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REVIEW ARTICLE

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Reliability and Maintenance Optimization for k-out-of-n Systems: A Systematic Review and Recent Advances in Theory and Practice

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Abstract

The k-out-of-n system is a fundamental redundancy model in reliability engineering, playing a critical role in safety-critical domains, such as power transmission, transportation network, communication networks and industrial production lines. This paper provides a systematic review of recent research advances in reliability modeling and maintenance strategy optimization for k-out-of-n systems. We first introduce the standard k-out-of-n model and its extensions, including multi-state, weighted, consecutive, and network configurations, along with their reliability evaluation methods. We then review maintenance models encompassing time-based, condition-based, and economically oriented maintenance strategies. Furthermore, we discuss intelligent maintenance approaches based on reinforcement learning and heuristic algorithms. Finally, we highlight industrial applications and practical challenges. This review aims to serve as a reference for both academic research and engineering practice in the field.

Keywords: k-out-of-n system, reliability, system modeling, maintenance strategy.

1 Introduction

In reliability engineering, system design often adopts redundancy technology to improve reliability and availability. Among various redundancy models, the k-out-of-n system is fundamental and critical: a system composed of n components functions normally if at least k of its components are working. The concept of redundancy design was applied in engineering projects as early as the V-1 rocket in the late stage of World War II. In the 1950s, as reliability engineering emerged as an independent discipline, the quantitative theory of system reliability laid the foundation for research on models including the k-out-of-n system. Early theoretical work established the framework for the k-to-l-out-of-n system, in which the system functions if the number of working components falls between k and l [1]. Classic models focus on quantity redundancy, but their assumptions are limited when applied to scenarios where the positions and connections of components are crucial, such as communication links and pipeline systems. This

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limitation prompted the exploration of structural redundancy models, leading to the derivation of the consecutive k-out-of-n system in the early 1980s—a model that emphasizes the continuity or adjacency of component failures [2]. The well-known term consecutive k-out-of-n system was first introduced by Chiang and Niu [3] (1981). The consecutive k-out-of-n system consists of n components arranged in a linear or circular sequence, and its key characteristic is that the system fails when the number of consecutive failed components reaches or exceeds k. This system has attracted widespread attention in reliability engineering.

The binary-state assumption, which classifies systems and components as either operational or failed, has been widely used in reliability analysis [4]. However, this assumption cannot capture the gradual degradation or multiple performance levels exhibited by many real-world systems. To overcome this limitation, researchers have extended the binary-state model to the multi-state system (MSS) framework, where both components and systems can occupy more than two states [5]. MSS models have been successfully applied to various engineering fields, including power systems [6], computer networks [7], and large-scale manufacturing and series-parallel systems [8]. Moreover, several studies have investigated maintenance strategies and optimization problems for multi-state systems [9, 10]. The MSS concept has also been extended to consecutive configurations. Huang [11] (2003) generalized the multi-state k-out-of-n model to multi-state consecutive k-out-of-n systems. Based on operational principles, such systems can be classified into consecutive k-out-of-n: F and consecutive k-out-of-n: G types.

Maintenance optimization is an important field for k-out-of-n systems. For k-out-of-n systems, maintenance strategies can be roughly divided into three categories according to the decision-making basis: time-based maintenance (TBM), condition-based maintenance (CBM), and economy-oriented maintenance. TBM relies on fixed schedules or component age to trigger preventive actions, but may lead to over-maintenance or under-maintenance [12]. CBM dynamically adjusts maintenance decisions based on real-time health monitoring, thereby reducing unnecessary interventions [13, 14]. Underpinning these strategies are two basic maintenance actions: corrective maintenance (CM), which is unpredictable, and preventive maintenance (PM), which is typically scheduled [15]. Economic maintenance aims to minimize total life cycle costs by exploiting dependencies among components, often implemented through opportunistic maintenance or group maintenance [16, 17]. The effectiveness of economic maintenance hinges on component dependencies, which are generally classified into three types: economic dependence, stochastic dependence, and structural dependence [18–20]. Moreover, practical considerations such as maintenance time, dynamic grouping of components, and selective maintenance further enrich the optimization landscape [10, 19].

In modern engineering fields, the performance of numerous infrastructure facilities and complex systems often shows a tendency of gradual degradation over time, which may eventually lead to system failures. Therefore, research on reliability modeling, evaluation, and optimization for degrading systems holds significant academic value from both system risk management and economic security perspectives. In the research on replacement strategies for multi-component systems, Song et al. [21] (2012) assumed that the replacement cost is constant; Eryilmaz and Devrim [22] defined the replacement cost based on the number of failed components; Kelkinnama and Lorvand [23] (2025) defined the replacement cost under operational states by combining the number of consecutive failed component groups, taking into account the characteristics of consecutive k-out-of-n systems. Therefore, constructing dynamic reliability models is of great significance for improving the predictive maintenance and resilience design of systems such as smart grids and rail transit loops. In smart grid applications, deep learning techniques have been integrated with situational awareness frameworks to enable real-time fault warning and predictive maintenance [24]. However, as system complexity and dimensionality increase, traditional degradation models and replacement policies face significant computational challenges, motivating the development of intelligent maintenance approaches based on reinforcement learning and heuristic algorithms.

The remainder of this paper is organized as follows. Section 2 reviews reliability evaluation methods for k-out-of-n systems, covering binary, multi-state, weighted, consecutive, and network variants. Section 3 discusses maintenance models and strategies, including TBM, CBM, and economic maintenance strategies. Section 4 presents intelligent maintenance

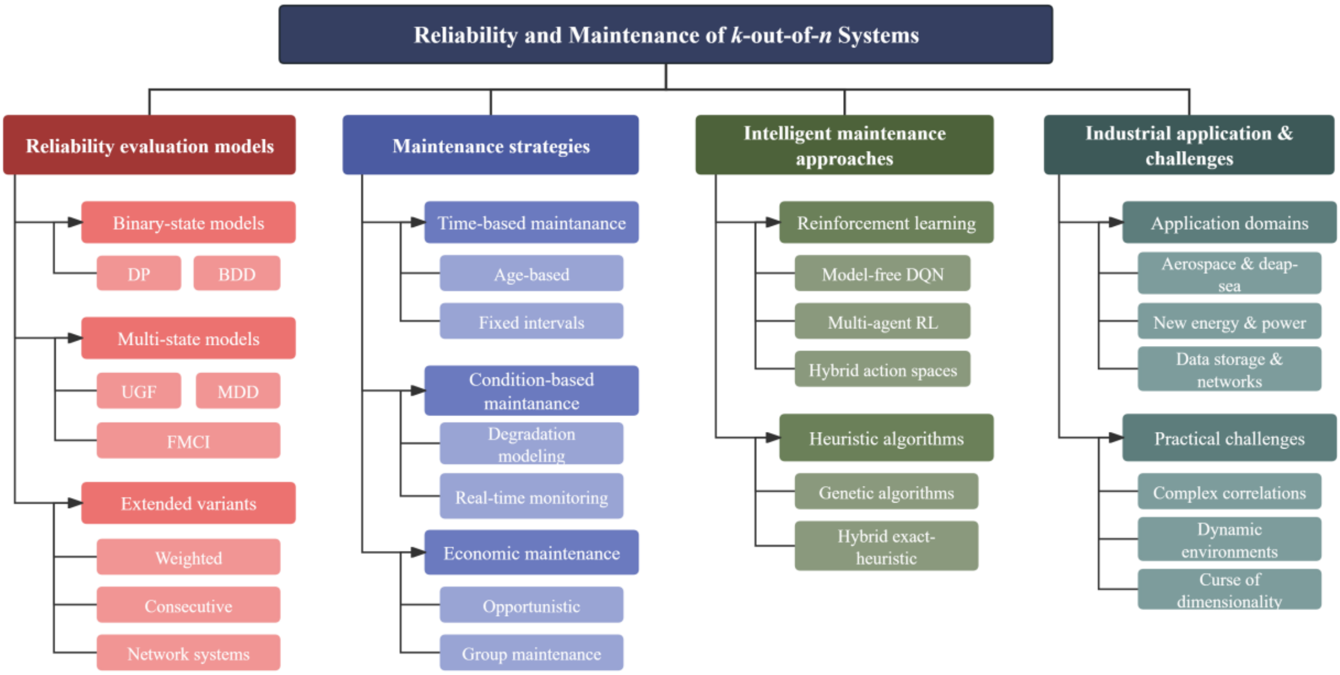

Figure 1. Overview of reliability and maintenance framework for k-out-of-n systems.

