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REVIEW ARTICLE

A Review of Reliability Assessment for Consecutive *k-out-of-n* Systems

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Abstract

As engineering systems grow in scale and complexity, the classical consecutive *k-out-of-n* model has evolved significantly to address the challenges of component dependence and dynamic maintenance. This paper presents a comprehensive survey of these advancements, categorized into model extensions and assessment methodologies. We synthesize key variants adapted for practical scenarios, including consecutive *k-within-m-out-of-n* systems, *m-consecutive-k-out-of-n* systems, (n, f, k) and $\langle n, f, k\rangle$ systems, sparse d systems, and complex models accounting for component dependence and repair mechanisms. Regarding reliability assessment, this paper evaluates the transition from analytical methods to scalable computational approaches like System Signatures and Universal Generating Functions (UGF). Furthermore, we identify a critical shift towards data-driven methodologies, highlighting how Graph Neural Networks (GNNs) are transforming the analysis of large-scale, multi-state systems. The review concludes by outlining a strategic roadmap for future research, emphasizing the integration of deep learning to achieve real-time reliability assessment and intelligent fault management.

Keywords: consecutive *k-out-of-n*, reliability assessment, multi-state systems, universal generating functions.

1 Introduction

A consecutive *k-out-of-n* system is a typical structural reliability model. The consecutive *k-out-of-n*: G system is defined by the successful operation of at least k consecutive components, while the consecutive *k-out-of-n*: F system is defined by k consecutive failures. This model is suitable for describing the operating characteristics of linear or circular structural systems, such as communication links, production lines, and multi-stage conveying systems. As a typical consecutive system, the study of consecutive *k-out-of-n* systems has become an active research area in reliability engineering since the 1980s. The fundamental premise—that system status depends on the state of k consecutive components—is widely applicable to engineering scenarios such as oil pipelines, communication networks, and nuclear facility monitoring.

Eryilmaz et al. [1] investigated the dynamic reliability evaluation of the consecutive-*k-within-m-out-of-n*: F system, which is an important extension of generalized consecutive-system models. Early research laid the

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theoretical framework for system classification (linear/circular, F/G type) and basic reliability assessment (recursive methods, combinatorial analysis). In recent years, with the increasing complexity, multi-state nature, and intelligence of engineering systems, traditional models and methods have faced challenges in meeting the new demands of heterogeneous components, complex dependence, and dynamic maintenance.

To more accurately characterize the reliability behavior of real-world systems, scholars have proposed a series of extended models, such as consecutive-*k-within-m-out-of-n*, *m-consecutive-k-out-of-n*, (n, f, k) and $\langle n, f, k\rangle$ systems, and sparse d systems. These system models retain the core logic of traditional systems while incorporating features such as multi-dimensional failure criteria, complex dependence structures, and repair mechanisms, significantly improving their applicability and practicality.

Simultaneously, reliability assessment methods have evolved from traditional analytical methods to modern computational and data-driven techniques. Markov chains, recursive methods, Binary Decision Diagrams (BDD), system signatures, and Universal Generating Functions (UGF) provide effective reliability analysis tools for systems with different structures. In recent years, artificial intelligence technologies such as machine learning and graph neural networks have also been introduced to this field, offering new possibilities for the efficient reliability assessment of large-scale complex systems.

The structure of this paper is as follows: Section 2 introduces the related system models and their extensions; Section 3 presents reliability assessment methods; and Section 4 summarizes the content of this paper and proposes future research directions.

Figure 1 illustrates a simple example of a linear consecutive *k-out-of-n* system. Unlike traditional *k-out-of-n* systems, the performance of a consecutive system depends not only on the number of working or failed components, but also on whether these components appear in consecutive positions. Consecutive *k-out-of-n* systems are generally classified into two types: consecutive *k-out-of-n*: G systems and consecutive *k-out-of-n*: F systems. Figure 1 also illustrates the basic operating logic of these two types of systems. For a 3-out-of-5: G system, the system functions if there are at least three consecutive working components; otherwise, the system is considered failed. For a 2-out-of-5: F system, the system can still work if there are no two consecutive failed components. However, the system fails when any two consecutive components fail.

