. The proof of Lemma 3.4- 3.6 is similar to Lemmas 3.1- 3.3, and the proof process is omitted here.

Data Availability Statement

Data will be made available on request.

Funding

This work was supported in part by the National Natural Science Foundation of China under Grant 72371030 and Grant 72001026; in part by the Natural Science Foundation of Inner Mongolia Autonomous Region under Grant 2025MS07010.

Conflicts of Interest

The authors declare no conflicts of interest.

Ethical Approval and Consent to Participate

Not applicable.

References

[1] Agrawal, V. V., Kavadias, S., & Toktay, L. B. (2016). The limits of planned obsolescence for conspicuous durable goods. *Manufacturing & Service Operations Management*, *18*(2), 216-226. [CrossRef]

[2] August, T., Chen, W., & Zhu, K. (2021). Competition among proprietary and open-source software firms: The role of licensing in strategic contribution. *Management Science*, *67*(5), 3041-3066. [CrossRef]

[3] Berrade, M. D., Calvo, E., & Badía, F. G. (2023). Maintenance of systems with critical components. Prevention of early failures and wear-out. *Computers & Industrial Engineering*, *181*, 109291. [CrossRef]

[4] Bian, Y., Xie, J., Archibald, T. W., & Sun, Y. (2019). Optimal extended warranty strategy: Offering trade-in service or not?. *European Journal of Operational Research*, *278*(1), 240-254. [CrossRef]

[5] Fu, W., Atasu, A., & Tereyağoğlu, N. (2022). Warranty length, product reliability, and secondary markets. *Manufacturing & Service Operations Management*, *24*(4), 2240-2255. [CrossRef]

[6] Automotive aftersales market value in selected countries 2020| Statista. (2021, July 1). Statista. Retrieved from https://www.statista.com/statistics/1267042/automotive-aftersales-market-value-in-selected-countries/.

[7] Hu, S., Zhu, S. X., & Fu, K. (2023). Optimal trade-in and refurbishment strategies for durable goods. *European Journal of Operational Research*, *309*(1), 133-151. [CrossRef]

[8] Hu, S., & Tang, Y. (2024). Impact of product sharing and heterogeneous consumers on manufacturers offering trade-in programs. *Omega*, *122*, 102949. [CrossRef]

[9] Huang, X., Atasu, A., Tereyağoğlu, N., & Toktay, L. B. (2023). Lemons, trade-ins, and certified pre-owned programs. *Manufacturing & Service Operations Management*, *25*(2), 737-755. [CrossRef]

[10] Jiang, B., & Zhang, X. (2011). How does a retailer's service plan affect a manufacturer's warranty? *Management Science*, *57*(4), 727-740. [CrossRef]

[11] Liang, X. J., Cui, L. R., & Wang, R. T. (2024) Non-renewable warranty cost analysis for dependent series configuration with distinct warranty periods. *Reliability Engineering and System Safety*, *246*, 110074. [CrossRef]

[12] Mitra, A. (2021). Warranty parameters for extended two-dimensional warranties incorporating consumer preferences. *European Journal of Operational Research*, *291*, 525-535. [CrossRef]

[13] Olsen, T. L., & Tomlin, B. (2020). Industry 4.0: Opportunities and challenges for operations management. *Manufacturing & Service Operations Management*, *22*(1):113-122. [CrossRef]

[14] Oraiopoulos, N, Ferguson, M. E., & Toktay, L. B. (2012). Relicensing as a secondary market strategy. Management Science, 58(5), 1022-1037. [CrossRef]

[15] Pei, J., Yan, P., & Kumar, S. (2023). No permanent friend or enemy: Impacts of the IIoT-based platform in the maintenance service market. *Management Science*, *69*(11), 6800-6817. [CrossRef]

[16] Peng, S. Z., Jiang, W., Huang, W. P., & Luo, Q. L. (2024). The impact of gamma usage processes on preventive maintenance policies under two-dimensional warranty. Reliability Engineering and System Safety, 242, 109743. [CrossRef]

[17] ReportLinker. (2022, February 9). Medical equipment maintenance global market report 2022. Yahoo Finance. Retrieved from https://finance.yahoo.com/news/medical-equipment-maintenance-global-market-114600182.html.