approaches based on reinforcement learning. Section 5 describes industrial applications and practical challenges. Section 6 concludes the paper.

As illustrated in Figure 1, this review systematically covers two main dimensions: reliability evaluation methods (including binary, multi-state, weighted, consecutive, and network configurations) and maintenance optimization strategies (time-based, condition-based, economic, and intelligent approaches), along with their industrial applications and practical challenges.

2 Reliability Evaluation of k-out-of-n Systems

This section provides a comprehensive overview of reliability evaluation methods for k-out-of-n systems, ranging from the classical binary-state model to more advanced extensions. We begin with the standard k-out-of-n formulation and its binary-state evaluation techniques, including dynamic programming and binary decision diagrams. The discussion then progresses to multi-state systems (MSS), where components and systems can occupy multiple performance levels, and presents key evaluation methodologies such as the universal generating function (UGF), recursive algorithms, multi-valued decision diagrams (MDD), and finite Markov chain imbedding (FMCI). Subsequently, we review three important model variants: weighted k-out-of-n systems, consecutive k-out-of-n systems, and network-based extensions. Each variant is discussed in terms of its definition, typical applications, and principal reliability assessment techniques.

2.1 Reliability of k-out-of-n systems

The k-out-of-n system is a foundational archetype in reliability engineering, representing a class of voting or redundancy structures [25]. It can be defined in two complementary ways: (1) k-out-of-n: G System: the system functions iff at least k constituent components are working; (2) k-out-of-n: F System: the system fails iff at least k components have failed. These definitions are equivalent, as a k/n (F) system is logically identical to an $(n-k+1)/n$ (G) system.

2.2 Reliability Evaluation of Binary k-out-of-n Systems

The binary k-out-of-n model, where components and the system exist in only two states (working or failed), serves as the cornerstone of reliability theory. Research has focused extensively on developing efficient computational algorithms, particularly for systems with non-identical components.

For systems with independent but non-identical components, dynamic programming (DP) offers an efficient exact solution. For instance, Arulmozhi [26] (2002) proposed an algorithm for calculation of the reliability of k-out-of-n: G systems, improving computational efficiency. For the binary k-out-of-n systems, numerous articles have developed methods to assess and improve algorithm efficiency and maximize system reliability, including analyses of the probability distribution of working component counts in correlated binary trials [27]. Furthermore, for larger or more structurally complex systems, Binary Decision Diagrams (BDD) have emerged as a powerful tool. The reliability computation is transformed into a recursive traversal of this directed acyclic graph, where subgraphs are shared, drastically reducing computational effort. Myers and Rauzy [28] (2008) proposed using BDD to efficiently calculate the reliability of redundant systems, which is particularly suitable for scenarios with imperfect fault coverage. Some studies address the optimal reassignment of degrading components to maximize system reliability, providing analytical frameworks for non-repairable systems under component heterogeneity [29].

2.3 Reliability Evaluation of Multi-state k-out-of-n Systems

The binary-state assumption is often a simplification. In practice, components and systems may exhibit multiple performance levels (or states), ranging from perfect function to complete failure. Multi-State Systems (MSS) model this spectrum, enabling a more precise analysis of systems like manufacturing lines, power networks, and rail systems where performance degrades gradually. On the component level, each component can have different states and affect the state of the whole system. For instance, assuming the system is composed of n components, and the state space of each component i is: $S_i = \{0, 1, 2, \ldots, M_i\}$, where state 0 indicates complete failure, state M_i represents optimal performance. On the system level, multiple states can be interpreted as multiple levels of system capacity or performance. For example, the state space of a system is: $S_{\text{sys}} = \{0, 1, 2, \ldots, M_{\text{sys}}\}$. A generalized multi-state k-out-of-n: G system consists of n multi-state components and is in state j or above iff at least k_j of its components are in states at or above their respective required threshold l_{ij} for that system state. Initially, Huang et al. [30] (2000) imposed a monotonicity constraint on the k_j sequence as the basic assumption. Zuo and Tian [31] (2006) generalized this by defining a flexible multi-state k-out-of-n: F model where k_j can follow any arbitrary pattern, significantly broadening applicability to non-intuitive system behaviors.

2.3.1 *Methodological developments of reliability evaluation of MSS*

There are diverse research efforts on the reliability evaluation of multi-state k-out-of-n systems with independently and identically distributed (i.i.d.) components. The reliability evaluation of MSS has spurred diverse methodological developments:

(1) **Universal Generating Function:** One of the most common methods is using the universal generating function (UGF) to calculate the probability distribution of the discrete random variable [32]. This efficient technique represents a component's state probability distribution as a polynomial. Li and Zio [33] (2012) extended the method to the HUGF (hybrid UGF) with random fuzzy variables (RFVs).

(2) **Recursive Algorithms:** The recursive method is also a commonly used computational approach. Huang et al. [30] (2000) put forward a basic enumerating algorithm. Zuo and Tian [31] (2006) found this algorithm inefficient, and instead proposed an efficient recursive algorithm based on minimal cut vectors. Building on the concept of minimal cut vectors, efficient recursive algorithms have been developed for the generalized model, offering exact solutions for systems of moderate size.

(3) **Multi-Valued Decision Diagram:** As an extension of BDD to multi-state variables, MDD provides a highly efficient framework for encoding the combinatorial logic of MSS. The MDD-based approach makes full use of clearly defined k-out-of-n structures, thereby reducing computational complexity compared to the recursive method [34]. Wang et al. [35] (2023) also applied MDD analysis to dynamic multi-state k-out-of-n: G systems.

(4) **Stochastic Process Methods:** Compared to the aforementioned evaluation methods, the Markov process method is more suitable for dynamic environment where systems have time-varying characteristics, such as component repeatability [36], time-dependent failure and repair rates, and dynamic interdependencies between components or subsystems [37]. For systems with consecutive-type failure criteria, the FMCI approach is particularly powerful.

The comparison of these methods in Table 1 highlights that the choice among UGF, recursive algorithms,

Table 1. Comparison of reliability evaluation methods for multi-state k-out-of-n systems.