2 Related Systems

Consecutive *k-out-of-n* systems have been widely used in communications, energy, the nuclear industry, and other fields. However, the structural complexity and diverse dependence in practical engineering have prompted the development of extended models. These variants extend the traditional scope by introducing multi-dimensional failure criteria and repair mechanisms while retaining the core consecutive logic. This section will systematically review several consecutive *k-out-of-n* system variants that have been extensively studied in recent years, and summarize their core definitions, research methods, and main achievements.

2.1 Consecutive k-within-m-out-of-n Systems

The core failure criterion for the consecutive *k-within-m-out-of-n*: F system is defined as follows: the system fails if and only if, within any consecutive m-component window of the n linearly or circularly arranged components, at least k components fail. The corresponding G system requires that at least k components function correctly within every consecutive m-component window. The model has significant applications in fields such as quality control and radar detection. It degenerates into the traditional consecutive *k-out-of-n* system when m = k, and becomes equivalent to the *k-out-of-n* system when m = n. Scholars have established analytical relationships between the reliability of the G-type and F-type systems, thereby enhancing the design flexibility of this model.

2.2 m-Consecutive-*k-out-of-n* Systems

The m-consecutive-*k-out-of-n*: F system fails if and only if at least m non-overlapping runs of k consecutive failed components occur within a sequence of n linearly or circularly arranged components. This model is particularly suited to scenarios requiring consideration of multiple failure chain effects, such as engineering systems maintained via spare parts or repairs, where failure risk depends more on the number of consecutive failure groups than on the total number of individual component failures. It degenerates into the traditional consecutive *k-out-of-n* system when m = 1 and is equivalent to the m-out-of-n system when $k = 1$.

