RESEARCH ARTICLE

On Strategic Planning of a Dynamic Allocation of Vehicles with Stochastic Breakdown to Destinations with Multiple Alternative Routes for Returns Maximization

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Abstract

This paper proposes a dynamic programming (DP) approach for a stochastic multi-period allocation problem, whereby fleet of vehicles are assigned from stations to destinations with multiple alternative routes in order to maximize returns, while the vehicles are subject to random failure. In the process of managing the business, the company is assumed to incur proportional management costs and pay tax to government. The expected returns is modelled as a function of random failure of vehicles due to bad roads and depreciation. The depreciation rate is assumed to follow a straight-line approach. The breakdown rate is modelled as function of the rate of bad roads on the fleets, depreciation rate and expiration time of the fleets. The sum of the probability rate of bad roads on the fleets and depreciation rate, is referred to in this paper, as "decay rate" of the fleets. This paper aim at: (i) modelling the breakdown rates of the vehicles over time; (ii) modelling a stochastic multi-period allocation of the vehicles from stations to destinations with multiple alternative routes and random breakdown of the vehicles; (iii) maximizing the expected net returns of the operations over a period of time; and (iv) determining the optimal management costs and tax payable to government over finite time horizon. Stochastic models and optimal policies of the fleet of vehicles allocation are considered, and problem is solved using DP approach. As a result, the optimal expected net returns from all the destinations and the sum total for all the stations, both for the absence and presence of stochastic break down are obtained. Also obtained are the optimal management costs and tax accrued to government from the investment process over time. Some sensitivity analysis are also carried out in this paper. It was found in this paper that as the breakdown rate of vehicles increases, the expected net returns decreases, and vice versa. Finally, the proposed models were validated using data from some of the transport companies in Nigeria.

Keywords: dynamic allocation of vehicles, stochastic breakdown, straight line depreciation, decay rate, destination.

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1 Introduction

This paper examines a stochastic multi-period allocation of vehicles from different stations to different destinations in the presence of multiple alternative routes in order to maximize returns using DP approach. The fleet of vehicles are assigned from stations to destinations with the policy of choosing the best possible available routes in order to maximize returns, while the vehicles are subject to random failures. These failures will render the vehicles unusable over a certain period of time, and will only be ready for use when all the faults on the vehicles are fixed. It is assumed that l number of vehicles with a capacity of x passengers are to be purchased by a transportation company at the beginning of the planning horizon. The vehicles are to be allocated to M number of destinations, from N number of stations under N number of policies and m number of alternative routes. These vehicles are subject to random breakdown due to usage, bad roads and depreciation. The expected returns from the stations are modelled as a function of breakdown rate, and the breakdown rate is modelled as function of depreciation rate and effect of bad roads on the vehicles over a period of time. The sum of the depreciation rate and rate of effect of bad roads on the vehicles is referred to as "decay rate" of the vehicles. It is expected that every broken-down vehicle will be repaired and await allocation. Since it is uncertain when the vehicles will breakdown, we assumed that the state of the vehicles is stochastic. In order to determine the best possible policies and effective allocation of the vehicles, there is the need for the vehicles to be distributed evenly to all the stations at the beginning of the planning horizon. At the early stage, it is expected that the benefits derived from the vehicles should be high, and declined as the vehicles ages. The decay rate is expected to be low at the early stage of the operation of the business and increase over time.

One of the major challenges facing the effective management of transportation companies in Nigeria is how to maximize returns in the presence of (i) random failure of vehicles, (ii) vehicle decay rate, (iii) management costs, and (iv) taxation. This now leads to the following question: 'How can a dynamic stochastic multi-period allocation techniques be used to solve vehicles routing problem in the presence of vehicles' random failure, decay rate, management costs and taxation? Hence, the aim of this paper is to determine the stations with optimal net returns putting into consideration the rate of breakdown of the vehicles from destinations under multiple alternative routes. The company estimated for the first one week, the expected returns each destinations will yield at the first stage of the operation by multiplying the number of vehicles assigned to a destination, the capacity of the vehicle, the number of trips, seven days and the price per passenger per trip. Also, the company estimated the rate of effect of bad roads on the vehicles allocated to each destinations. When a vehicle breaks down, it goes in for a repair, after which it returns back to the station for re-allocation at the beginning of the next period.

Investment in transportation business could be a lucrative one if it is properly managed and the vehicles are allocated strategically and routes are effectively chosen so as to maximize returns. It is imperative to note that a station for one maybe a destination for another, and vice versa. It is assumed that vehicles allocated from 'A' station to 'B' station and from 'B' station to 'A' station may not yield the same returns as a result of the following factor: the amount charge from each stations may vary, due to difference in demand. It is also assumed that each of the destinations may have a sub-station. Again, it is assumed that the business will incur proportional management costs and pay tax to government over time. The tax rate in this paper, is assumed to be constant in time.

The static formulation of the problem of empty car allocation is based on average time data, so it has many deficiencies. From the practical point of view, the dynamic presentation of this problem, shaped as a deterministic network transshipment is interesting, see Smith [31]. But, dynamic presentation of this problem in a stochastic certain, will be much more fascinating.

White et al. [35] studied a multi-period transportation problem in which they introduced a space-time diagram which allows explicit consideration of the time dimension, and construction of the given network under study. Powell [25] considered the stochastic vehicle allocation problem which addresses the movement of vehicles between locations over a given period of time. It was assumed that the demand for the vehicles to carry loads between locations was uncertain, and vehicles were to handle several loads over the course of the planning horizon. The techniques required tracking the movement of both loaded and empty vehicles, resulting in a network with stochastic flows. Bertsimas et al. [7] introduced and analyzed a model for stochastic and dynamic vehicle routing in which a single, incapacitated vehicle traveling at a certain constant velocity in a Euclidean space must service demands whose time of arrival, location and on-site service are stochastic. The objective was to find a policy to service demands over an infinite planning horizon that minimizes the expected system time of the demands. Bertsimas et al. [8] extended the analysis in several directions by analyzing the problem of several identical vehicles with unlimited capacity. They show that in heavy traffic, the system time was reduced by a certain factor over the single-server cases. The policies were found to improve by a factor of two on the best known system time for the single-server case. They further examined the case in which each vehicle can serve at most certain number of customers before returning to a station. Bojovic [11] examined the nature of the problem involving railroad empty car distribution and the interrelationship between problem solving algorithm techniques and development of practical application systems. The paper gave an exemplified presentation of successfully applied optimal techniques for railroad empty car distribution. The paper highlighted the fleet management problem from different modelling and solving perspectives using optimization methods for empty car allocation and fleet management in freight transportation. Ralphs et al. [27] considered the vehicle routing problem, in which a fixed fleet of delivery vehicles of the same capacity must service deterministic customer demands for a single commodity from a common station at minimum transportation cost. Their combinatorial analysis contains both the Bin Packing Problem and the Traveling salesman problem which served as special cases and conceptually lies at the intersection of these two well-known and studied problems. Smith [31] considered the problems in distributing task allocation and in dynamic vehicle routing problem. In task allocation, they considered a target assignment problem in which a group of vehicles must be allocated to a set of target (or tasks). In dynamic vehicle routing, the vehicles must complete spatially distributed to tasks that arrive sequentially in time. First, they considered a problem in which the vehicles have different capabilities, and each target requires a set of vehicles for its completion, secondly, they considered a problem in which targets have different levels of urgency, and thus the vehicles must be order according to how importance (or urgency) the tasks will be. The urgent tasks must be completed with minimal delay while simultaneously considering and completing less urgent tasks.

But, the problem of vehicle allocation problems with failure (or breakdown) has not been addressed by the above literature (or elsewhere) of dynamic routing problems. As we know, Vehicles are fixed assets subject to depreciation. This paper intend to study this problem by assuming that the vehicles are subject to a straight line approach of depreciation.

In decentralized task allocation the goal is generally to assign vehicles to spatially distributed tasks while maximizing returns. Gerkey et al. [16] gave a classification of task allocation problems, dividing problems into groups based on the number of tasks a vehicle can execute, and the number of vehicles required for a task, for more on this, see Smith [31], Jin et al. [19], Schumacher et al. [30], Godwin et al. [17], Alighanbari et al. [1], Brunet et al. [12], Bethke et al. [9], Moore et al. [21]. Castanon et al. [14] studied the problem of reassigning vehicles in a dynamic environment as new tasks arrive and old tasks expire. Arslan et al. [3] formulated a task allocation problem as a multi-player game and seek to optimize a global utility. Zavlanos et al. [36] developed a task allocation algorithm based on hybrid systems tools, see Smith [31]. For the case of centralized task allocation, see Korte et al. [20] and Burkard [13].

In the case of classic vehicle routing problem which involved planning optimal vehicle routes that provide services to a given set of customers. The routes are planned with the mind that information of the customers are completely known, and thus, the optimization is static, and indeed, combinatorial, see Toth et al. [32]. The most famous vehicle routing problem is the traveling salesperson problem (TSP), see Applegate et al. [2]. TSP involves a fixed number of customer locations (or tasks) with the goal to find the shortest tour which visits each location exactly once, and returns to the starting point, for more of TSP, see Savla et al. [29], Savla et al. [28], Bansal et al. [6], Cordeau et al. [15], Blum et al. [10], Golden et al. [18].