Method	Principle	Advantages and Disadvantages	Literature
Universal Generating Function (UGF)	Represents component state distribution as polynomial; applies composition operators	**Advantages:** 1) Efficient for systems with many components; 2) State-dependent weights **Disadvantages:** Difficult to model complex dependencies	Li and Zio [33] (2012); Li *et al.* [32] (2014)
Recursive Algorithms	Based on minimal cut/path vectors; recursively computes system state probabilities	**Advantages:** 1) Exact solutions for moderate-sized systems; 2) Conceptually simple **Disadvantages:** 1) Exponential complexity in system size; 2) Limited scalability	Huang *et al.* [30] (2000); Zuo and Tian [31] (2006)
Multi-Valued Decision Diagram (MDD)	Extends BDD to multi-state variables; shares subgraphs to reduce complexity	**Advantages:** 1) High computational efficiency; 2) Exploits system structure **Disadvantages:** 1) Requires careful variable ordering; 2) Construction can be complex	Mo *et al.* [34] (2015); Wang *et al.* [35] (2023)
Stochastic Process Methods (Markov)	Models time-varying behavior with state transition rates	**Advantages:** 1) Suitable for dynamic environments; 2) Captures repairability and time dependencies **Disadvantages:** 1) State space explosion for large systems; 2) Assumption may be restrictive	Ruiz-Castro [36] (2020); Wu and Cui [37] (2022)

MDD, and stochastic process methods depends on system size, state granularity, and the need for dynamic modeling.

The selection among these reliability evaluation methods depends on system characteristics and analytical requirements. UGF is preferable for systems with numerous components and state-dependent weights due to its computational efficiency, though it struggles with complex dependencies. Recursive algorithms provide exact solutions for moderate-sized systems but suffer from exponential complexity, making them unsuitable for large-scale applications. MDD offers the highest computational efficiency for structurally well-defined systems, yet requires careful variable ordering. Stochastic process methods are indispensable for dynamic environments with time-varying failure rates and repairability, despite the state-space explosion challenge. Researchers should weigh these trade-offs based on system size, state granularity, and the need for dynamic modeling.

2.3.2 *Expansions and Advanced Models of Multi-State k-out-of-n Systems*

Building upon the foundational generalized multi-state models, recent research has focused on capturing more intricate, realistic behaviors that arise from component interactions, system-level equilibria, and dynamic operational environments. These expansions move beyond the assumption of independent component contributions towards a more holistic system-view.

A significant leap in modeling fidelity is the introduction of balance constraints. A balanced system requires that the performance levels of its components remain within a specified range relative to each other. This system is employed in critical applications where safety is paramount, including military weapons, aerospace, and advanced energy storage technologies. Cui et al. [38] (2018) initially constructed four reliability models for the k-out-of-n: F balanced system with m sectors. Wang et al. [39] (2022) analyzed such a system using continuous-time Markov processes, deriving key reliability metrics and optimizing maintenance strategies under this novel constraint. Further extending this paradigm, Dong et al. [40] (2025) investigated an optimal load adjustment policy for multi-state k-out-of-n balanced systems with self-healing mechanisms. Extending this idea to networked systems, Gao et al. [41] (2025) proposed a model evaluating multi-state k-out-of-n: F network systems with balance dependence. Here, the balance requirement is not merely a global constraint but a local dependence mechanism that affects the performance and failure behavior of the entire network.

These advanced models, balanced systems, self-healing mechanisms, and network systems with balance dependence, extend the classical

k-out-of-n framework toward greater realism at the cost of analytical tractability. Balance constraints are essential in applications where component performance uniformity is safety-critical, such as aerospace and energy storage, but introduce complex interdependencies that preclude simple recursive evaluation. Self-healing mechanisms are preferable for systems where autonomous recovery is feasible between maintenance windows, though optimal load adjustment policies require dynamic programming or RL solutions. Networked balance dependence captures local interaction effects but demands graph-based or Bayesian network evaluation methods. Researchers should select among these extensions based on whether the primary concern is global performance uniformity, autonomous recovery capability, or local interaction effects.

2.4 Extended system variants and their reliability

The standard k-out-of-n model relies on simplifying assumptions that often limit its direct applicability to complex real-world systems. To bridge this gap, significant research has generalized the core model along several key dimensions. This section reviews three pivotal extensions: weighted, consecutive (including sparsely connected), and networked systems. For each variant, we outline its defining characteristics, typical applications, and the principal methods developed for its reliability assessment.

2.4.1 Reliability evaluation of weighted k-out-of-n systems

The weighted k-out-of-n system is designed to capture the reality that components often contribute unequally to system function. In 1994, Wu and Chen proposed a system with n components, each with a positive integer weight (total system weight equals w), such that the system functions (failed) if the total weight of working (failed) components is at least threshold k.

The reliability evaluation of these systems hinges on efficiently calculating the probability that the random sum of weights of working components reaches the threshold k. For binary-state with fixed weight, dynamic programming offers exact solutions by recursively computing the cumulative weight distribution. This approach was extended to more complex application scenarios, such as two-stage weighted systems with shared components, providing methods for generating minimal paths and cuts for bound derivations [42]. For systems where component weights are random variables (i.e., fluctuating capacities), recursive formulas and Monte Carlo simulation algorithms based on ordered component lifetimes can be used to assess reliability and system performance duration [43].

Furthermore, the model has been extended to the multi-state domain, where both components and the system exhibit multiple performance levels, and weight may be state-dependent. Li and Zuo [44] (2008) proposed distinct multi-state weighted models and evaluated them using recursive algorithms and UGF. Subsequent research generalized the concept further to multi-performance weighted multi-state systems where components contribute to multiple, possibly correlated, performance criteria [45]. For dynamic and repairable multi-state weighted systems, the Lz-transform approach, integrated with continuous-time Markov chains, has proven effective for computing a comprehensive set of reliability indices [46]. Recently, some advancements consider weighted systems operating in harsh, dynamic environments. Wang et al. [47] (2022) introduced a mixed shock model for a multi-state weighted system, where components undergo a two-stage degradation process and their resistance to external shocks deteriorates.

2.4.2 Reliability evaluation of consecutive k-out-of-n systems

Consecutive k-out-of-n systems impose a critical spatial or sequential dependency on the classical model. This section focuses on reliability evaluation methods for such systems, particularly the linear and circular configurations. This model is fundamental for analyzing systems where localized failure clusters are catastrophic, such as pipeline networks, sensor arrays, and communication lines.

The reliability evaluation of these systems, particularly the linear and circular consecutive-k-out-of-n models, has been extensively studied. A key methodology is the Finite Markov Chain Imbedding (FMCI) approach, which embeds the system lifetime into a small Markov chain tracking the progression toward a fatal consecutive failure pattern. This method provides exact and recursive solutions, avoiding the state-space explosion of generic Markov models [48]. The approach has been generalized to multi-state consecutive systems, where components and the system can reside in multiple performance levels. For multi-state consecutive k-out-of-n systems, where components and the system can occupy multiple performance levels, reliability evaluation becomes more involved. The finite Markov chain imbedding (FMCI) approach has been extended to such settings

Table 2. Reliability evaluation methods for different k-out-of-n system variants

System Variant	Typical Methods	Key Features	Literature
Binary k-out-of-n	DP, BDD, inclusion-exclusion	Exact solutions and efficient for i.i.d. or independent components	Arulmozhi [26] (2002); Myers and Rauzy [28] (2008)
Multi-state k-out-of-n	UGF, recursive algorithms, MDD	Handles performance degradation and flexible state spaces	Li & Zio [33] (2012); Mo et al. [34] (2015)
Weighted k-out-of-n	DP (fixed weights), UGF, Lz-transform	Accounts for heterogeneous component contributions	Wu & Chen [56] (1994); Li & Zuo [44] (2008)
Consecutive k-out-of-n	FMCI, recursive formulas, Bayesian networks	Models spatial or sequential failure dependency	Zhao et al. [48] (2007); Gao et al. [49] (2021)
Network k-out-of-n	Minimal path/cut, BAT, integer programming	Evaluates connectivity or flow reliability	Kozyra [53] (2024); Yeh [55] (2025)

by defining a multi-dimensional state space that tracks both the degradation levels of individual components and the progression of consecutive failures [49]. Alternatively, the universal generating function (UGF) method can be adapted to multi-state consecutive systems by representing component state distributions as polynomials and applying consecutive failure criteria through specialized composition operators [11]. Bayesian networks have also been employed to capture complex dependencies in multi-state consecutive systems [50]. The choice of method depends on system size, state space granularity, and whether component lifetimes are independent or correlated.