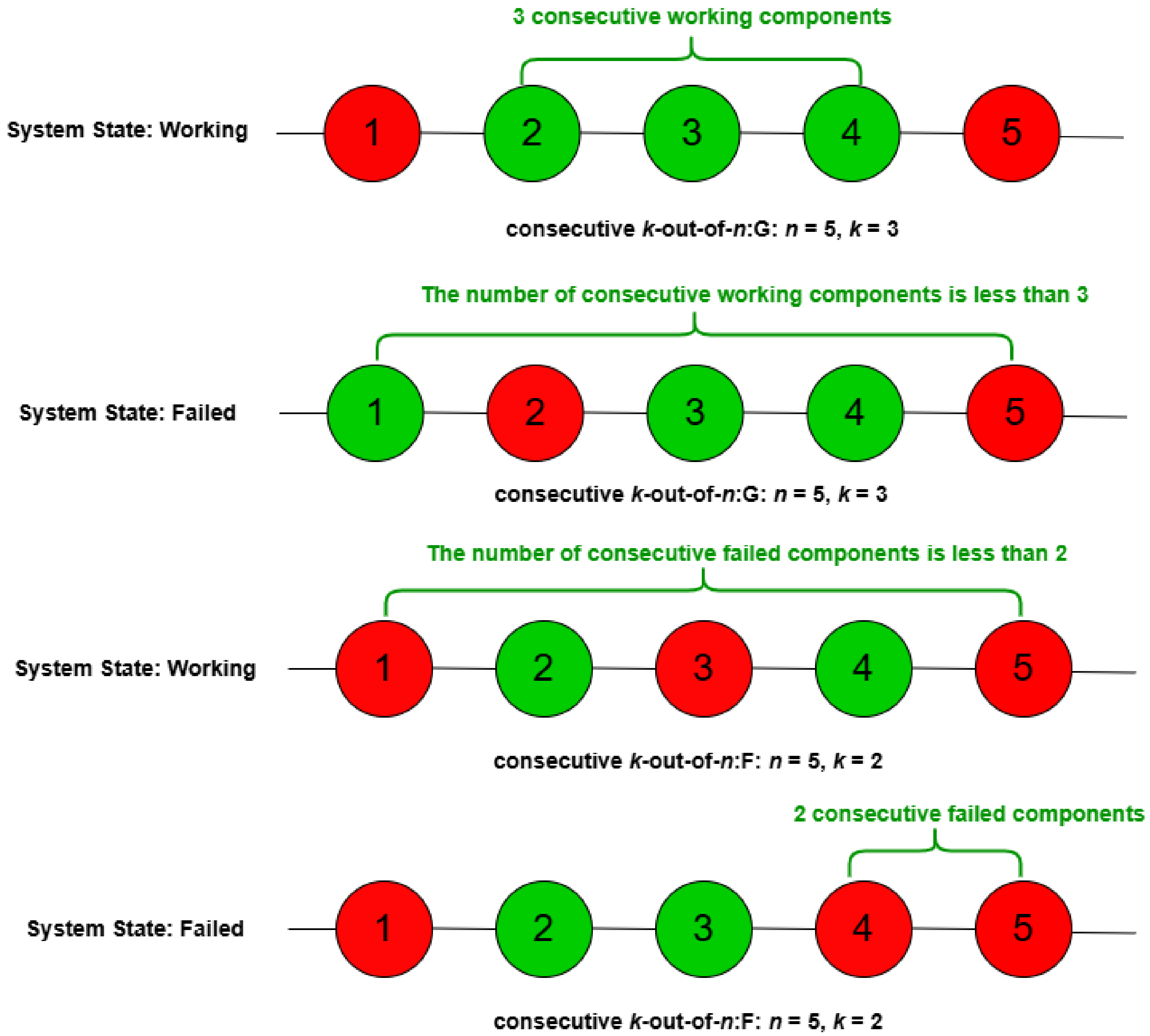

Figure 1. Illustration of Consecutive *k-out-of-n* System.

First proposed by Griffith [2], this system generalizes both the standard consecutive *k-out-of-n*: F system (when m = 1) and the m-out-of-n: F system (when $k = 1$) [3–5], proving valuable in maintenance contexts where the sequence of failing components is critical. In recent research, Eryilmaz et al. [6] derived reliability formulas for linear and circular systems, representing an early theoretical contribution to the "m-extension" framework. Zhu et al. [7] investigated the reliability and importance measures of an m-consecutive-k/l-out-of-n system with non-homogeneous Markov-dependent components, thereby extending the study of m-consecutive systems to dependent components. Wang et al. [8] introduced heuristic and evolutionary algorithms from optimization theory to address design optimization and maintenance scheduling problems for consecutive systems.

2.3 (n, f, k) and $\langle n, f, k\rangle$ Systems

The (n, f, k) and $\langle n, f, k\rangle$ systems are composite systems that combine two criteria: the "total number of failures" and the "number of consecutive failures," distinguishing them from traditional single-criterion models. The failure logic of the (n, f, k) system corresponds to an OR relationship: the system fails if the total number of failed components is at least f or if there are at least k consecutive failed components. Conversely, the failure logic of the $\langle n, f, k\rangle$ system corresponds to an AND relationship: the system fails only when the total number of failed components is at least f and there are at least k consecutive failed components. These models have been applied to practical scenarios such as automated banking systems and industrial furnaces, providing a more comprehensive description of their failure mechanisms. Chang et al. [9] first applied the (n, f, k): F system to automated banking systems, while Zuo et al. [10] extended it to the reliability assessment of furnace systems. Cui et al. [11] derived recursive reliability formulas for these two fundamental systems and their dual systems under independent non-identically distributed (INID) components.

Demir [12] obtained explicit reliability formulas for Markov-dependent components. Subsequently, Eryilmaz [13] analyzed the system under exchangeable dependence. Eryilmaz et al. [14] extended the (n, f, k) system to include weighted components. More recently, Cui et al. [15] combined the two criteria into a "Class II composite system," providing analytical and recursive reliability expressions for combined scenarios. This reflects the recent trend of expanding from single models towards multi-criteria combinations and shared components.

2.4 Sparse d Systems

Sparse d systems extend the traditional model by introducing a sparsity parameter d, which limits the number of components between consecutively failed (or operational) components. For sparse d-F systems, the failure criterion is that at least k consecutive failed components exist, and no more than d operational components lie between any two adjacent failed components. The corresponding sparse d-G system requires at least k consecutive operational components, subject to the sparse constraint. This model was further extended to multi-state scenarios. In multi-state sparse d systems, the sparsity parameter dj varies with system state j, requiring at least kl consecutive components in state $l \geq j$, with no more than dl operational components between any two adjacent failed components.