The problem of decentralized vehicle stations whereby the stations are spatially located and the vehicles (with load of passengers) are assigned from the each station to routes. At the new station, the vehicles are loaded and assigned in the same manner. This kind of decentralized station has not been reported in the literature and this paper will examine this problem. Nwozo et al. [24] considered the distribution of goods from stations using dynamic programming principles with error bounds. Nwozo et al. [23], used DPP to consider the allocation of vehicles from single station to different routes in Nigeria for profit maximization. Nkeki [22] considered the allocation of vehicles from different (multiple) stations to different routes putting into consideration the rate of breakdown of the vehicles during the allocation, but failed to put into consideration some fundamental challenging ingredients in transportation business, such as the depreciation of the vehicles over time, management costs, effect taxation on the returns of the investment portfolio, and providing the right allocation policies for the transportation companies. This paper intend to address these challenges. Bakach et al. [4] addresses a vehicle routing problem with a make span objective incorporating both stochastic and correlated travel times. As an alternative to simulation, they developed an approach based on extreme-value theory to estimate the expected make span and shown how this approach can be embedded within an existing routing heuristic. Zhang et al. [37] studied dynamic vehicle routing problems (DVRP) arising in several applications such as technician routing, meal delivery, and parcel shipping. They considered the DVRP with stochastic customer requests (DVRPSR), in which vehicles must be routed dynamically with the goal of maximizing the number of served requests. Also, they modeled the DVRPSR as a multi-stage optimization problem, where the first-stage decision defines route plans for serving scheduled requests. Ruszczyński et al. [34] considers a stochastic programming in vehicle routing problems addresses the inherent uncertainty in real-world logistics, such as fluctuating travel times, varying customers' demands and unpredictable traffic conditions. The methods was found to balance exploration and exploitation, enabled dynamic adjustments to route as new information becomes available, enhanced the reliability and performance of transportation systems. Venkateshan et al. [33] examined the problem of designing delivery routes for vehicles where the vendor has the choice of how much of the demand from a customer to fulfill. For a given a route, the objective was to decide at each customer location, how much demand to satisfy so as to maximize expected profit taking into account a linear penalty cost for unfulfilled demand and the vehicle routing costs. Beatrici et al. [5] considers the joint optimization of fleet size and mix, along with vehicle routing, under uncertain customer demand. They proposed a two-stage stochastic mixed-integer programming model, where first-stage decisions concern the composition of the delivery fleet and the design of consistent baseline routes. In the second stage, approximate recourse actions were introduced to adapt the initial routes in response to realized customer demands. Their objective was to minimize the total delivery cost, including vehicle acquisition, travel distance, and penalty costs for unserved demand. In this paper, we consider the allocation of vehicles from different stations to different destinations with multiple alternative routes while the vehicles are subject to stochastic failure. We further consider the effect of decay rate of the vehicles on the investment portfolio, proportional management costs, taxation on the returns of the investment, and providing the right allocation policies for the transportation companies.

The highlights of this paper are

1. model the breakdown rate as a function of decay rate and life-time of the vehicle.
2. model a stochastic multi-period allocation of vehicles from stations to destinations with multiple alternative routes in the presence of random breakdown.
3. model the expected returns as functions of vehicle failure and decay rate.
4. maximize the expected net returns of the fleet of vehicles over a period of time.
5. determine the management costs of the stations and destinations over a period of time.
6. determine amount of tax to be paid to government from each stations and destination over a period of time.
7. numerical results of optimal net returns for all the stations in the absence and presence of breakdown of vehicles, management costs and amount of tax to be paid are obtained.

The following will tremendously contributes to the literature of stochastic allocation of fleet of vehicles. (1) Stochastic allocation of vehicles with proportional management cost and taxation; (2) Vehicles' allocation with stochastic breakdown; and (3) Vehicles decay rate in optimal allocation of fleet of vehicles. All these are the contributions of this paper.

The remaining parts of the paper is structured as follows: section 2 presents the definitions of some notations and assumptions. In section 3, we present the models and problem formulation, multi-period optimization of the gross return function, dynamic programming formulation of the problem and expected net return as a function of depreciation rate. In section 4, we consider numerical results of the problem. Section 5 concludes the paper.

2 Some Notations and Assumptions

In this section, we present the definitions of some of the basic notations, and assumptions. The notations and assumptions:

2.1 Notations

- $\eta \Rightarrow$ discount factor, $0 < \eta < 1$.
- α the percentage of breakdown (but repaired) vehicles that are expected to join the functioning vehicles.
- $S \Rightarrow$ the set of all vehicles from all the stations.
- $a_y \Rightarrow$ tax rate.
- $T \Rightarrow$ terminal period in the planning horizon.
- $\pi_t^i \Rightarrow$ admissible policy that represents the decision policy of station i at time t.
- $\Pi \Rightarrow$ set of all admissible policy such that $\pi_t^i \in \Pi$.
- $y_t^{\pi_t^i} \Rightarrow$ number of vehicles allocated from station i to destination k under the policy π_t^i at period t.
- $y_{t-1}^{\pi_t^i} \Rightarrow$ number of available vehicles to destination k from station i at period $t-1$ under the policy π_t^i.
- $S_t \Rightarrow$ the state space of the vehicles at the stations at time t.
- $s_t^i \Rightarrow$ number of functioning vehicles at station i at period t; $s_t^i \in S$.
- $\omega_t^i \Rightarrow$ number of random breakdown vehicles in station i at period t.
- $s_0^i \Rightarrow$ number of vehicles for station i at the beginning of the planning horizon.
- v the capacity of a vehicle.
- $x_t^{i,k}$ the number of passengers that board a vehicle to destination k in station i at period t.
- $p_t^{i,k}$ the price paid per passenger for destination k in station i at period t.
- $\tau_t^{i,k}$ the number of trips from station i to destination k at period t.
- $Q_t^{\pi_t^i}(\mathbf{s}_t) \Rightarrow$ objective function of the system, $\mathbf{s}_t \in S_t$ under the policy π_t^i.
- $P(i,k;t) \Rightarrow$ the stochastic rate of breakdown vehicles from station i to destination k via route j at time t.
- $A^i(j,k;t) \Rightarrow$ the transition probability matrix of vehicle from station i taking route j to destination k at time t.

2.2 Assumptions

1. the number of vehicles to be allocated from station i at period t is determined at period $t-1$.
2. the number of vehicles allocated from station i to destination k depends on the available vehicles in station i.
3. all the vehicles have equal capacity and the passengers are readily available to take the spaces.
4. the number of passengers is deterministic.
5. allocated vehicle from station i that successfully get to its destination k is said to have completed a trip, and vice versa.
6. at the beginning of the planning horizon, all the vehicles are allocated, and are all functional.
7. there are no breakdown at the beginning of the planning horizon.

In next section, we define the stochastic multi-period expected return function and formulate the problem as a dynamic program.

3 The Models and Problem Formulation

In this section, we give the model formulation. We assume that the company knows the number of vehicles available for the next period before decision will be made on how to allocate them to various destinations. The number of vehicles to be allocated to the destinations in the next period depends on the number of breakdown at the end of the previous period. The aim of this paper is to maximize the total expected net return, determine the proportional management costs and amount of tax to be paid over a time-horizon.

3.1 Vehicle breakdown as function of decay rate and expected life-time of the vehicle

In this subsection, we consider the rate of breakdown of vehicles as functions of depreciation rate process, negative effect of bad roads on the vehicles and expected life-time of the vehicle at time t. Vehicles are fixed assets and are subject to depreciation. But, one of the factors that brings about rapid reduction in the value of vehicles other than depreciation, is bad roads. Bad roads have contributed tremendously to rapid reduction of vehicles lifespan, hence the need to put it into account. Here, we assume that the rate of vehicle breakdown is a function of the vehicle decay rate. The vehicle decay rate is the sum of the depreciation rate and probability rate of breakdown of the vehicle due to bad roads. The rate of breakdown of the vehicles due to bad roads are determined based on the company historical information and the nature of the roads. It is expected that vehicle's breakdown rate will be monotone increasing in time. Here, we assume that the depreciation rate of the vehicles follows straight line approach. The depreciation rate of the vehicles is valued on a daily basis. We now give the following definitions:

Definition 3.1. *Let ν_0 be the salvage value (in percentage) of a vehicle and D number of days, then depreciation rate ν of a vehicle is defined as*

$$\nu = \frac{100\% - \nu_0}{D} \text{per day.} \tag{1}$$

Definition 3.2. *Let $\tilde{\delta}_{i,j,k}$ be the probability rate that a vehicle from station i to destination k via route j will reduce its lifespan due to bad roads and thereby increase the rate of decay of vehicles, hence, we define the decay rate of vehicle from station i to destination k via route j as the sum of depreciation rate and the probability rate of the effect of bad road on the vehicle, that is,*

$$\begin{aligned} \delta_{i,j,k} &= \nu + \tilde{\delta}_{i,j,k}, i = 1, 2, \dots, N; \\ j &= 1, 2, \dots, m; k = 1, 2, \dots, M. \end{aligned} \tag{2}$$

Remark 3.1. $0 \leq \delta_{i,j,k} < 1$, $i = 1, 2, \dots, N; j = 1, 2, \dots, m; k = 1, 2, \dots, M$.

Definition 3.3. *Given that T the terminal period or expected life-time of the vehicle, and $t \in [1, T]$, we define the stochastic rate of vehicle breakdown, $\gamma_{i,j,k}(T, t)$ from station i to destination k via route j at period t as*

$$\begin{aligned} &\gamma_{i,j,k}(T, t) = e^{-\delta_{i,j,k}(T-t)}, i = 1, 2, \dots, N; \\ &j = 1, 2, \dots, m; k = 1, 2, \dots, M, t \in [1, T]. \end{aligned} \tag{3}$$

Remark 3.2. (*i*) *Since vehicle is a fixed asset, no matter how high its decay rate, it will still has a positive value over time. Hence, the suitable way to model the vehicle breakdown probability is to assume exponential form that will remain positive or zero irrespective of the value of $0 \leq \delta_{i,j,k} < 1$, $i = 1, 2, \dots, N; j = 1, 2, \dots, m; k = 1, 2, \dots, M$.*

(*ii*) *Since T is the expected life-time of the vehicle, we have $\gamma_{i,j,k}(T, T) = 1$. It implies that at time T, the vehicles will attained their salvage value.*

3.2 Multi-Period Stochastic Optimization of Net Return Function

An investment company may want to know at time t, how many vehicles are readily available in station i to be assigned to destination k at time $t-1$. This to a great extent, will inform them on how to make allocation at time t. Note that the destination with the highest breakdown of vehicles may have fewer or no allocations. If vehicles must be allocated to such destinations, it may attract high transport charges on the consumers.