Research has further extended the model to capture greater realism by relaxing key assumptions. Arbitrary component dependencies and non-identical component lifetimes have been incorporated, with studies deriving formulas for reliability characteristics like mean time to failure and residual lifetime, and providing approximations for survival functions [22]. Structural generalization includes the k-out-of-r-from-n system with multiple failure criteria and systems with shared components between adjacent subsystems. Further structural generalizations include the linear zigzag and circular polygon configurations proposed by Lin et al. [51] (2016), which extend the consecutive-k-out-of-n: F model to more complex topologies. An important generalization is the sparsely connected consecutive-k system (or d-sparse system), where up to d working components are allowed between failed ones within the critical cluster [48, 49, 52]. For efficient and practical evaluation methods, some research has derived non-recursive closed-form expressions for systems with heterogeneous components and integrated the model with Bayesian Networks to enhance modeling capabilities, especially for complex multi-state systems [50].

2.4.3 *Reliability evaluation of network systems*

Extending the k-out-of-n paradigm to networked systems addresses the reliability of infrastructures with explicit topological interconnections. The "k-out-of-n" concept is typically interpreted as the requirement for a certain level of connectivity or flow capacity between critical nodes. This transforms the problem into evaluating k-terminal connectivity reliability or flow reliability.

Early work focused on efficient exact methods for binary-state networks, such as redefining minimal cuts in terms of node sets to simplify computation. For the more complex multi-state flow networks (MFNs), research has progressed along two key fronts: developing efficient evaluation schemes and tackling broader optimization problems. Advanced approximation methods using integer linear programming and binary-state minimal cuts have been proposed for reliability-redundancy allocation, while parallel algorithms that simultaneously find all multi-state minimal paths and decompose the state space have significantly enhanced computational efficiency [53]. Recently, an evolving trend is the refinement of the BAT (Branching and Aggregation Technique). Developments include a hybrid inequality BAT for comprehensive identification of all-level demand-based minimal paths in MFNs [54]. This algorithmic evolution culminates in the first BAT specifically designed for one-output k-out-of-n binary-state networks, which incorporates reduction rules and a cut-based layered-search algorithm to efficiently verify the critical path connectivity condition [55].

Table 2 summarizes the reliability evaluation methods for different k-out-of-n system variants, illustrating how each extension requires tailored computational approaches.

3 Maintenance models and strategies for k-out-of-n systems

Maintenance optimization, the ability of a system to be restored to an operational state after failure, is a critical factor influencing long-term availability, operational costs, and system risk, particularly in redundant systems where maintenance delays can deplete redundancy and lower the failure threshold [57]. As engineering systems evolve toward multi-state, load-sharing, degrading, and multi-fault configurations, traditional static failure models become inadequate. Recent research integrates maintenance behaviors, priorities, resource constraints, and operational uncertainties into unified frameworks, with opportunistic maintenance, condition-based maintenance, and multi-level repair strategies progressively incorporated into k-out-of-n system analysis [58]. Leveraging tools such as Markov processes, multi-state system theory, simulation, and impact processes, numerous studies have demonstrated that maintainability analysis has shifted from reactive repair to proactive planning, strategy optimization, and system-level collaborative design [59].

3.1 Maintenance model

Based on the decision-making basis, maintenance models for k-out-of-n systems are generally classified into three categories: time-based maintenance (TBM), condition-based maintenance (CBM), and economic maintenance. TBM relies on component age or fixed schedules; CBM uses real-time health monitoring to trigger actions; economic maintenance aims to minimize lifecycle costs by exploiting dependencies among components, often implemented through opportunistic or group maintenance [17]. These three types of models form the basic framework for maintenance strategy analysis and optimization of k-out-of-n systems. Three commonly adopted maintenance approaches are corrective maintenance, preventive maintenance, and opportunistic maintenance [16]. Among them, PM can be implemented via TBM or CBM, and opportunistic maintenance (OM) is typically an implementation of economic maintenance.

3.1.1 Time-based maintenance

Time-based maintenance, also known as age-based maintenance or life-based preventive replacement strategy, is a maintenance approach based on the "service time/age" of equipment or components. When equipment reaches a preset threshold of usage time, preventive maintenance or replacement is performed even if no failure has occurred. The objective is to reduce future failure risks, minimize unplanned downtime, and lower overall maintenance costs [60]. Age-based maintenance strategies typically assume that the failure rate of system components gradually increases over time, making equipment more prone to failure as it ages.

In practical engineering applications, age-based maintenance is suitable for systems where usage time is closely correlated with failure rates, such as mechanical structural components, long-running pump stations, and wind power equipment. Such failure behavior typically exhibits a wear-induced growth trend and is less directly influenced by condition monitoring data. The strategy's advantages lie in its low implementation threshold and relatively simple modeling, facilitating engineering adoption.

Theoretically, age-based maintenance models are typically grounded in reliability theory and stochastic process frameworks. Traditional approaches include employing renewal processes, virtual age models, and classical replacement models to describe system failure and replacement behavior over time. These models derive metrics such as average cost rate, mean time to failure, and long-term availability, enabling the determination of optimal replacement age or intervals. In multi-component or redundant systems, age-based maintenance models can be extended using tools like system structure functions to evaluate preventive maintenance effectiveness and cost-benefit at the system level. For systems with arbitrarily dependent components, reliability properties under age-based strategies can be derived analytically [61].

For k-out-of-n systems, where overall functionality depends on at least k components operating effectively, age-based maintenance strategies for individual components directly impact system-level reliability and downtime risk. Consequently, researchers have integrated age-based maintenance strategies with k-out-of-n system architectures, proposing replacement or maintenance threshold decision methods aligned with system structure and cost functions [62, 63]. Zhang and Fang [64] (2025) recently explored optimal age-based replacement strategies for k-out-of-n systems considering task/work cycle duration, comprehensively optimizing maintenance decisions based on system lifetime and task duration.

Furthermore, extended models for age-based maintenance exist, such as discrete-time age replacement strategies. These models can integrate inspection intervals and system failure distributions for comprehensive analysis within specific systems, yielding unique optimal solutions for preventive maintenance cycles [65].

3.1.2 Condition-based maintenance

First proposed in 1975, condition-based maintenance (CBM) aims to maximize the effectiveness of preventive maintenance with limited resources. Modern CBM frameworks increasingly rely on data-driven health prognostics to estimate remaining useful life and trigger maintenance decisions [66]. This strategy is typically applicable to engineering systems equipped with condition monitoring capabilities compared with TBM [13]. By deploying sensors and online monitoring devices, real-time operational data such as vibration, temperature, pressure, shock intensity and frequency [67], wind speed [68, 69], and degradation degree [70] are collected. Bloch and Geitner [71] (2012) noted that 99% of equipment exhibits precursor signs prior to failure, making condition monitoring a viable approach for preventive maintenance. Combined with signal processing and health assessment methods, this approach enables continuous status updates. When monitored indicators reach preset thresholds or predicted remaining life falls below safe levels, the system triggers preventive maintenance or replacement decisions [72]. Jardine et al. [14] (2006) defined CBM as a maintenance decision-making scheme that provides maintenance recommendations by monitoring component information. This dynamic, condition-based maintenance mechanism finds extensive application in rotating machinery, energy equipment, and high-reliability industrial systems.