Proposed by Zhao et al. [16], this type of system generalizes the standard consecutive-k system (when $d = 0$) and is suitable for modeling cold reserve repairable systems and communication transmission systems. Scholars have extended this concept to develop sparse dm-consecutive-*k-out-of-n*: F systems and sparse d-(n, f, k): F systems. Zhu et al. [17] explicitly incorporated sparse d into the m-consecutive-*k-out-of-n* framework, demonstrating the method's feasibility for engineering-scale problems through numerical examples. More recently, Yi et al. [18] extended sparse d to scenarios involving "multiple systems sharing or operating in parallel."

2.5 Complex Dependent Systems

In practical engineering, components often exhibit complex dependence rather than mutual independence, motivating researchers to investigate system models accounting for such dependence. These models focus on failure mechanisms involving component dependence, which significantly improves their adaptability to practical engineering applications.

Fang et al. [19] used copula function to establish joint distributions of component degradation and failure times, systematically explaining how copula function links marginal degradation processes to calculate reliability and failure probabilities at the system level. Yi et al. [20] noted that for complex dependent systems with component dependence, system reliability can be decomposed using survival signatures, and the joint distribution of components can be expressed by a copula function.

For weighted *k-out-of-n* systems with component dependence, Ozkut [21] adopted copula function to express the joint distribution and correlated survival functions between two classes of components. They also analyzed the influence of dependence strength on reliability. Yin et al. [22] investigated scenarios where subsystems share components, extending the analysis to linear sawtooth and circular polygonal structures. Rykov et al. [23] found that in *k-out-of-n* systems where component failures trigger load sharing, the increased load on surviving components shortens their remaining lifetimes. In recent years, scholars have studied systems operating in stochastic environments, describing environmental states through continuous-time Markov chains. They established that component failure probabilities are dependent on these environmental states. These studies introduce predictive concepts, achieving a comprehensive assessment of reliability, remaining useful life (RUL), and availability by integrating environmental data with observation updates.

2.6 Repairable Systems

Traditional studies on consecutive *k-out-of-n* systems mainly focused on non-repairable settings, whereas many practical engineering systems are repairable. Therefore, repairable consecutive systems have gradually become an important topic in reliability research. Gökdere et al. [24] investigated the time-dependent reliability of repairable consecutive-*k-out-of-n*: F systems. In addition, related studies have considered *k-out-of-n* systems in random environments and integrated prognostic analysis with reliability-based measures [25].

3 Reliability Assessment Methods

Different reliability assessment methods are suitable for different modeling assumptions and engineering requirements. To make the review more practical for researchers, the methods surveyed in this section are compared from several perspectives, including component dependence, binary versus multi-state modeling, repairable versus non-repairable settings, computational scalability, and typical application scenarios.

3.1 Markov Chain Methods

The core of the Markov chain methods is to define the system state as a stochastic process. The system's future state depends solely on the current state and is independent of past states (the Markov property). This method models the system state evolution as a stochastic state transition process and analyzes the system's reliability characteristics over time by defining the state space and the transition rate matrix.

Fu [26] first proposed using this method to study the reliability of consecutive *k-out-of-n* systems. Li et al. [27] extended this method to repairable circular *k-out-of-n*: F retrial systems. They defined the generalized transition probability of the retrial system and the priority maintenance rule for key components. Furthermore, they derived the availability and reliability indices of repairable circular consecutive *k-out-of-n*: F retrial systems using the supplementary variable method and the Laplace transform. Related studies also extended Markov-type modeling ideas to repairable consecutive systems with more complex maintenance assumptions. For example, Guan et al. [28] investigated a repairable consecutive-*k-out-of-n*: F system with fuzzy states and derived important reliability and availability measures under repairman-based maintenance mechanisms.

3.2 Recursive Methods

The recursive method breaks the reliability problem of the entire system down into two or more mutually exclusive and simpler subproblems by conditioning on the working state of the first (or a key) component in the system. The system size of these subproblems is reduced, or the failure condition is modified. This process is repeated until the problem is decomposed into basic cases with known solutions. Then, the reliability of the original system is calculated in reverse using these basic cases.