Let $s_t^{i,k}$ be the number of functioning vehicles in station i to be allocated to destination k at period t (note that the decision on how to allocate the vehicles to destinations is yet to be taken at this point, since the decision makers need to know the readily available vehicles for the next period, and destinations with more breakdown vehicles) and $\omega_t^{i,k}$ is the random breakdown of vehicles in station i from destination k at period t, it then follows from assumption 1 that

$$\begin{aligned} s_t^{i,k} = s_{t-1}^{i,k} - \omega_{t-1}^{i,k}, i = 1, 2, \cdots, N; \\ k = 1, 2, \cdots, M; t \in [1, T]. \end{aligned} \tag{4}$$

Note that (4) gives the readily available functional vehicles in station i to be allocated to destination k at time t. This is the difference between the total allocated vehicles from station i to destination k at time $t-1$ and the number of breakdown vehicles in station i that was assigned to destination k at time $t-1$.

It is imperative to note that the company policy is to find the best possible routes to destinations that will minimize the risk of breakdown of the vehicles at time t. Hence, the vehicles are allow to transit from one route to another for the purpose of finding the best possible routes to a destination.

Next, the company will now decides on how to allocate $s_t^{i,k}$ under the policy π_t^i. Since $y_t^{\pi_t^i}(s_t^{i,k})$ is the number of vehicles allocated in station i to destination k under the policy π_t^i at time t, $x_t^{i,k}$ the number of passengers that board a vehicle from station i to destination k at period t, $p_t^{i,k}$ the price paid per passenger from station i to destination k at period t, and $\tau_t^{i,k}$ the number of trips from station i to destination k at period t, then the total amount to be generated by station i from destination k under policy π_t^i per period of time t is given by the quantity $y_t^{\pi_t^i}(s_t^{i,k})x_t^{i,k}p_t^{i,k}\tau_t^{i,k}\tau_t^{i,k}$, $i = 1, 2, \cdots, N; ; k = 1, 2, \cdots, M$, $t \in [1, T]$. It therefore follows that

$$y_t^{\pi_t^i}(s_t^{i,k})x_t^{i,k}p_t^{i,k}\tau_t^{i,k} = (y_t^{\pi_t^i}(s_{t-1}^{i,k}) - \omega_{t-1}^{i,k})x_t^{i,k}p_t^{i,k}\tau_t^{i,k}, \quad i = 1, \cdots, N; k = 1, \cdots, M, t \in [1, T]. \tag{5}$$

It therefore follows that

$$y_t^{\pi_t^i}(s_t^{i,k}) = y_t^{\pi_t^i}(s_{t-1}^{i,k}) - \omega_{t-1}^{i,k}, \quad i = 1, \cdots, N; k = 1, \cdots, M, t \in [1, T]. \tag{6}$$

From (5), we have that station i will lose the sum of $\omega_{t-1}^{i,k}x_t^{i,k}p_t^{i,k}$, $i = 1, 2, \cdots, N; k = 1, 2, \cdots, M, t \in [1, T]$ to vehicle breakdown from destination k at time t. If there is no breakdown of vehicles from station i at time t, (5) becomes

$$\begin{aligned} &y_t^{\pi_t^i}(s_t^{i,k})x_t^{i,k}p_t^{i,k}\tau_t^{i,k} \\ &= y_t^{\pi_t^i}(s_{t-1}^{i,k})x_t^{i,k}p_t^{i,k}\tau_t^{i,k}, \\ &i = 1, \cdots, N; k = 1, \cdots, M, t \in [1, T]. \end{aligned} \tag{7}$$

Note that if the vehicles in the stations have full capacity at all time period t, then (5) becomes

$$\begin{aligned} v y_t^{\pi_t^i}(s_t^{i,k})p_t^{i,k}\tau_t^{i,k} = v y_t^{\pi_t^i}(s_{t-1}^{i,k})p_t^{i,k}\tau_t^{i,k} \\ -v\omega_{t-1}^{i,k}p_t^{i,k}\tau_t^{i,k}, \\ i = 1, 2, \cdots, N; k = 1, 2, \cdots, M, t \in [1, T]. \end{aligned} \tag{8}$$

It then follows from (5) and (8) that

$$\begin{aligned} &y_t^{\pi_t^i}(s_t^{i,k})(v - x_t^{i,k})p_t^{i,k}\tau_t^{i,k} \\ &= (v - x_t^{i,k})(y_t^{\pi_t^i}(s_{t-1}^{i,k}) - \omega_{t-1}^{i,k})p_t^{i,k}\tau_t^{i,k}, \\ &i = 1, 2, \cdots, N; k = 1, 2, \cdots, M, t \in [1, T]. \end{aligned} \tag{9}$$

(9) is the amount lost by station i for not having full capacity or complete passengers to destination k under the policy π_t^i at period t. If all the vehicles have full capacity at all time period, it follows that $v = x_t^{i,k}$. It implies that the company will have more returns if $v = x_t^{i,k}$ and less returns if $v > x_t^{i,k}$. Note that it is not possible for $v < x_t^{i,k}$, since the highest capacity of the vehicles is v.

By assumption 2, we can see that the gross returns for station i to destination k depend on the number of functioning vehicles, availability of passengers and price per destinations.

Next, we consider the management costs of the investment at time t. Let $c_t^{i,k}$ be the costs set aside for the management of the business (costs of maintenance, repair of the vehicles and administrative costs, e.t.c.) for station i to destination k at period t and ξ the proportion set aside from the total returns generated by station i from destination k, as management costs, then the expected costs of managing $y_t^{\pi_t^i}(s_t^{i,k})$ number of vehicles under the policy π_t^i at period t is given by

$$\begin{aligned} &c_t^{i,k} = \xi y_t^{\pi_t^i}(s_t^{i,k})x_t^{i,k}p_t^{i,k}\tau_t^{i,k}, i = 1, 2, \cdots, N; \\ &k = 1, 2, \cdots, M, t \in [1, T]. \end{aligned} \tag{10}$$

Remark 3.3. *Note that* $c_t^{i,k} = \xi x_t^{i,k}p_t^{i,k}\tau_t^{i,k}(y_t^{\pi_t^i}(s_{t-1}^{i,k}) - \omega_{t-1}^{i,k})$. *This means that the quantity* $\xi x_t^{i,k}p_t^{i,k}\tau_t^{i,k}y_t^{\pi_t^i}(s_{t-1}^{i,k})$ *is the amount set aside by station* i *for management costs at time* t. *The quantity* $\xi x_t^{i,k}p_t^{i,k}\tau_t^{i,k}\omega_{t-1}^{i,k}$ *is the amount accrued to station* i *as management costs, but lost due to breakdown of the vehicles at time* t.

We now have the following fundamental definitions.

Definition 3.4. *Let* $\Lambda_{t-1}^{i,k}$ *be the returns-after-management-costs (RAMC) function at period* t. *The RAMC for station* i *from destination* k *at period* t *is defined as*

$$\begin{aligned} &\Lambda_{t-1}^{i,k}(y_t^{\pi_t^i}(s_t^{i,k}), x_t^{i,k}, p_t^{i,k}, \tau_t^{i,k}, c_t^{i,k}) \\ &= (1 - \xi)x_t^{i,k}p_t^{i,k}\tau_t^{i,k}(y_t^{\pi_t^i}(s_{t-1}^{i,k}) - \omega_{t-1}^{i,k}), \\ &i = 1, 2, \cdots, N; k = 1, 2, \cdots, M, t \in [1, T]. \end{aligned} \tag{11}$$

Remark 3.4. *Observe that if* $\xi = 0$, *it implies that there is no management costs incurred. If* $\xi = 1$, *it implies that all the returns are used for vehicles maintenance and administrative expenses which is unrealistic. To avoid these trivial cases, we take* $0 < \xi < 1$. *Again, observe that return is zero if and only if* $y_t^{\pi_t^i}(s_{t-1}^{i,k}) = \omega_{t-1}^{i,k}$. *This simply implies that the returns for station* i *will be zero if all the vehicles are broken down at time* t *under the policy* π_t^i.