Regarding modeling approaches, condition-based maintenance models are typically constructed based on stochastic degradation processes-such as Gamma processes, Wiener processes, or multi-state Markov processes-to describe the random evolution of equipment performance over time [73–75]. By integrating state observation mechanisms with maintenance decisions, researchers can establish optimization models based on thresholds or policy spaces, with long-term average cost rates or system availability as optimization objectives [14, 76]. Compared to traditional age-based maintenance models, these approaches demonstrate greater adaptability under uncertain conditions [77]. For k-out-of-n systems, system functionality depends on at least k components being operational. Consequently, the degradation and maintenance behavior of individual components directly impacts system-level reliability and availability. Against this backdrop, condition-based maintenance strategies have been introduced into k-out-of-n system analysis to coordinate maintenance timing across multiple components, preventing rapid depletion of redundant capacity due to delayed maintenance or simultaneous failures [58, 78]. Recent studies indicate that integrating component degradation states with system structural characteristics can significantly enhance the long-term availability of k-out-of-n systems while reducing their full lifecycle costs [79, 80].

Furthermore, some studies have addressed practical engineering challenges such as failures during maintenance, state-dependent maintenance costs, and joint maintenance of multiple components, constructing more realistic condition-based maintenance models [73, 81]. These findings indicate that condition-based maintenance research for k-out-of-n systems is evolving from single-component state decisions toward system-level collaborative maintenance and resource optimization.

3.1.3 Economic maintenance

Economic maintenance is a decision-making approach aimed at minimizing the total life cycle cost of a system by jointly considering maintenance costs, failure repair costs, downtime losses, preventive replacement costs, and indirect economic consequences of failures, seeking an optimal balance between reliability and cost-effectiveness. This strategy emphasizes the "economic rationality", meaning activities are only initiated when the expected benefits outweigh the implementation costs [82]. This approach is particularly suited for engineering systems with high demands for operational continuity and cost control, such as power systems, communication systems, and redundant industrial installations. In these systems, unplanned downtime often incurs economic losses far exceeding the maintenance costs, making fixed-interval or simple condition-threshold strategies suboptimal [14, 73]. By incorporating downtime loss functions and maintenance cost functions, economic maintenance models better reflect real-world engineering decision environments [83].

Two common implementations of economic maintenance are opportunistic maintenance and group maintenance. Opportunistic maintenance means that when a component is maintained, it creates an opportunity to service other components that have reached their opportunistic maintenance thresholds, saving setup costs and exploiting economic dependence. Group maintenance performs joint maintenance on a set of components. Positive economic dependence occurs when the joint maintenance cost is lower than the sum of individual costs; group maintenance can exploit this to reduce overall costs [84]. Conversely, negative economic dependence arises when simultaneous maintenance is more expensive. Positive economic dependence is generally applicable to series systems [84]. Dynamic grouping strategies further exploit this dependence by jointly scheduling maintenance for components based on their operational conditions and interdependencies [85]. Pham and Wang [17] (2000) investigated both positive and negative economic dependence in k-out-of-n systems. However, as system structures become increasingly complex with intricate connection modes, traditional group maintenance models lose their applicability, calling for more refined grouping methods and standardized criteria. The increasing structural complexity is evident in safety-critical systems such as aircraft fault-tolerant architectures, where redundancy management must account for multiple degraded system states [86].

In terms of model construction, economic maintenance is typically grounded in stochastic processes and reliability models, coupling system state evolution with cost structures. Common approaches include average cost rate models based on renewal processes, long-term expected cost optimization models based on Markov decision processes, and cost evaluation models incorporating simulation methods [82, 87]. The optimization objective is typically set to minimize either the expected cost per unit time or the system's total lifetime cost. Based on this, the optimal maintenance threshold, replacement strategy, or maintenance combination is determined [83].

For k-out-of-n systems, since system functionality depends on at least k components being operational, the maintenance or replacement of a single component can have amplified effects on system availability and economic losses. Therefore, researchers integrate economic maintenance strategies with k-out-of-n system architectures, focusing on analyzing the interactions among component failures, maintenance decisions, and system downtime costs [78, 80]. Related studies indicate that in k-out-of-n systems,

Table 3. Comparison of maintenance strategies for k-out-of-n systems.

Strategy	Decision Basis	Advantages and Limitations	Typical Applications
Time-Based Maintenance (TBM)	Component age or fixed schedule	**Advantages:** 1) Simple; 2) Low monitoring cost **Limitations:** 1) Over or under-maintenance; 2) Ignores actual condition	Mechanical structural components; pump stations; wind turbines
Condition-Based Maintenance (CBM)	Real-time health monitoring	**Advantages:** 1) Adaptive; 2) Reduces unnecessary maintenance **Limitations:** 1) High sensor/data processing cost; 2) Needs accurate degradation models	Rotating machinery; energy equipment; high-reliability systems
Economic Maintenance	Life cycle cost minimization; exploits component dependencies	**Advantages:** 1) Optimizes cost-benefit; 2) Reduces total cost **Limitations:** 1) Needs accurate cost models; 2) Dependency characterization hard	Power systems; communication networks; redundant industrial installations
Opportunistic Maintenance	Maintenance opportunities from other components	**Advantages:** 1) Cut setup costs; 2) Exploits positive economic dependence **Limitations:** 1) coordination; 2) Threshold optimization complex	Multi-component systems with economic dependence
Group Maintenance	Joint maintenance of multiple components	**Advantages:** Lowers total cost via bundling **Limitations:** 1) coordination; 2) Threshold optimization complex	Series systems; systems with positive economic dependence

rationally setting cost-based maintenance strategies can significantly reduce long-term average system costs while effectively controlling the high downtime risks associated with redundancy depletion [58].

In recent years, some studies have further integrated economic maintenance with constraints such as limited maintenance personnel, failures during maintenance, and spare parts inventory decisions, constructing joint optimization models closer to practical engineering applications [79, 82]. Such research indicates that in k-out-of-n systems, making economic maintenance decisions solely from a single-component perspective often fails to achieve system-level optimal results. Instead, coordinated maintenance and cost optimization are required under system structural constraints.

The optimal maintenance strategy for k-out-of-n systems depends on monitoring infrastructure, cost structure, and system scale. TBM is preferable when monitoring capabilities are limited and component age strongly correlates with failure risk, despite its inherent over-maintenance risk. CBM becomes essential when real-time degradation data is available and degradation processes can be accurately modeled, though it demands higher sensor and computational investments. Economic maintenance excels in multi-component systems with strong positive economic dependence, where joint maintenance significantly reduces setup costs. Opportunistic maintenance is particularly advantageous when system downtime windows are rare but maintenance readiness costs are high. Group maintenance is optimal for series-structured systems or subsystems with synchronized degradation patterns. In practice, hybrid strategies that combines CBM triggers with economic grouping often yield superior cost-effectiveness for large-scale k-out-of-n systems, though their optimization requires balancing real-time responsiveness with combinatorial complexity.