Zuo [29] proposed a recursive equation to analyze the reliability of linear and circular systems. The system model proposed by Zuo is a generalization of linear consecutive *k-out-of-n* systems. In order to improve the calculation speed of this recursive equation, Kuo et al. [30] proposed using a matrix representation of the recursive equation to calculate the k-th order Fibonacci numbers. Byun et al. [31] proposed a fast matrix exponentiation method to accelerate the reliability calculation of linear consecutive *k-out-of-n* systems based on the matrix defined by Gries and Levin. Ghoraf [32] evaluated the reliability of the circular r-consecutive-*k-out-of-n*: F system using the reliability formula for linear systems. Kamalja [33] proposed a computational method emphasizing algorithmic implementation for generalized *k-out-of-n* systems. They used a recursive/block strategy to define multidimensional tables and iterative update rules, thereby transforming complex multi-event dependence situations into a stepwise computation process. This approach is conducive to programming implementation and scale expansion. The proposed model degenerates into traditional *k-out-of-n*, consecutive *k-out-of-n*, (n, f, k), serial, and parallel systems, covering single-component, multi-module, and single/multi-failure criteria scenarios. Furthermore, it can calculate not only static reliability but also extend to time-domain scenarios where component lifetimes are exponentially distributed. From a broader perspective, recursive reliability evaluation has long been recognized as an efficient exact-computation strategy for *k-out-of-n* systems. A classical example is the recursive algorithm proposed by Jain et al. [34], which provides a simple and memory-efficient exact procedure for evaluating *k-out-of-n*: G system reliability.

3.3 Binary Decision Diagram Methods

A Binary Decision Diagram (BDD) is a directed acyclic graph data structure that efficiently represents and manipulates Boolean functions by sharing subgraphs. In system reliability assessment, the BDD method first represents the system failure logic as a Boolean function, then constructs the corresponding BDD model, and finally calculates the system reliability by traversing the BDD.

Akers [35] first proposed the BDD algorithm. Chang et al. [36] developed an efficient algorithm based on OBDDs, which can be used to evaluate the reliability of non-repairable systems and the availability of repairable systems subject to imperfect fault coverage mechanisms. Prescott et al. [37] proposed a new phased mission analysis method based on BDDs. This technique facilitates the rapid construction of reliability models for phased missions and achieves efficient quantification through the BDD structure.

3.4 Multi-valued Decision Diagram Methods

A Multi-valued Decision Diagram (MDD) is a natural extension of the BDD for multi-state systems. Unlike BDDs, it allows nodes to have multiple outgoing edges (representing different states) instead of just two. The core of this method is to associate each performance level of the system with an MDD and calculate the overall performance distribution of the system by synthesizing these MDDs.

Xing et al. [38] first proposed the MDD method to analyze multi-state systems with multi-state components and compared it with BDD-based methods. Empirical results show that, compared with BDD-based methods, this method offers lower computational complexity and a simpler model assessment algorithm. Mo et al. [39] proposed a performance analysis method for multi-state linear consecutive-*k-out-of-n* systems (MLC(k, n) systems) based on MDDs. This method focuses on providing an efficient MDD model generation algorithm to model and evaluate the performance of MLC(k, n) systems with heterogeneous components following arbitrary lifetime distributions. The effectiveness of the MDD method is demonstrated by analyzing specific examples of non-repairable and repairable MLC(k, 10) systems. More recently, Zaitseva et al. [40] developed a new MDD-based method for the reliability analysis of multi-state systems under epistemic uncertainty, further demonstrating the flexibility of MDDs in handling more realistic and uncertain system-performance descriptions.

3.5 Universal Generating Functions Methods

The Universal Generating Functions (UGF) method is a powerful mathematical tool that utilizes polynomial algebra to efficiently compute the reliability and performance indices of multi-state systems. It achieves a concise and modular solution to the static reliability problem of complex systems by encoding probability distributions into polynomials and employing combinatorial operators based on the system structure.

The UGF method was first proposed by Levitin [41]. Levitin [42] subsequently extended this method and proposed a new model that generalizes the consecutive *k-out-of-n* system, namely the linear consecutive k-out-of-r-from-n system. In this model, the system consists of n independent and identically distributed components. If any group containing at least k failed components has an interval of less than m components between r consecutive components, the system fails. They also proposed a system reliability assessment algorithm based on the extended UGF. Similarly, Xiang et al. [43] proposed an algorithm based on the UGF method for the reliability assessment of the new model system. The new model generalizes the linear multi-state serial connection system model by introducing allowable gaps. Liu et al. [44] proposed an improved UGF algorithm for sensing data fusion. They established a bidirectional *k-out-of-n*: F system model for a linear topology WSN, redefined the UGF expression and combination operator of the sensing node in the improved algorithm, and derived the reliability expression. The applicability of the UGF framework has also been extended beyond independent-component settings. Levitin [45] presented a universal generating functions approach for multi-state systems with dependent components, showing that UGF-based reliability analysis can be adapted to more complex dependence structures.