It is expected that RAMC should be tax by government, and we then have the following definition:

Definition 3.5. *Let* a_y *be a proportional tax rate on RAMC. The net returns (i.e., RAMC after tax) for station* i *from destination* k *under the policy* π_t^i *at period* t *is defined as*

$$\begin{aligned} &\Psi_{t-1}^{i,k}(y_t^{\pi_t^i}(s_t^{i,k}), x_t^{i,k}, p_t^{i,k}, \tau_t^{i,k}, c_t^{i,k}) \\ &= (1 - a_y)\Lambda_{t-1}^{i,k}(y_t^{\pi_t^i}(s_t^{i,k}), x_t^{i,k}, p_t^{i,k}, \tau_t^{i,k}, c_t^{i,k}) \\ &= (1 - a_y)(1 - \xi)x_t^{i,k}p_t^{i,k}\tau_t^{i,k}(y_t^{\pi_t^i}(s_{t-1}^{i,k}) - \omega_{t-1}^{i,k}), \\ &i = 1, 2, \cdots, N; k = 1, 2, \cdots, M, t \in [1, T]. \end{aligned} \tag{12}$$

The sum total net returns for allocating the vehicles from station i to destination k at period t is define as

$$\begin{aligned} &\textstyle\sum_{k=1}^{M} \Psi_{t-1}^{i,k}(y_t^{\pi_t^i}(s_{t-1}^{i}(\omega_{t-1}^{i}))) : \\ &= \textstyle\sum_{k=1}^{M} \Psi_{t-1}^{i,k}(y_t^{\pi_t^i}(s_t^{i,k}), x_t^{i,k}, p_t^{i,k}, \tau_t^{i,k}, c_t^{i,k}), \\ &i = 1, 2, \cdots, N; t \in [1, T]. \end{aligned}$$

3.3 Expected Discounted Net Returns Function

Since the returns are the amount to be received sometime in future, we discount it with a discount rate η. The expected maximum discounted net return function putting into consideration breakdown rate of vehicles obtained over time under policy π_t^i, $i = 1, 2, \ldots, N$, $t \in [1, T]$ and $\varphi = 1 - \eta$, is

$$\tilde{Q}_t^{\pi_t^i}(s_t^i) := E_t\left[\sum_{k=1}^{M} \varphi^t \Psi_{t-1}^{i,k}(y_t^{\pi_t^i}(s_{t-1}^i(\omega_{t-1}^i)))\right], \quad s_t^i \in S_t^i, i = 1, 2, \ldots, N; t = 1, ..., T \tag{13}$$

subject to

$$\sum_{k=1}^{M} y_t^{\pi_t^i}(s_{t-1}^i(\omega_{t-1}^i)) \le s_t^i, t = 1, ..., T; i = 1, \ldots, N;$$
$$y_t^{\pi_t^i} \ge 0, t = 1, ..., T; \text{for } k = 1, \ldots, M \text{ and } i = 1, \ldots, N.$$

Observe that the breakdown rate of vehicles reduces the net returns function over time, which is an expected result.

Let $Y(s_t^i)$ be the set of possible solution of (13). Conditioning (13) on $s_t^i \in S$, we have the following optimization problem

$$Q^{\pi_t^i}(s_t^i) = E\left[\max_{y_{t'}^{\pi_t^i} \in Y(s_t^i)} \left\{\sum_{t'=t}^{T} \sum_{k=1}^{M} \varphi^{t'} \Psi_{t'}^{i,k}(y_{t'}^{\pi_{t'}^i}(s_{t'-1}^i(\omega_{t'-1}^i)))\right\} |s_t^i\right], \quad i = 1, 2, \cdots, N; t = 1, 2, \cdots, T. \tag{14}$$

(14) maximizes the expected net returns over $Y(s_t^i)$ subject to

$$\sum_{k=1}^{M} y_t^{\pi_t^i}(s_{t-1}^i(\omega_{t-1}^i)) \le s_t^i,$$
$$t = 1, ..., T; y_t^{\pi_t^i} \ge 0, k = 1, ..., M, \text{and } i = 1, \ldots, N.$$

For the return function $\Psi : S^N \longrightarrow \mathbb{R}^N$, if we accumulate the return of the first T-stage and add to it, the terminal return $\phi_T^i(s_T^i) = \sum_{k=1}^{M} \Psi_T^{i,k}(y_T^{\pi_T^i}(s_T^T(\omega_T^i))))$, $i = 1, 2, \cdots, N$, then (14) becomes

$$Q_t^{\pi_t^i}(s_t^i) = E[\max_{y_t^{\pi_t^i} \in Y(s_t^i)} \left\{\sum_{t'=t}^{T-1} \sum_{k=1}^{M} \varphi^{t'} \Psi_{t'}^{i,k}(y_t^{\pi_t^i}(s_{t'-1}^i(\omega_{t'-1}^i)))\right\} + \varphi^T \phi_T^i(s_T^i)|s_t^i], i = 1, \ldots, N \tag{15}$$

subject to

$$\sum_{k=1}^{M} y_t^{\pi_t^i}(s_{t-1}^i(\omega_{t-1}^i)) \le s_t^i, \quad y_t^{\pi_t^i} \ge 0, t = 1, ..., T; i = 1, 2, \cdots, N. \tag{16}$$

Remark 3.5. *(15) and (16) tell us that it is possible to obtain the optimal returns for each of the N stations.*

3.4 Dynamic Programming Formulation

Let s_t^i be a vector representing the state variable of the system at period t and S the state space, we can formulate the problem as a dynamic program. The rate of breakdown vehicles from station i to destination k via route j under the policy π_t^i at period t is given by $A^i(j, k; t)e^{-\delta_{i,j,k}(T-t)}$.

Hence, total rate of breakdown vehicles from the station i under the policy π_t^i at period t is given by $\sum_{k=1}^{M} A^i(j, k; t))e^{-\delta_{i,j,k}(T-t)}$, $i = 1, 2, \cdots, N$, $j = 1, 2, \cdots, m$, $t = 1, 2, \ldots, T$. By definition, it implies that

$$\omega_t^i = \sum_{k=1}^{M} A^i(j, k; t)e^{-\delta_{i,j,k}(T-t)} y_{k,t-1}^{\pi_t^i},$$
$$i = 1, 2, \ldots, N, \ t = 1, 2, \ldots, T.$$

Since α is the percentage of breakdown (but repaired) vehicles that are expected to join the functioning vehicles in period t, then we have

$$s_t^i = s_{t-1}^i - (1 - \alpha) \sum_{k=1}^{M} A^i(j, k; t))e^{-\delta_{i,j,k}(T-t)} y_{k,t-1}^{\pi_t^i}, \quad i = 1, 2, \cdots, N; j = 1, 2, \cdots, m; t = 1, ..., T, \tag{17}$$

which is our transformation equation and s_t^i is stochastic. Setting $1 - \alpha = \beta$ in (17), we obtain,

$$s_t^i = s_{t-1}^i - \beta \sum_{k=1}^{M} A^i(j, k; t))e^{-\delta_{i,j,k}(T-t)} y_{k,t-1}^{\pi_t^i}, \quad i = 1, 2, \cdots, N; j = 1, 2, \cdots, m; t = 1, ..., T, \tag{18}$$

where β is the fraction or percentage of breakdown vehicles that are still under repairs.

Since the policy of company is to make sure the vehicles take the best possible routes to their destinations, hence, we set $y_{k,t}^{\pi_t^i} = A^i(j, k; t))e^{-\delta_{i,j,k}(T-t)} y_{k,t-1}^{\pi_t^i}$, where $A^i(j, k; t)$ is the transition probability matrix of vehicles in station i moving via route j to destination k at time t. Then. (18) compactly becomes

$$s_t^i = s_{t-1}^i - \beta \sum_{k=1}^{M} A^i(j, k; t)P(i, k; t) y_{k,t-1}^{\pi_t^i}, \quad i = 1, 2, \cdots, N; j = 1, 2, \cdots, N; t = 1, ..., T, \tag{19}$$

where

$$A^i(j, k; t) = \begin{pmatrix} A^i(1, 1; t) & A^i(1, 2; t) & A^i(1, 3; t) & \ldots & A^i(1, M; t) \\ A^i(2, 1; t) & A^i(2, 2; t) & A^i(2, 3; t) & \ldots & A^i(2, M; t) \\ A^i(3, 1; t) & A^i(3, 2; t) & A^i(3, 3; t) & \ldots & A^i(3, M; t) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A^i(m, 1; t) & A^i(m, 2; t) & A^i(m, 3; t) & \ldots & A^i(m, M; t) \end{pmatrix},$$

and $P(i, k; t) := \gamma_{i,j,k}(T, t) = e^{-\delta_{i,j,k}(T-t)}$; $i = 1, 2, \cdots, N$; $k = 1, 2, \cdots, M$; $t = 1, ..., T$.

Equivalently,

$$s^i_t = s^i_{t-1} - \beta \begin{pmatrix} A^i(1,1;t) & A^i(1,2;t) & A^i(1,3;t) & \dots & A^i(1,M;t) \\ A^i(2,1;t) & A^i(2,2;t) & A^i(2,3;t) & \dots & A^i(2,M;t) \\ A^i(3,1;t) & A^i(3,2;t) & A^i(3,3;t) & \dots & A^i(3,M;t) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A^i(m,1;t) & A^i(m,2;t) & A^i(m,3;t) & \dots & A^i(m,M;t) \end{pmatrix} \begin{pmatrix} P(i,1;t)y^{\pi^i_t}_{1,t} \\ P(i,2;t)y^{\pi^i_t}_{2,t} \\ P(i,3;t)y^{\pi^i_t}_{3,t} \\ \vdots \\ P(i,M;t)y^{\pi^i_t}_{M,t} \end{pmatrix},$$
$$i = 1, 2, \cdots, N; ; t = 1, ..., T.$$

In this case, the optimal value can be found by computing the value functions $G^{\pi^i_t} : S \to \mathbb{R}^N$, through the optimization problem

$$G^{\pi^i_t}_t(s^i_t) = \left[\max_{y^{\pi^i_t}_t \in Y(s^i_t)} \left\{\sum_{k=1}^M \Psi^{i,k}_{t-1}(y^{\pi^i_t}_{k,t}(s^i_{t-1}(\omega^i_{t-1})))\right\} + \varphi E(G_{t-1}(s^i_t)) | s^i_t\right] \tag{20}$$

subject to

$$\sum_{k=1}^M y^{\pi^i_t}_{k,t}(s^i_{t-1}(\omega^i_{t-1})) \leq s^i_t, y^{\pi^i_t}_{k,t} \geq 0, \quad t = 1, ..., T; k = 1, ..., M, i = 1, \ldots, N. \tag{21}$$

(21) can be re-written as follows:

$$z^i_t + \sum_{k=1}^M y^{\pi^i_t}_{k,t}(s^i_{t-1}(\omega^i_{t-1})) = s^i_t, \tag{22}$$

$z^i_t \geq 0$; $y^{\pi^i_t}_{k,t} \geq 0$, for $t = 1, ..., T$;
for $k = 1, ..., M$ and $i = 1, \ldots, N$.