As summarized in Table 3, the choice among TBM, CBM, economic maintenance, opportunistic maintenance, and group maintenance depends on monitoring infrastructure, cost structure, and system scale.

3.2 System-level maintenance strategy and optimization

In k-out-of-n systems, the design objective of maintenance strategies typically aims to simultaneously ensure both system-level availability and cost-effectiveness. Since the system maintains functionality as long as at least k components remain operational, maintenance decisions shift from single-component optimization to cooperative optimization under system structural constraints. Examples include determining whether to perform corrective maintenance on a single degraded/failed component, execute preventive replacement upon reaching an age threshold, or conduct joint interventions on multiple components during scheduled downtime. For such multi-component systems, maintenance optimization research typically employs long-term average cost rates, steady-state availability, or multi-objective trade-offs as primary evaluation criteria. Relevant modeling and classification frameworks can be referenced in systematic reviews of the maintenance optimization field [82].

From a strategy perspective, time/age-based preventive maintenance remains a key baseline approach for k-out-of-n systems. Its core principle involves replacing or repairing components upon reaching a predetermined age threshold, with the strategy's impact on cost rates for k-out-of-n structures evaluated at the system level. Eryilmaz [80] (2020) presented an age-maintenance cost rate representation for binary coherent systems and further specialized the method for structures like consecutive-k-out-of-n, providing an operational theoretical tool for jointly analyzing time-threshold strategies with system structures. Conversely, other studies integrate age-based replacement with more operationally relevant actions like partial repairs into k-out-of-n: F system strategy design, enhancing the flexibility and implementability of preventive maintenance [78].

With advancements in condition monitoring and data acquisition capabilities, CBM and inspection-based maintenance optimization have seen significant growth within k-out-of-n systems. A typical approach involves characterizing component health through degradation processes and triggering maintenance when the system approaches failure thresholds, thereby achieving a dynamic balance between risk and cost. Zhang et al. [58] (2020) developed a CBM strategy for k-out-of-n degradation systems, explicitly accounting for failure dependencies and impact effects between components. This approach reflects the coupled degradation phenomena common in engineering systems, providing a representative modeling paradigm for CBM optimization in structurally redundant systems. Furthermore, for larger k-out-of-n systems with limited observability, recent research has explored CBM planning centered on partial observability combined with economic constraints, reflecting a trend toward extending these approaches to complex scenarios [88].

Opportunistic maintenance yields superior economics performance in scenarios where downtime opportunities are available but maintenance readiness and downtime costs are high. For example, Arabzadeh Jamali and Pham [79] (2022) proposed an opportunistic maintenance model for load-sharing k-out-of-n systems, integrating partial failure, corrective, and preventive actions into a unified framework. This demonstrates the practical advantages of opportunistic maintenance in cost and risk control for redundant systems. Concurrently, research on integrated optimization of inspection and maintenance continues to advance. By optimizing inspection intervals, trigger thresholds, and maintenance action combinations, more refined system-level maintenance decisions are achieved [89]. Wang et al. [90] (2023) developed an inspection-based replacement planning in consideration of state-driven imperfect inspections.

In addition to the above strategies, practical considerations such as maintenance time, dynamic grouping of components based on their status and dependencies, and selective maintenance under limited resources further enrich the optimization landscape [10, 19]. For example, in safety-critical systems with components of varying reliability degrees, Jia and Cui [91] (2011) demonstrated that joint maintenance strategies must balance cost minimization with stringent safety constraints. Furthermore, Qiu et al. [92] (2019) developed an optimal inspection policy that considers "neglected down time", where short interruptions are not considered failures.

4 Intelligent Maintenance Models and Strategies for k-out-of-n Systems

While Section 3 focused on traditional maintenance policies based on time, condition, and economic criteria, this section reviews data-driven intelligent approaches, particularly reinforcement learning and heuristic algorithms, which have emerged to handle the complexity and uncertainty of large-scale k-out-of-n systems. The maintenance optimization of k-out-of-n systems has evolved from parametric structural approaches toward sequential decision-making formulations using Markov decision process (MDP) frameworks. This transition enables greater policy flexibility by learning maintenance strategies rather than optimizing predetermined policy parameters. Recent research increasingly integrates reinforcement learning with MDP formulations to address computational challenges in large-scale systems. This section reviews optimization objective formulations, exact solution algorithms, and intelligent heuristic approaches.

4.1 Optimization Objective Function Construction

Cost minimization with discounting dominates objective formulations, though significant variations exist in cost component inclusion and system dependency modeling. The infinite-horizon discounted cost framework represents the mainstream approach, balancing immediate expenses against future consequences through discount factors [93]. Cost structures typically aggregate inspection expenses, preventive and corrective maintenance costs, inventory charges when applicable, and downtime penalties. For instance, Wang and Zhu [93] (2021) minimized discounted long-run system maintenance and spare inventory costs for k-out-of-n: F systems, while Zhao et al. [94] (2025) adopted cost rate minimization for series k-out-of-n load-sharing configurations, where the objective function explicitly captures setup costs, imperfect repair costs, replacement costs, and downtime expenses.

Three fundamental modeling choices differentiate formulation approaches across the literature. First, component dependency assumptions: while Wang and Zhu [93] (2021) assumed independent components to simplify state transitions, Xu et al. [95] (2021) introduced Copula functions to model stochastic dependencies among component degradation processes, capturing complex joint probability distributions. In contrast, load-sharing formulations capture degradation rate dependencies on operational component counts, substantially increasing state space complexity [96]. Second, maintenance effectiveness representation: binary restoration assumptions in earlier threshold-based models have evolved toward continuous representations. Third, decision timing mechanisms: periodic inspection using standard MDP [97] versus event-driven inspection using semi-MDP for systems with self-announcing failures [98]; the latter captures variable sojourn times but increasing analytical complexity.

Beyond traditional maintenance-cost objectives, specialized system structures require tailored modeling approaches. Gao et al. [49] (2021) developed reliability modeling for sparsely connected consecutive-k-out-of-n: G systems using finite Markov chain imbedding methods, addressing systems where component connectivity follows specific topological patterns. For mission-oriented systems operating in dynamic shock environments, Zhao et al. [99] (2025) investigated reliability and mission abort strategies for multi-state k-out-of-n: F systems, proposing competing mission termination criteria that incorporate environmental information and component state distributions.

Guidance on selecting cost parameter weights remains inadequate, and sensitivity to these parameters is often overlooked. Multi-objective formulations explicitly representing competing stakeholder priorities are scarce, and parameter uncertainty is typically neglected—gaps that limit practical deployment.

4.2 Exact Solution Algorithms

Value iteration and policy iteration within MDP frameworks provide guaranteed optimal solutions for finite-state problems, but face fundamental scalability barriers from exponential state-action space growth. Dynamic programming approaches iteratively solve Bellman optimality equations until convergence, with Wang and Zhu [93] (2021) employing value iteration for joint maintenance-inventory optimization. Zheng et al. [100] (2024) quantified the computational challenge: combinatorial explosion in component count, state discretization levels, inventory capacities, and supplier configurations renders exact enumeration impractical beyond approximately five components, with computation time escalating from minutes to days.

Three mitigation strategies exist, each with inherent limitations. State space discretization enables value iteration application to continuous degradation processes. For instance, Xu et al. [95] (2021) discretized continuous state spaces by defining discrete levels for each component at regular intervals, then applied value iteration with Monte Carlo simulation to compute transition probabilities. However, discretization introduces approximation errors that compound with dimension and may violate Markov properties. Problem decomposition exploits special structure: Liu et al. [101] (2024) leveraged memoryless exponential distribution properties to sequentially optimize inspection intervals through dynamic programming, though this approach applies only to restricted problem classes with specific structural properties.