[18] Ruan, Y., Luo, X., Ding, X., & Dai, A. (2023). Non-periodic preventive maintenance policies for complex multi-component products sold with loss-based two-dimensional warranties. *Computers & Industrial Engineering*, *185*, 109626. [CrossRef]

[19] Sun, M. Y., Ng, C. T., Yang, L., & Zhang, T. H. (2024). Optimal after-sales service offering strategy: Additive manufacturing, traditional manufacturing, or hybrid? *International Journal of Production Economics*, *268*, 109116. [CrossRef]

[20] VMR. (2022). Global aviation MRO market size by

organization type, by sevice type, by geographic scope and forecast. Retrieved from, http://www.verifiedmarketresearch.com-/product/aviation-mro-market/.

[21] Waldman, M. (2003). Durable goods theory for real world markets. *Journal of Economic Perspectives*, *17*(1), 131-154. [CrossRef]

[22] Wang, X. L., & Xie, W. (2018). Two-dimensional warranty: A literature review. *Proceedings of the Institution of Mechanical Engineers Part O-Journal of Risk and Reliability*, *232*(2), 284-307. [CrossRef]

[23] Wu, S. M. (2014). Construction of asymmetric copulas and its application in two-dimensional reliability modelling. *European Journal of Operational Research*, *238*, 476-485. [CrossRef]

[24] Wu, S. (2024). A copula-based approach to modelling the failure process of items under two-dimensional warranty and applications. *European Journal of Operational Research*, *314*(3), 854-866. [CrossRef]

[25] Xu, J., Liang, Z. L., Li, Y. F., & Wang, K. B. (2021). Generalized condition-based maintenance optimization for multi-component systems considering dependency and imperfect maintenance. *Reliability Engineering & System Safety*, *211*, 107592. [CrossRef]

[26] Japan: Extended warranty service sales value 2030 | Statista. (2022, September 26). Statista. Retrieved from, https://www.statista.com/statistics/1350433/japan-extended-warranty-service-sales-value/.

[27] Zhang, S., Tong, X., & Jin, X. (2023). Contract design and comparison under the opportunity cost of working capital: Buyback vs. revenue sharing. *European Journal of Operational Research*, *309*(2), 845-856. [CrossRef]

[28] Zhao, X., Liu, B., Xu, J., & Wang, X. L. (2023). Imperfect maintenance policies for warranted products under stochastic performance degradation. *European Journal of Operational Research*, *308*(1), 150-165. [CrossRef]

Xiaojun Liang received the Ph.D. degree in pure mathematics from the School of Mathematical Sciences, Sichuan Normal University, Chengdu, China, in 2017. He is currently an Associate Professor with the School of Mathematics & Physics, Hulunbuir College, China and engages in Post-Doctor Research Station, Qingdao University, China in 2022 to 2024. His recent research interests are system reliability, quality and reliability engineering, optimization, game theory, and applications of probability and statistics. (Email: xiaojunliang@hlbec.edu.cn)

Li Yang received the Ph.D. degree in systems engineering from Beihang University, Beijing, China, in 2008. He is currently an Associate Professor with the School of Reliability and System Engineering, Beihang University. Prior to joining Beihang University, he was a Postdoctoral Fellow with the Department of Mechanical of Industrial Engineering, University of Toronto, ON, Canada. His research interests include intelligent maintenance, system reliability, prognostic, and risk analysis. (Email: yanglirass@buaa.edu.cn)

Qingan Qiu received the Ph.D. degree in management science and engineering from the School of Management and Economics, Beijing Institute of Technology, Beijing, China, in 2020. He is currently an Associate Professor with the School of Management, Beijing Institute of Technology. His research interests include stochastic modeling, decision-making under uncertainty, and risk analysis. (Email: qiu_qingan@bit.edu.cn)