We therefore make the following proposition.

Proposition 3.1. *If all the functioning vehicles are to be allocated at time period t, then the slack variable $z^i_t = 0$, for all i, so that $s^i_t = \sum_{k=1}^M y^{\pi^i_t}_{k,t}(s^i_{t-1}(\omega^i_{t-1}))$, $i = 1, 2, \cdots, N$, $t = 1, 2, \cdots, T$.*

Proof. The result follows immediately by setting z_i in (22) to zero. Proposition 3.1 states that since all the available functioning vehicles from all the stations must be allocated in the next period, then, z^i_t must be zero for all i and $t \in [1, T]$. This is the slack variable of the problem.

Remark 3.6. *Let s^i_t be the number of vehicles to be allocated from station i that captures the relevant history up to time t, and let $Q^{\pi^i_t}_t(s^i_{t+1})$ be some function measured at $t' \geq t + 1$ conditional on the random variable vector s^i_t, then,*

$$E[E\{Q^{\pi^i_t}_{t'} | s^i_{t+1}\} | s^i_t] = E[Q^{\pi^i_t}_{t'} | s^i_t].$$

It can be shown that the optimization problem $Q^{\pi^i_t}_t(s^i_t)$ is equivalent to the optimality equation $G^{\pi^i_t}_t(s^i_t)$, see Powell [26].

For any given objective function, we desire to find the best policy, π^i_t, that optimizes it, that is, we search for

$$Q^*_t(s^i_t) = \max_{\pi^i_t \in \Pi} Q^{\pi^i_t}_t(s^i_t), t = 1, \ldots, T; i = 1, \ldots, N.$$

This is obtained by solving the optimality equation for station i at period t under the policy π^i_t and is given by

$$G^{\pi^i_t}_t(s^i_t) = \max_{y^{\pi^i_t} \in Y(s^i_t)} \left(\sum_{k=1}^M \Psi^{i,k}_t(y^{\pi^i_t}_{k,t}(s^i_{t-1}(\omega^i_{t-1}))) + \varphi E\left[G^{\pi^i_t}_{t-1}(s^i_t)\right]\right),$$
$$i = 1, 2, \cdots, N; t = 1, 2, \cdots, T. \tag{23}$$

Remark 3.7. *The optimization problem $Q^{\pi^i_t}_t(s^i_t)$ is equivalent to the optimality equation $G^{\pi^i_t}_t(s^i_t)$.*

4 Numerical Results

In this section, we consider the numerical computation on our derived model using MatLab and data collected from an anonymous transportation company in Nigeria. The transport price are obtained from Intercity.ng[1]; Peace Mass Transit (PMT)[2]; ABC Transport PLC[3]. The dynamic programming model for our multi-period and multi-state expected returns function is given as

$$\begin{aligned} G^{\pi^i_t}_t(s^i_t) &= \max_{y^{\pi^i_t} \in Y} \left[\sum_{k=1}^M \Psi^{i,k}_{t-1}(y^{\pi^i_t}_{k,t}(s^i_{t-1}(\omega^i_{t-1}))) + \varphi E(G^{\pi^i_t}_{t-1}(s^i_{t-1}))\right], \\ &= \max_{y^{\pi^i_t}_t \in Y} [\sum_{k=1}^M \Psi^{i,k}_{t-1}(y^{\pi^i_t}_{k,t}(s^i_{t-1}(\omega^i_{t-1}))) \\ &+ \varphi G^{\pi^i_t}_{t-1}\left(s^i_{t-1} - \frac{3}{20}\sum_{k=1}^M A^i(j,k;t)P(i,k;t)y^{\pi^i_t}_{k,t-1}\right)], \end{aligned}$$
$$t = 1, \ldots, T, i = 1, 2, \cdots, N, \tag{24}$$

and

$$y^{\pi^i_t}_{k,t-1} \geq 0, t = 1, 2, \ldots, Ti = 1, 2, \ldots, N; k = 1, 2, \ldots, M.$$

Note that $N = 6$ and $M = 9$.
Therefore, for $\mathbf{s}_t = s^1_t, s^2_t, \ldots, s^6_t$, we have the explicit vsersion of (24) as follows:

$$G^{\pi_t}_t(\mathbf{s}_t) = \max \left(\begin{matrix} \left[\left(\Psi^{1,k}_{t-1}(y^{\pi^1_t}_{k,t}(s^1_{t-1}) + \varphi G^{\pi^1_t}_{t-1}\left(s^1_{t-1} - \frac{3}{20}A^1(j,k;t)P(1,k;t)y^{\pi^1_t}_{k,t-1}\right)\right)\right] \\ \left[\left(\Psi^{2,k}_{t-1}(y^{\pi^2_t}_{k,t}(s^2_{t-1}) + \varphi G^{\pi^2_t}_{t-1}\left(s^2_{t-1} - \frac{3}{20}A^2(j,k;t)P(2,k;t)y^{\pi^2_t}_{k,t-1}\right)\right)\right] \\ \left[\left(\Psi^{3,k}_{t-1}(y^{\pi^3_t}_{k,t}(s^3_{t-1}) + \varphi G^{\pi^3_t}_{t-1}\left(s^3_{t-1} - \frac{3}{20}A^3(j,k;t)P(3,k;t)y^{\pi^3_t}_{k,t-1}\right)\right)\right] \\ \left[\left(\Psi^{4,k}_{t-1}(y^{\pi^4_t}_{k,t}(s^4_{t-1}) + \varphi G^{\pi^4_t}_{t-1}\left(s^4_{t-1} - \frac{3}{20}A^4(j,k;t)P(4,k;t)y^{\pi^4_t}_{k,t-1}\right)\right)\right] \\ \left[\left(\Psi^{5,k}_{t-1}(y^{\pi^5_t}_{k,t}(s^5_{t-1}) + \varphi G^{\pi^5_t}_{t-1}\left(s^5_{t-1} - \frac{3}{20}A^5(j,k;t)P(5,k;t)y^{\pi^5_t}_{k,t-1}\right)\right)\right] \\ \left[\left(\Psi^{6,k}_{t-1}(y^{\pi^6_t}_{k,t}(s^6_{t-1}) + \varphi G^{\pi^6_t}_{t-1}\left(s^6_{t-1} - \frac{3}{20}A^4(j,k;t)P(6,k;t)y^{\pi^6_t}_{k,t-1}\right)\right)\right] \end{matrix} \right),$$
$$t = 1, \ldots, T; j = 1, 2, \ldots, 9; k = 1, 2, \ldots, 9. \tag{25}$$

[1] https://intercity.ng
[2] https://pmt.ng
[3] https://www.abctransport.com

Table 1. The stations and their corresponding destinations.

Benin to	Lagos to	Warri to	Abuja to	PH to	Ibadan to
Sapele R_1^1	Abuja R_1^2	Benin R_1^3	Lagos R_1^4	Benin R_1^5	Benin R_1^6
Ekpoma R_2^1	Benin R_2^2	Lagoa R_2^3	Benin R_2^4	Abuja R_2^5	PH R_2^6
Auchi R_3^1	Ibadan R_3^2	Ibadan R_3^3	Kaduna R_3^4	Lagos R_3^5	Warri R_3^6
Warri R_4^1	Ilorri R_4^2	Abuja R_4^3	Kano R_4^4	Warri R_4^5	Abuja R_4^6
PH R_5^1	Minna R_5^2	Kano R_5^3	Onitcha R_5^4	Onitcha R_5^5	Lagoa R_5^6
Lagos R_6^1	Warri R_6^2	Kaduna R_6^3	PH R_6^4	Ibadan R_6^5	Minna R_6^6
Ibadan R_7^1	PH R_7^2	Onitcha R_7^3	Ibadan R_7^4	Sapele R_7^5	Ilorri R_7^6
Ilorri R_8^1	Kaduna R_8^2	Enugu R_8^3	Minna R_8^4	Auchi R_8^5	Kano R_8^6
Abuja R_9^1	Kano R_9^2	Abia R_9^3	Ilorri R_9^4	Agbor R_9^5	Kaduna R_9^6

Note: PH stands Port Harcourt.

4.1 The data, information for the model and its description

Let $l = 120$ be the number of vehicles (buses) with capacity of $x = 18$ passengers that are purchased to be allocated to six stations at first stage of the operation. These buses are to ply nine different destinations with nine alternative routes from six different stations, and they are subject to random breakdown. The first station is Benin (S_1), follow by Lagos (S_2), follow by Warri (S_3), follow by Abuja (S_4), follow by Port Harcourt (S_5), follow by Ibadan (S_6).

Table 1 shows the six stations and their corresponding nine destinations. That is, R_k^i means destination from station i to destination k. The data used in this paper were collected from one of a transport companies in Nigeria. The data are obtained through interview and interaction with some of the management and staff of an anonymous transport company in Nigeria. The data collected are number of buses allocated to each destination, number of trip a buses undertake each day, the price per trip per passenger, decay rate of a bus, rate of breakdown buses from a given destination, and probability of a bus transiting from one route to another before getting to its final destination.

The expected returns from the first period of operation from station i to destination k are presented in Table 2.

The interpretation of the entries in Table 2:

The first entry gives the information of Benin Station (S_1) that allocate 3 vehicles to destinations, R_1^1, which is Benin to Sapele. The vehicles have capacity of 18 passengers each. The period is 7 days and the fare is 4,500 naira per trip per passenger, which gives a total of 243,000 naira per day, and 1,701,000 naira per week. Similar interpretations also go for the remaining entries.

Table 2. Expected returns (in Naira (N)) of station i from destination k at the beginning of the planning horizon.