Exact methods remain constrained to small benchmark problems and assume perfect parameter knowledge. Approximation quality-computation trade-offs in discretization require theoretical characterization, and dimension reduction techniques that preserve problem structure warrant investigation.

4.3 Intelligent Heuristic Algorithms

Reinforcement learning addresses computational intractability through function approximation and adaptive learning, with algorithmic innovations targeting specific challenges in state space dimensionality, action space complexity, and sample efficiency. Two distinct research streams emerge: model-free approaches eliminating explicit system model requirements, and model-based methods leveraging known or learned transition dynamics.

4.3.1 Model-free DQN-based methods

For discrete finite state-action spaces, Q-learning and its deep learning extensions dominate model-free applications. Yang et al. [102] (2022) addressed redundant systems with arbitrary structures using Deep Q-network with BP neural networks, introducing pre-learning and re-learning processes to improve convergence: pre-learning initializes with heuristic strategies rather than random policies, while re-learning records historical optimal networks and reassigns them periodically during training. For multi-component systems where standard DQN struggles with large state spaces, architectural innovations prove critical. Zheng et al. [100] (2024) developed Hybrid Deep Reinforcement Learning combining Double DQN to reduce overestimation bias with dueling network architectures separating state value and action advantage estimation, where the Q-value function equals the sum of state value and the centered action advantage. Their algorithm integrates prioritized experience replay, that assigns sampling probabilities proportional to temporal-difference errors raised to a power, and incorporates threshold control initialization plus greedy strategies in early training stages. Zhao et al. [94] (2025) applied Double-Dueling Deep Q-network with action masking mechanisms to handle invalid actions, particularly for systems with self-announcing failures requiring semi-MDP formulations.

4.3.2 Multi-agent RL methods

Multi-agent decomposition addresses exponentially large action spaces through subsystem-level learning with coordination. Zhao et al. [98] (2025) proposed Multi-Agent Reinforcement Learning for series k-out-of-n load-sharing systems, assigning agents to control individual subsystems while minimizing the discounted cumulative penalty, expressed as an infinite sum of future penalties weighted by a discount factor. Their framework employs Branching Dueling Network structures decomposing high-dimensional action spaces, with cooperative learning through shared global rewards and coordinated network architectures. Critically, they designed heuristic-enhanced penalty functions incorporating domain knowledge about maintenance dependencies and downtime costs—ablation studies demonstrated this enhancement achieved 40.65% cost reduction compared to standard penalty functions.

4.3.3 Hybrid action space methods

For hybrid discrete-continuous action spaces, Zhang et al. [96] (2022) customized Proximal Policy Optimization through multi-task learning architectures maintaining separate discrete and continuous policy networks while sharing representations, enabling simultaneous optimization of maintenance type selection and intensity parameters.

4.3.4 Model-based RL methods

Model-based approaches assume explicit transition functions from prior knowledge or data-driven learning. Zhang et al. [103] (2021) investigated model-based reinforcement learning combining learned models with planning for maintenance optimization in large state spaces, though computational complexity remains substantial. In particular, Zhang et al. [96] (2022) introduced continuous maintenance intensity parameters in hybrid action spaces, enabling explicit control over maintenance degree. When transition functions are

Table 4. Reinforcement Learning Approaches for k-out-of-n System Maintenance Optimization.

RL Approach	Description	Advantages and Disadvantages	Literature
Model-free DQN-based methods	Apply Q-learning with neural network approximation without explicit transition models	**Advantages:** 1) No system model required; 2) Polynomial time complexity scaling **Disadvantages:** 1) No optimality guarantee; 2) Hyperparameter sensitive	Yang *et al.* [102] (2022); Zheng *et al.* [100] (2024); Zhao *et al.* [94] (2025)
Multi-agent RL methods	Decompose problems into subsystems with coordinated agents through shared rewards	**Advantages:** 1) Addresses exponentially large action spaces; 2) Incorporates domain knowledge via heuristics **Disadvantages:** 1) Limited convergence analysis; 2) Highly sensitive to hyperparameters	Zhao *et al.* [98] (2025)
Hybrid action space methods	Employ policy optimization with separate networks for discrete and continuous actions	**Advantages:** 1) Handles hybrid discrete-continuous spaces; 2) Enables explicit maintenance degree control **Disadvantages:** 1) No optimality guarantee; 2) Lacks cross-problem transferability	Zhang *et al.* [96] (2022)
Model-based RL methods	Leverage known or learned transition dynamics combined with planning	**Advantages:** 1) Can incorporate prior knowledge; 2) Enables forward planning **Disadvantages:** 1) Substantial computational complexity; 2) Discretization introduces approximation errors	Zhang *et al.* [103] (2021)

known, Xu et al. [95] (2021) applied value iteration after discretizing continuous state spaces, though this introduces the previously discussed discretization limitations.

Empirical evaluations reveal consistent performance-efficiency trade-offs. Zheng et al. [100] (2024) reported that HDRL increases costs 3.86% on average compared to value iteration for comparable small-scale instances, while reducing computation time by at least 95.99% when component count exceeds three. Yang et al. [102] (2022) demonstrated polynomial rather than exponential time complexity scaling with system size for their improved DQN approach. Zhao et al. [98] (2025) showed MARL achieved 7.38% lower minimum cost rates than standard DQN and 5.27% lower than D3QN for large action space problems, though convergence analysis for cooperative MARL remains limited and performance appears highly sensitive to hyperparameters.

The choice among RL approaches for k-out-of-n system maintenance depends on model availability, state-action space complexity, and coordination requirements. The model-free DQN-based methods are preferable when system dynamics are unknown or difficult to model, offering polynomial time complexity scaling but lacking optimality guarantees. Multi-agent RL becomes essential when the system comprises multiple interacting subsystems with large action spaces, as it decomposes the problem through coordinated agents, though convergence analysis remains limited. Hybrid action space methods are required when maintenance decisions involve both discrete action selection and continuous intensity control. Model-based RL is optimal when accurate transition models can be learned from data or prior knowledge, enabling forward planning, but it demands substantial computational resources and suffers from discretization errors. In practice, for large-scale systems with unknown dynamics, hybrid approaches that combine model-free learning with model-based planning for critical subsystems may offer the best balance of scalability and reliability. Table 4 compares the reinforcement learning approaches discussed above, highlighting their respective advantages and disadvantages for k-out-of-n system maintenance optimization.

Several fundamental limitations persist across reinforcement learning approaches. No optimality guarantees or approximation bounds exist for most algorithms, convergence depends critically on architecture and hyperparameter selection; learned policies lack cross-problem transferability, and uncertainty quantification remains absent. Research gaps include developing hybrid methods combining RL scaling with exact guarantees for critical subsystems, establishing transfer learning frameworks to reduce retraining costs, integrating safety constraints for high-reliability applications, and advancing theoretical understanding of algorithm selection criteria. Standardized benchmark problems enabling rigorous comparison remain notably absent—current fragmentation across custom problem instances hinders cumulative progress assessment.