3.6 Sum of Disjoint Products Methods

The core of the Sum of Disjoint Products (SDP) method is to disjoint the overlapping failure/success events of the system, so as to calculate the exact system reliability through probability summation. This method simplifies the complexity of system reliability calculation by handling overlapping failure events, allowing the exact calculation of the entire system's reliability through the disjointed probability terms of system failure/success events. In this method, system reliability is typically evaluated by summing the probabilities of disjoint failure (or success) paths.

Yin et al. [46] used the minimal cut sets to perform disjoint transformation, obtaining the failure probability calculation formulas for two-state linear and circular consecutive *k-out-of-n*: F systems, respectively. Similarly, Özbey et al. [47] used the minimal cut sets to perform disjoint transformation to study the non-repairable linear consecutive *k-out-of-n*: F system. Rushdi et al. [48] used the SDP method to study the explicit calculation formulas for linear and circular *k-out-of-n*: F and *k-out-of-n*: G systems.

However, for modern complex systems, this method incurs high computational complexity, and it is difficult to manage dynamic and multi-state systems. Therefore, this method is generally suitable only for small-scale systems.

3.7 Algebraic Methods

This method utilizes abstract algebraic structures (such as groups, rings, fields, and Boolean algebras) to describe the logical functions and working states of the system, thereby deriving the system's reliability through algebraic operations and analysis.

Pascual-Ortigosa et al. [49] first applied the algebraic reliability method to the analysis of various multi-state *k-out-of-n* systems. Based on the algebraic method of monomial ideals and Hilbert series, they conducted algebraic reliability analysis on sparse and non-sparse multi-state consecutive *k-out-of-n* systems and multi-state weighted *k-out-of-n* systems, laying an important foundation for subsequent algebraic methods in the reliability research of related systems.

3.8 Inclusion-Exclusion Principle Methods

The core idea of the inclusion–exclusion principle is to first sum the probabilities of all basic events that lead to system failure, then subtract the probabilities of the intersections of every pair of events, and subsequently add back the probabilities of the intersections of every triplet of events. This alternating process of addition and subtraction is repeated to eliminate the over-counting caused by event overlaps, finally yielding the exact solution for the system failure probability.

Fu et al. [50] used the 3-state *k-out-of-n*: G system as an example to derive the system reliability formula using minimal path sets, minimal cut sets, and the inclusion-exclusion principle. However, the corresponding computational complexity is high. For classical *k-out-of-n* systems, Heidtmann [51] proposed an improved inclusion-exclusion approach that avoids many canceling terms and thereby reduces the computational burden, which illustrates both the exactness and the computational challenges of this class of methods.

3.9 System Signature Methods

The system signature method describes the failure behavior of the entire system using a concise probability vector. This method assumes that the system consists of n components (typically assumed to be independent and identically distributed), and the system signature is an n-dimensional vector, where the i-th element represents the probability that the

Table 1. Comparison of reliability assessment methods for consecutive *k-out-of-n* systems.

Method	Binary / multi-state	Repairable / non-repairable	Advantages	Disadvantages	Typical Applications
Markov chain methods	Binary, multi-state	Especially repairable; also non-repairable	Captures dynamics well	State-space explosion	Dynamic and repairable systems
Recursive methods	Mainly binary	Mainly non-repairable	Exact and efficient for regular structures	Limited for complex dependence	Regular linear/circular systems
BDD methods	Mainly binary	Non-repairable; some repairable cases	Compact and exact	Sensitive to variable ordering	Binary systems with clear logic
MDD methods	Mainly multi-state	Both	Natural for multi-state modeling	Can be complex for large systems	Multi-state systems
UGF methods	Mainly multi-state	Mainly non-repairable	Modular and efficient for multi-state systems modeling	Weak for strong dependence	Performance–reliability analysis
SDP methods	Mainly binary	Mainly non-repairable	Exact and interpretable	Combinatorial explosion	Small systems and validation
Inclusion–exclusion methods	Mainly binary	Mainly non-repairable	Strong theoretical insight	Less practical in engineering	Theoretical analysis
Algebraic methods	Mainly multi-state	Both	Elegant structure-based analysis	Restricted by assumptions	Symmetric or exchangeable systems
System signature methods	Binary, multi-state	Both	Scalable and flexible	Restricted by i.i.d. component assumption	Large complex dependent systems