	R_1^1	R_2^1	R_3^1	R_4^1	R_5^1	R_6^1	R_7^1	R_8^1	R_9^1
PPS	×N4500	×N4500	×N8000	×N5500	×N10000	×N12000	×N21000	×N20000	×N17500
S_1	(3)(18)(4)	(3)(18)(4)	(2)(18)(4)	(1)(18)(3)	(2)(18)(2)	(1)(18)(2)	(4)(18)(2)	(3)(18)(2)	(1)(18)(2)
	R_1^2	R_2^2	R_3^2	R_4^2	R_5^2	R_6^2	R_7^2	R_8^2	R_9^2
PPS	×36500	×27500	×5750	×7250	×25000	×33000	×32500	×15000	×30000
S_2	(2)(18)(2)	(3)(18)(2)	(3)(18)(4)	(2)(18)(2)	(4)(18)(2)	(1)(18)(2)	(2)(18)(1)	(2)(18)(2)	(1)(18)(1)
	R_1^3	R_2^3	R_3^3	R_4^3	R_5^3	R_6^3	R_7^3	R_8^3	R_9^3
PPS	×4000	×35000	×15000	×22500	×30000	×48750	×15000	×35000	×14000
S_3	(5)(18)(4)	(1)(18)(2)	(1)(18)(2)	(3)(18)(1)	(2)(18)(1)	(4)(18)(1)	(2)(18)(4)	(1)(18)(2)	(1)(18)(3)
	R_1^4	R_2^4	R_3^4	R_4^4	R_5^4	R_6^4	R_7^4	R_8^4	R_9^4
PPS	×34500	×15000	×7500	×11500	×23000	×48000	×22500	×4500	×15000
S_4	(1)(18)(1)	(3)(18)(2)	(2)(18)(4)	(4)(18)(4)	(3)(18)(2)	(2)(18)(1)	(2)(18)(1)	(2)(18)(5)	(1)(18)(2)
	R_1^5	R_2^5	R_3^5	R_4^5	R_5^5	R_6^5	R_7^5	R_8^5	R_9^5
PPS	×10000	×27000	×43000	×11500	×14000	×27500	×11500	×20000	×13500
S_5	(2)(18)(2)	(1)(18)(1)	(4)(18)(1)	(3)(18)(4)	(4)(18)(3)	(3)(18)(1)	(1)(18)(4)	(1)(18)(2)	(1)(18)(4)
	R_1^6	R_2^6	R_3^6	R_4^6	R_5^6	R_6^6	R_7^6	R_8^6	R_9^6
PPS	×11500	×30000	×27500	×11500	×6500	×12500	×5500	×22500	×12500
S_6	(3)(18)(2)	(2)(18)(1)	(4)(18)(1)	(1)(18)(2)	(3)(18)(5)	(2)(18)(2)	(2)(18)(4)	(2)(18)(2)	(1)(18)(2)

Note: $S_i = (s_0^{i,k})(x^i, k_1)(\tau_1^{i,k})$, $k = 1, 2, \cdots, 9$, $i = 1, 2, \cdots, 6$ and R_k^i stands for destination from station i to destination k. PPS stands for Price Per Seat for a vehicle, and N denotes Naira.

Table 3. Optimal net returns, optimal management costs and optimal tax of nine destinations from Benin Station in the absence of breakdown vehicles at time T (in week), given that $\nu_0 = 20\%$, $\xi = 60\%$, $a_y = 5\%$, and $\eta = 30\%$.

S	T	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	R_9
		N	N	N	N	N	N	N	N	N
	1	1.8099e+006	2.0110e+006	2.1450e+006	5.5301e+005	1.3406e+006	8.0438e+005	5.6307e+006	4.0219e+006	1.1731e+006
	2	6.8775e+006	7.6416e+006	8.1511e+006	2.1015e+006	5.0944e+006	3.0567e+006	2.1397e+007	1.5283e+007	4.4576e+006
	3	2.1067e+007	2.3408e+007	2.4968e+007	6.4371e+006	1.5605e+007	9.3630e+006	6.5541e+007	4.6815e+007	1.3654e+007
	4	6.0797e+007	6.7552e+007	7.2056e+007	1.8577e+007	4.5035e+007	2.7021e+007	1.8915e+008	1.3510e+008	3.9405e+007
	5	1.7204e+008	1.9116e+008	2.0390e+008	5.2568e+007	1.2744e+008	7.6463e+007	5.3524e+008	3.8231e+008	1.1151e+008
S_1	6	4.8353e+008	5.3725e+008	5.7307e+008	1.4774e+008	3.5817e+008	2.1490e+008	1.5043e+009	1.0745e+009	3.1340e+008
	7	1.3557e+009	1.5063e+009	1.6067e+009	4.1424e+008	1.0042e+009	6.0253e+008	4.2177e+009	3.0126e+009	8.7868e+008
	8	3.7977e+009	4.2197e+009	4.5010e+009	1.1604e+009	2.8131e+009	1.6879e+009	1.1815e+010	8.4394e+009	2.4615e+009
	20	8.8213e+014	9.8015e+014	1.0455e+015	2.6954e+014	6.5343e+014	3.9206e+014	2.7444e+015	1.9603e+015	5.7175e+014
	26	1.5182e+017	1.6869e+017	1.7993e+017	4.6389e+016	1.1246e+017	6.7475e+016	4.7232e+017	3.3737e+017	9.8401e+016
	27	4.2509e+017	4.7232e+017	5.0381e+017	1.2989e+017	3.1488e+017	1.8893e+017	1.3225e+018	9.4465e+017	2.7552e+017
	1	2.8577e+006	3.1752e+006	3.3869e+006	8.7318e+005	2.1168e+006	1.2701e+006	8.8906e+006	6.3504e+006	1.8522e+006
	2	1.0859e+007	1.2066e+007	6.7738e+006	3.3181e+006	8.0438e+006	4.8263e+006	3.3784e+007	2.4132e+007	7.0384e+006
	3	3.3263e+007	3.6959e+007	3.9423e+007	1.0164e+007	2.4640e+007	1.4784e+007	1.0349e+008	7.3919e+007	2.1560e+007
	4	9.5995e+007	1.0666e+008	1.1377e+008	2.9332e+007	7.1108e+007	4.2665e+007	2.9865e+008	2.1332e+008	6.2219e+007
	5	2.7164e+008	3.0183e+008	3.2195e+008	8.3002e+007	2.0122e+008	1.2073e+008	8.4512e+008	6.0365e+008	1.7607e+008
C_1	6	7.6346e+008	8.4829e+008	9.0484e+008	2.3328e+008	5.6553e+008	3.3932e+008	2.3752e+009	1.6966e+009	4.9484e+008
	7	2.1405e+009	2.3784e+009	2.5369e+009	6.5406e+008	1.5856e+009	9.5136e+008	6.6595e+009	4.7568e+009	1.3874e+009
	8	5.9964e+009	6.6627e+009	7.1068e+009	1.8322e+009	4.4418e+009	2.6651e+009	1.8655e+010	1.3325e+010	3.8866e+009
	20	1.3928e+015	1.5476e+015	1.6508e+015	4.2559e+014	1.0317e+015	6.1904e+014	4.3333e+015	3.0952e+015	9.0277e+014
	26	2.3971e+017	2.6635e+017	2.8410e+017	7.3246e+016	1.7757e+017	1.0654e+017	7.4577e+017	5.3270e+017	1.5537e+017
	27	6.7120e+017	7.4577e+017	7.9549e+017	2.0509e+017	4.9718e+017	2.9831e+017	2.0882e+018	1.4915e+018	4.3503e+017
	1	9.5256e+004	1.0584e+005	1.1290e+005	2.9106e+004	7.0560e+004	4.2336e+004	2.9635e+005	2.1168e+005	6.1740e+004
	2	3.6197e+005	4.0219e+005	4.2900e+005	1.1060e+005	2.6813e+005	1.6088e+005	1.1261e+006	8.0438e+005	2.3461e+005
	3	1.1088e+006	1.2320e+006	1.3141e+006	3.3879e+005	8.2132e+005	4.9279e+005	3.4495e+006	2.4640e+006	7.1865e+005
	4	3.1998e+006	3.5554e+006	3.7924e+006	9.7773e+005	2.3703e+006	1.4222e+006	9.9551e+006	7.1108e+006	2.0740e+006
	5	9.0548e+006	1.0061e+007	1.0732e+007	2.7667e+006	6.7073e+006	4.0244e+006	2.8171e+007	2.0122e+007	5.8689e+006
T_1	6	2.5449e+007	2.8276e+007	3.0161e+007	7.7760e+006	1.8851e+007	1.1311e+007	7.9174e+007	5.6553e+007	1.6495e+007
	7	7.1352e+007	7.9280e+007	8.4565e+007	2.1802e+007	5.2853e+007	3.1712e+007	2.2198e+008	1.5856e+008	4.6246e+007
	8	1.9988e+008	2.2209e+008	2.3689e+008	6.1074e+007	1.4806e+008	8.8836e+007	6.2185e+008	4.4418e+008	1.2955e+008
	20	4.6428e+013	5.1587e+013	5.5026e+013	1.4186e+013	3.4391e+013	2.0635e+013	1.4444e+014	1.0317e+014	3.0092e+013
	26	7.9904e+015	8.8783e+015	9.4701e+015	2.4415e+015	5.9188e+015	3.5513e+015	2.4859e+016	1.7757e+016	5.1790e+015
	27	2.2373e+016	2.4859e+016	2.6516e+016	6.8363e+015	1.6573e+016	9.9437e+015	6.9606e+016	4.9718e+016	1.4501e+016

S_1, C_1 and T_1 represent net returns, management costs, and taxes payable, respectively, from the nine destinations taking by vehicles in Benin Station, and N denotes Naira.