5 From Theory to Practice: Industrial Applications and Challenges

The ultimate goal of theoretical research on k-out-of-n system is to address reliability and maintenance decision-making problems in practical engineering. With the advancement of Industry 4.0, the application scope of this model has rapidly expanded from traditional aerospace fields to emerging critical infrastructure sectors such as deep-sea exploration, new energy power generation, and large-scale distributed storage. However, a gap still exists between the simplified assumptions of theoretical models and the "unstructured" characteristics of engineering environments.

5.1 Typical Application Field Cases

5.1.1 Aerospace and Deep-Sea Engineering

Aerospace is a traditional stronghold for k-out-of-n systems, with a focus on fault-tolerant architectures and phased-mission systems reliability. For instance, Rehage et al. [86] (2005) analyzed redundancy management in aircraft fault-tolerant architectures and utilized k-out-of-n logic to ensure system survivability under partial component failures. This logic is also applied to satellite or missile systems performing complex tasks [104].

Notably, the application of this model in deep-sea engineering has increased significantly in recent years. Deep-sea oil and gas production systems operate in extreme high-pressure and corrosive environments, where key components (e.g., subsea connectors, control modules) are typically redundant. Given the scarcity of operational data in deep-sea settings, Cai et al. [105] (2012) introduced Bayesian networks to assess the reliability of deep-sea k-out-of-n systems, efficiently handling uncertain information.

Additionally, for autonomous underwater vehicles (AUVs) used in subsea pipeline monitoring, Rykov et al. [106] (2021) explored a location-dependent preventive maintenance strategy for k-out-of-n systems, pointing out that the maintenance window for deep-sea equipment is extremely narrow. Thus, precise optimization of the k-value threshold is required to balance mission success rates and recovery costs.

5.1.2 *New Energy and Power Systems*

With the advancement of the "dual carbon" goals, reliability modeling based on k-out-of-n logic plays a pivotal role in wind power and photovoltaic (PV) systems. Wang et al. [39] (2022) established a multi-state k-out-of-n: F balanced system model with a rebalancing mechanism to address cell inconsistency in battery energy storage packs. Once the state difference between the maximum and minimum values exceeds the threshold, the system performs peak shaving and valley filling through circuit reconstruction. For floating offshore wind turbines, Zhou and Yin [107] (2019) used k-out-of-n logic to optimize condition-based maintenance strategies to reduce high offshore operation and maintenance costs.

5.1.3 *Big Data Storage and Communication Networks*

In information technology, the k-out-of-n concept has evolved into erasure coding in distributed storage systems. Compared with traditional replication, erasure coding splits data into n fragments, and the original data can be recovered as long as any k fragments are available. To meet the high reliability requirements of cloud data centers, Plank [108] (2005) and subsequent researchers established data durability assessment models based on the (n, k) erasure coding mechanism. Essentially, these models represent the application of k-out-of-n systems at the discrete data block level, focusing on mitigating data loss risks caused by correlated node failures. Lu et al. [109] (2023) extended their research to smart streetlight systems, using the finite Markov chain imbedding method to calculate the joint signature of the system, addressing the cumulative impact of adjacent node failures on overall lighting performance.

Table 5 provides representative industrial applications of k-out-of-n systems across aerospace, new energy, and data storage domains, demonstrating the broad practical relevance of the reviewed models.

5.2 Challenges in Practice

5.2.1 *Accurate Characterization of Complex Correlations*

In practical engineering, the dependencies between components far exceed the simple assumption of statistical independence. Dinh et al. [18] (2022) studied multi-component systems with economic dependencies and disassembly effects, proposing a multi-level opportunistic predictive maintenance strategy. However, the deeper challenge lies in functional dependency, where the degradation of one subsystem alters the load distribution of another k-out-of-n subsystem. In PV or power electronic systems, when some components fail, the remaining components must bear higher current loads, accelerating their degradation. Although Zhang et al. [96] (2022) investigated opportunistic maintenance under load sharing, quantifying the impact of nonlinear load increases on residual life remains a key challenge.

5.2.2 *Dynamic Environments and Incomplete Information*

Most existing models assume static environments or fully observable system states, whereas actual systems often operate in dynamic environments with limited monitoring capabilities. Traditional offline models struggle to respond to sudden shocks. Systems typically face competing failure processes involving internal degradation and external shocks. Zhao et al. [99] (2025) pointed out that in dynamic shock environments, mission abort strategies need to be adjusted based on real-time environmental information and component state distributions, which imposes extremely high requirements on real-time monitoring and decision-making. Research by Wang et al. [35] (2023) demonstrated that efficient performability analysis of dynamic multi-state k-out-of-n: G systems requires careful treatment of data quality and system state estimation to avoid erroneous maintenance decisions.

5.2.3 *"Curse of Dimensionality" and Computational Efficiency in Large-Scale Systems*

As n increases (e.g., in large-scale server clusters or battery packs), the state space expands exponentially. Traditional Markov methods become ineffective when $n > 100$. However, deep reinforcement learning (DRL) shows potential. For example, Zheng et al. [100] (2024) used DRL to solve the joint maintenance and spare parts ordering problem for multi-component systems. However, Ogunfowora and Najjaran [110] (2023) noted, DRL models in industrial applications face the "cold start" problem and the risk of violating safety

Table 5. Industrial applications of k-out-of-n systems.

Application Domain	**Typical System**	Role of k-out-of n Logic	Representative Studies
Aerospace & Deep-sea	Aircraft fault-tolerant architecture AUVs	Redundancy management, system survivability	Rehage *et al.* [86] (2005); Rykov *et al.* [106] (2021)
New Energy & Power	Battery storage packs Offshore wind turbines	Balanced system, condition-based maintenance	Wang *et al.* [39] (2022); Zhou and Yin [107] (2019)
Data Storage & Networks	Distributed storage Smart streetlights	Data durability, joint signature analysis	Plank [108] (2005); Lu *et al.* [109] (2023)

constraints. Specifically, AI may attempt dangerous actions that cause system downtime to explore optimal solutions.

6 Conclusion

In this paper, we have provided a systematic review of reliability modeling and maintenance strategy optimization for k-out-of-n systems. We first introduced the standard k-out-of-n model and its extensions, including multi-state, weighted, consecutive, and network configurations, along with their reliability evaluation methods. We then reviewed maintenance models encompassing time-based maintenance, condition-based maintenance, and economically oriented maintenance strategies. Furthermore, we discussed intelligent maintenance approaches based on reinforcement learning and heuristic algorithms. Finally, we highlighted industrial applications and practical challenges in aerospace, power systems, and data storage.

Despite the significant progress, several limitations remain in the current literature. First, most reliability models assume static environments or fully observable system states, whereas real-world systems often operate under dynamic conditions with incomplete information. Second, the dependencies among components, economic, stochastic, and structural, are frequently simplified or ignored, limiting the practical applicability of existing models. Third, the "curse of dimensionality" in large-scale k-out-of-n systems remains a major computational barrier, and current reinforcement learning approaches lack optimality guarantees and cross-problem transferability. Fourth, standardized benchmark problems for evaluating and comparing maintenance policies are notably absent.

Future research should focus on developing scalable and provably reliable methods for large-scale k-out-of-n systems. Promising directions include integrating domain knowledge (e.g., physics-informed or causality-informed learning) into data-driven maintenance models, designing hybrid frameworks that combine reinforcement learning with exact optimization for critical subsystems, and establishing benchmark datasets to facilitate reproducible research. Additionally, human-in-the-loop maintenance decision-making and the coupling between data trustworthiness and model trustworthiness warrant further investigation. We hope this review will inspire researchers in reliability engineering and operations research to contribute to this important and evolving field.

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The authors declare no conflicts of interest.

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