i-th component failure causes the system failure. A key advantage of the signature vector is its exclusive dependence on system structure, which is independent of component lifetime distributions. Once the system signature is obtained, it can be combined with the lifetime distribution of the components to express the overall reliability function of the system through a concise inner product form.

Samaniego [52] first proposed the application of the system signature method in engineering reliability. In recent years, Eryilmaz et al. [53] extended the system signature to the m-consecutive-*k-out-of-n*: F system, studied the reliability characteristics of the m-consecutive-*k-out-of-n*: F system with exchangeable components, derived the exact formula and recursive relationship of the system signature, and established the closed-form expression of the survival function and lifetime distribution as a mixture of order statistics distributions. Eryilmaz [54] extended the system signature method to repairable systems, using system signatures to derive characteristic expressions for the steady-state availability, failure rate, and mean time to failure (MTTF) of repairable systems. He pointed out that repairable systems with arbitrary structures can be evaluated using signatures.

3.10 Artificial Intelligence Methods

Traditionally, reliability assessment has relied on analytical methods. However, with the rapid development of artificial intelligence (AI) technologies such as machine learning, reliability assessment is shifting towards modern data-driven and AI-based methods. This represents a significant trend in the field.

van Nooten et al. [55] applied Graph Neural Networks (GNNs) as a machine learning method to model and analyze large power grids. GNNs capture grid topology and spatial dependence, enabling scalable reliability analysis under faults. By modeling the system as a graph and automatically capturing complex component dependence via message passing mechanisms, GNNs can effectively handle large-scale complex network systems that pose challenges for traditional analytical methods.

To provide a clearer comparison framework for method selection, the main reliability assessment methods reviewed in Section 3 are summarized in Table 1 from the perspectives of dependence modeling, state-space characteristics, repairability, advantages, limitations, and typical application scenarios.

As shown in Table 1, no single method is universally optimal for all consecutive *k-out-of-n* systems. Markov-chain-based methods are more suitable for repairable and dynamic systems, recursive and SDP methods are more effective for relatively small or structurally regular binary systems, BDD / MDD methods are advantageous for logical and multi-state representations, UGF methods are attractive for multi-state performance-reliability analysis, and AI-based methods show strong potential for large-scale systems with complex dependence. Therefore, the choice of method should depend on the dependence structure, state-space type, repairability, and computational scale of the target system.

4 Conclusions and Future Research Directions

This paper systematically reviews the research progress on consecutive *k-out-of-n* systems and their extended systems. These models significantly enhance the adaptability of traditional systems to complex engineering scenarios by incorporating features such as multiple failure criteria, component dependence, and repair mechanisms.

Regarding reliability assessment methods, this paper summarizes the research results of various analytical and numerical methods, including Markov chains, recursive methods, BDD, UGF, and system signatures. These methods each have advantages in modeling accuracy, computational efficiency, and application scope, providing powerful analytical tools for systems with different structures. Notably, the recent introduction of artificial intelligence technologies, such as graph neural networks (GNNs), signifies a gradual shift in reliability assessment from traditional analytical methods to data-driven and intelligent modeling, demonstrating the potential to handle large-scale, high-dimensional dependent systems.

It is evident that the consecutive system family has broad application prospects in many engineering fields, and its unique structural characteristics have given rise to many research questions worthy of in-depth exploration. Future research should focus on integrating advanced AI paradigms, particularly GNNs and deep learning, into multi-state and repairable systems to enable real-time reliability assessment and fault prediction.

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Conflicts of Interest

The authors declare no conflicts of interest.

AI Use Statement

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Ethical Approval and Consent to Participate

Not applicable.

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