Table 4. Optimal net returns, optimal management costs and optimal tax of nine routes for Benin Station in the presence of breakdown vehicles at time T (in week), given that $\nu_0 = 20\%$, $\xi = 60\%$, $a_y = 5\%$, $\beta = 15\%$, $\beta = 15\%$ and $\eta = 30\%$.

		R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	R_9
S	T	N	N	N	N	N	N	N	N	N
	1	1.8099e+006	2.0110e+006	2.1450e+006	5.5301e+005	1.3406e+006	8.0438e+005	5.6307e+006	4.0219e+006	1.1731e+006
	2	6.3061e+006	7.0067e+006	7.4738e+006	1.9269e+006	4.6712e+006	2.8027e+006	1.9619e+007	1.4013e+007	4.0873e+006
	3	1.7206e+007	1.9118e+007	2.0392e+007	5.2574e+006	1.2745e+007	7.6471e+006	5.3530e+007	3.8236e+007	1.1152e+007
	4	4.2978e+007	4.7753e+007	5.0937e+007	1.3132e+007	3.1835e+007	1.9101e+007	1.3371e+008	9.5506e+007	2.7856e+007
	5	1.0224e+008	1.1360e+008	1.2117e+008	3.1240e+007	7.5733e+007	4.5440e+007	3.1808e+008	2.2720e+008	6.6266e+007
S_1	6	2.3419e+008	2.6021e+008	2.7756e+008	7.1558e+007	1.7347e+008	1.0408e+008	7.2859e+008	5.2042e+008	1.5179e+008
	7	5.1696e+008	5.7440e+008	6.1270e+008	1.5796e+008	1.3922e+007	2.2976e+008	1.6083e+009	1.1488e+009	3.3507e+008
	8	1.0956e+009	1.2174e+009	1.2985e+009	3.3478e+008	8.1159e+008	4.8695e+008	3.4087e+009	2.4348e+009	7.1014e+008
	20	2.5540e+008	2.8378e+008	3.0270e+008	7.8039e+007	1.8918e+008	1.1351e+008	7.9458e+008	5.6755e+008	1.6554e+008
	26	−2.2063e+010	−2.4514e+010	−2.6149e+010	−6.7414e+009	−1.6343e+010	−9.8057e+009	−6.8640e+010	−4.9029e+010	−1.4300e+010
	27	1.2109e+011	1.3454e+011	1.4351e+011	3.6998e+010	8.9693e+010	5.3816e+010	3.7671e+011	2.6908e+011	7.8481e+010
	1	2.8577e+006	3.1752e+006	3.3869e+006	8.7318e+005	2.1168e+006	1.2701e+006	8.8906e+006	6.3504e+006	1.8522e+006
	2	9.9569e+006	1.1063e+007	1.1801e+007	3.0424e+006	7.3755e+006	4.4253e+006	3.0977e+007	2.2127e+007	6.4536e+006
	3	2.7167e+007	3.0186e+007	3.2198e+007	8.3012e+006	2.0124e+007	1.2074e+007	8.4521e+007	6.0372e+007	1.7609e+007
	4	6.7860e+007	7.5400e+007	8.0426e+007	2.0735e+007	5.0266e+007	3.0160e+007	2.1112e+008	1.5080e+008	4.3983e+007
	5	1.6143e+008	1.7937e+008	1.9133e+008	4.9326e+007	1.1958e+008	7.1747e+007	5.0223e+008	3.5874e+008	1.0463e+008
C_1	6	3.6977e+008	4.1086e+008	4.3825e+008	1.1299e+008	2.7391e+008	1.6434e+008	1.1504e+009	8.2172e+008	2.3967e+008
	7	8.1626e+008	9.0695e+008	9.6741e+008	2.4941e+008	6.0463e+008	3.6278e+008	2.5395e+009	1.8139e+009	5.2905e+008
	8	1.7300e+009	1.9222e+009	2.0503e+009	5.2860e+008	1.2815e+009	7.6887e+008	5.3821e+009	3.8444e+009	1.1213e+009
	20	4.0326e+008	4.4807e+008	4.7794e+008	1.2322e+008	2.9871e+008	1.7923e+008	1.2546e+009	8.9614e+008	2.6137e+008
	25	−3.4836e+010	−3.8707e+010	−4.1287e+010	−1.0644e+010	−2.5805e+010	−1.5483e+010	−1.0838e+011	−7.7414e+010	−2.2579e+010
	27	1.9119e+011	2.1243e+011	2.2659e+011	5.8418e+010	1.4162e+011	8.4972e+010	5.9480e+011	4.2486e+011	1.2392e+011
	1	9.5256e+004	1.0584e+005	1.1290e+005	2.9106e+004	7.0560e+004	4.2336e+004	2.9635e+005	2.1168e+005	6.1740e+004
	2	3.3190e+005	3.6878e+005	3.9336e+005	1.0141e+005	2.4585e+005	4.0248e+005	1.0326e+006	7.3755e+005	2.1512e+005
	3	9.0558e+005	1.0062e+006	1.0733e+006	2.7671e+005	6.7080e+005	1.4751e+005	2.8174e+006	2.0124e+006	5.8695e+005
	4	2.2620e+006	2.5133e+006	2.6809e+006	6.9116e+005	1.6755e+006	1.0053e+006	7.0373e+006	5.0266e+006	1.4661e+006
	5	5.3810e+006	5.9789e+006	6.3775e+006	1.6442e+006	3.9859e+006	2.3916e+006	1.6741e+007	1.1958e+007	3.4877e+006
T_1	6	1.2326e+007	1.3695e+007	1.4608e+007	3.7662e+006	9.1302e+006	5.4781e+006	3.8347e+007	2.7391e+007	7.9889e+006
	7	2.7209e+007	3.0232e+007	3.2247e+007	8.3137e+006	2.0154e+007	1.2093e+007	8.4649e+007	6.0463e+007	1.7635e+007
	8	5.7665e+007	6.4073e+007	6.8344e+007	1.7620e+007	4.2715e+007	2.5629e+007	1.7940e+008	1.2815e+008	3.7376e+007
	20	1.3442e+007	1.4936e+007	1.5931e+007	4.1073e+006	9.9571e+006	5.9743e+006	4.1820e+007	2.9871e+007	8.7125e+006
	26	−1.1612e+009	−1.2902e+009	−1.3762e+009	−3.5481e+008	−8.6015e+008	−5.1609e+008	−3.6126e+009	−2.5805e+009	−7.5263e+008
	27	6.3729e+009	7.0810e+009	7.5531e+009	1.9473e+009	4.7207e+009	2.8324e+009	1.9827e+010	1.4162e+010	4.1306e+009

S_1, C_1 and T_1 represent net returns, management costs, and taxes payable, respectively, from the nine destinations taking by vehicles in Benin Station, and N denotes Naira.

Table 5. Optimal net returns, optimal management costs and optimal tax of nine routes for Lagos Station in the presence of breakdown vehicles at time T (in week), given that $\nu_0 = 20\%$, $\xi = 60\%$, $a_y = 5\%$, $\beta = 15\%$ and $\eta = 30\%$.

		R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	R_9
S	T	N	N	N	N	N	N	N	N	N
	1	4.8933e+006	5.5301e+006	2.3126e+006	9.7196e+005	6.7032e+006	2.2121e+006	2.1785e+006	2.0110e+006	1.0055e+006
	2	1.7049e+007	1.9268e+007	8.0576e+006	3.3865e+006	2.3355e+007	7.7073e+006	7.5905e+006	7.0066e+006	3.5033e+006
	3	4.6397e+007	5.2435e+007	2.1927e+007	9.2159e+006	6.3558e+007	2.0974e+007	2.0656e+007	1.9067e+007	9.5337e+006
	4	1.1514e+008	1.3013e+008	5.4416e+007	2.2871e+007	1.5773e+008	5.2050e+007	5.1262e+007	4.7318e+007	2.3659e+007
	5	2.7067e+008	3.0589e+008	1.2792e+008	5.3763e+007	3.7078e+008	1.2236e+008	1.2050e+008	1.1123e+008	5.5617e+007
S_2	6	6.0819e+008	6.8734e+008	2.8743e+008	1.2080e+008	8.3313e+008	2.7493e+008	2.7077e+008	2.4994e+008	1.2497e+008
	7	1.3040e+009	1.4737e+009	6.1628e+008	2.5901e+008	1.7863e+009	5.8948e+008	5.8055e+008	5.3589e+008	
	8	2.6485e+009	2.9931e+009	1.2517e+009	5.2607e+008	3.6280e+009	1.1973e+009	1.1791e+009	1.0884e+009	5.4421e+008
	20	1.1711e+009	1.3235e+009	5.5345e+008	2.3261e+008	1.6042e+009	5.2939e+008	5.2137e+008	4.8126e+008	2.4063e+008
	26	−6.4797e+012	−7.3229e+012	−3.0623e+012	−1.2871e+012	−8.8763e+012	−2.9292e+012	−2.8848e+012	−2.6629e+012	−1.3314e+012
	27	7.3550e+013	8.3122e+013	3.4760e+013	1.4609e+013	1.0075e+014	3.3249e+013	3.2745e+013	3.0226e+013	1.5113e+013
	1	7.7263e+006	8.7318e+006	3.6515e+006	1.5347e+006	1.0584e+007	3.4927e+006	3.4398e+006	3.1752e+006	1.5876e+006
	2	2.6920e+007	3.0423e+007	1.2723e+007	5.3471e+006	3.6877e+007	1.2169e+007	1.1985e+007	1.1063e+007	
	3	7.3259e+007	8.2793e+007	3.4622e+007	1.4551e+007	1.0035e+008	3.3117e+007	3.2615e+007	3.0106e+007	1.5053e+007
	4	1.8180e+008	2.0546e+008	8.5920e+007	3.6111e+007	2.4904e+008	8.2185e+007	8.0939e+007	7.4713e+007	3.7357e+007
	5	4.2737e+008	4.8299e+008	2.0198e+008	8.4889e+007	5.8544e+008	1.9320e+008	1.9027e+008	1.7563e+008	8.7817e+007
C_2	6	9.6030e+008	1.0853e+009	4.5384e+008	1.9074e+008	1.3155e+009	4.3411e+008	4.2753e+008	3.9464e+008	1.9732e+008
	7	2.0590e+009	2.3269e+009	9.7307e+008	4.0897e+008	2.8205e+009	9.3076e+008	9.1666e+008	8.4614e+008	4.2307e+008
	8	4.1818e+009	4.7260e+009	1.9763e+009	8.3063e+008	5.7285e+009	1.8904e+009	1.8618e+009	1.7185e+009	8.5927e+008
	20	1.8491e+009	2.0897e+009	8.7387e+008	3.6728e+008	2.5329e+009	8.3587e+008	8.2321e+008	7.5988e+008	3.7994e+008
	25	−1.0231e+013	−1.1563e+013	−4.8352e+012	−2.0322e+012	−1.4015e+013	−4.6250e+012	−4.5549e+012	−4.2045e+012	−2.1023e+012
	27	1.1613e+014	1.3125e+014	5.4884e+013	2.3067e+013	1.5909e+014	5.2498e+013	5.1703e+013	4.7726e+013	2.3863e+013
	1	2.5754e+005	2.9106e+005	1.2172e+005	5.1156e+004	3.5280e+005	1.1642e+005	1.1466e+005	1.0584e+005	5.2920e+004
	2	8.9734e+005	1.0141e+006	4.2408e+005	1.7824e+005	1.2292e+006	4.0564e+005	3.9950e+005	3.6877e+005	1.8438e+005
	3	2.4420e+006	2.7598e+006	1.1541e+006	4.8505e+005	3.3452e+006	1.1039e+006	1.0872e+006	1.0035e+006	5.0177e+005
	4	6.0601e+006	6.8487e+006	2.8640e+006	1.2037e+006	8.3015e+006	2.7395e+006	2.6980e+006	2.4904e+006	1.2452e+006
	5	1.4246e+007	1.6100e+007	6.7326e+006	2.8296e+006	1.9515e+007	6.4399e+006	6.3423e+006	5.8544e+006	2.9272e+006
T_2	6	3.2010e+007	3.6176e+007	1.5128e+007	6.3581e+006	4.3849e+007	1.4470e+007	1.4251e+007	1.3155e+007	6.5774e+006
	7	6.8632e+007	7.7563e+007	3.2436e+007	1.3632e+007	9.4016e+007	3.1025e+007	3.0555e+007	2.8205e+007	1.4102e+007
	8	1.3939e+008	.5753e+008	6.5878e+007	2.7688e+007	1.9095e+008	6.3013e+007	6.2059e+007	5.7285e+007	2.8642e+007
	20	6.1635e+007	6.9656e+007	2.9129e+007	1.2243e+007	8.4432e+007	2.7862e+007	2.7440e+007	2.5329e+007	1.2665e+007
	26	−3.4104e+011	−3.8542e+011	−1.6117e+011	−6.7740e+010	−4.6717e+011	−1.5417e+011	−1.5183e+011	−1.4015e+011	−7.0076e+010
	27	3.8711e+012	4.3748e+012	1.8295e+012	7.6891e+011	5.3028e+012	1.7499e+012	1.7234e+012	1.5909e+012	7.9543e+011

S_2, C_2 and T_2 represent net returns, management costs, and taxes payable, respectively, from the nine destinations taking by vehicles in Benin Station, and N denotes Naira.

Table 6. Optimal net returns, optimal management costs and optimal tax of nine routes for Warri Station in the presence of breakdown vehicles at time T (in week), given that $\nu_0 = 20\%$, $\xi = 60\%$, $a_y = 5\%$, $\beta = 15\%$ and $\eta = 30\%$.

		R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	R_9
S	T	N	N	N	N	N	N	N	N	N
	1	2.6813e+006	2.3461e+006	1.0055e+006	2.2623e+006	2.0110e+006	6.1334e+006	4.0219e+006	2.3461e+006	1.4077e+006
	2	1.1054e+007	9.6721e+006	4.1452e+006	9.3267e+006	8.2904e+006	2.5286e+007	1.6581e+007	9.6721e+006	5.8033e+006
	3	3.6528e+007	3.1962e+007	1.3698e+007	3.0821e+007	2.7396e+007	8.3559e+007	5.4793e+007	3.1962e+007	1.9177e+007
	4	1.1194e+008	9.7950e+007	4.1979e+007	9.4452e+007	8.3957e+007	2.5607e+008	1.6791e+008	9.7950e+007	5.8770e+007
	5	3.2823e+008	2.8720e+008	1.2309e+008	2.7695e+008	2.4617e+008	7.5083e+008	4.9235e+008	2.8720e+008	1.7232e+008
S_3	6	9.2530e+008	8.0964e+008	3.4699e+008	7.8072e+008	6.9398e+008	2.1166e+009	1.3880e+009	8.0964e+008	4.8578e+008
	7	2.4979e+009	2.1856e+009	9.3670e+008	2.1076e+009	1.8734e+009	5.7139e+009	3.7468e+009	2.1856e+009	1.3114e+009
	8	6.4026e+009	5.6023e+009	2.4010e+009	5.4022e+009	4.8020e+009	1.4646e+010	9.6040e+009	5.6023e+009	3.3614e+009
	20	4.3457e+010	3.8025e+010	1.6296e+010	3.6667e+010	3.2592e+010	9.9407e+010	6.5185e+010	3.8025e+010	2.2815e+010
	26	−2.0601e+015	−1.8026e+015	−7.7253e+014	−1.7382e+015	−1.5451e+015	−4.7124e+015	−3.0901e+015	−1.8026e+015	−1.0815e+015
	27	3.8953e+016	3.4084e+016	1.4607e+016	3.2867e+016	2.9215e+016	8.9106e+016	5.8430e+016	3.4084e+016	2.0450e+016
	1	4.2336e+00	63.7044e+006	1.5876e+006	3.5721e+006	3.1752e+006	9.6844e+006	6.3504e+006	3.7044e+006	2.2226e+006
	2	1.7453e+007	1.5272e+007	6.5451e+006	1.4726e+007	1.3090e+007	3.9925e+007	2.6180e+007	1.5272e+007	9.1631e+006
	3	5.7676e+007	5.0467e+007	2.1629e+007	4.8664e+007	4.3257e+007	1.3193e+008	8.6515e+007	5.0467e+007	3.0280e+007
	4	1.7675e+008	1.5466e+008	6.6282e+007	1.4913e+008	1.3256e+008	4.0432e+008	2.6513e+008	1.5466e+008	9.2795e+007
C_3	5	5.1826e+008	4.5348e+008	1.9435e+008	4.3728e+008	3.8870e+008	1.1855e+009	7.7739e+008	4.5348e+008	2.7209e+008
	6	1.4610e+009	1.2784e+009	5.4788e+008	1.2327e+009	1.0958e+009	3.3420e+009	2.1915e+009	1.2784e+009	7.6703e+008
	7	3.9440e+009	3.4510e+009	1.4790e+009	3.3277e+009	2.9580e+009	9.0219e+009	5.9160e+009	3.4510e+009	2.0706e+009
	8	1.0109e+010	8.8457e+009	3.7910e+009	8.5298e+009	7.5821e+009	2.3125e+010	1.5164e+010	8.8457e+009	5.3074e+009
	20	6.8616e+010	6.0039e+010	2.5731e+010	5.7895e+010	5.1462e+010	1.5696e+011	1.0292e+011	6.0039e+010	3.6023e+010
	26	−3.2528e+015	−2.8462e+015	−1.2198e+015	−2.7445e+015	−2.4396e+015	−7.4407e+015	−4.8791e+015	−2.8462e+015	−1.7077e+015
	27	6.1505e+016	5.3817e+016	2.3064e+016	5.1895e+016	4.6129e+016	1.4069e+017	9.2258e+016	5.3817e+016	3.2290e+016
	1	1.4112e+005	1.2348e+005	5.2920e+004	1.1907e+005	1.0584e+005	3.2281e+005	2.1168e+005	1.2348e+005	7.4088e+004
	2	5.8178e+005	5.0906e+005	2.1817e+005	4.9088e+005	4.3634e+005	1.3308e+006	8.7267e+005	5.0906e+005	3.0544e+005
	3	1.9225e+006	1.6822e+006	7.2096e+005	1.6221e+006	1.4419e+006	4.3978e+006	2.8838e+006	1.6822e+006	1.0093e+006
	4	5.8917e+006	5.1553e+006	2.2094e+006	4.9712e+006	4.4188e+006	1.3477e+007	8.8376e+006	5.1553e+006	3.0932e+006
	5	1.7275e+007	1.5116e+007	6.4783e+006	1.4576e+007	1.2957e+007	3.9517e+007	2.5913e+007	1.5116e+007	9.0696e+006
T_3	6	4.8700e+007	4.2613e+007	1.8263e+007	4.1091e+007	3.6525e+007	1.1140e+008	7.3050e+007	4.2613e+007	2.5568e+007
	7	1.3147e+008	1.1503e+008	4.9300e+007	1.1092e+008	9.8600e+007	3.0073e+008	1.9720e+008	1.1503e+008	6.9020e+007
	8	3.3698e+008	2.9486e+008	1.2637e+008	2.8433e+008	2.5274e+008	7.7084e+008	5.0547e+008	2.9486e+008	1.7691e+008
	20	2.2872e+009	2.0013e+009	8.5770e+008	1.9298e+009	1.7154e+009	5.2319e+009	3.4308e+009	2.0013e+009	1.2008e+009
	26	−1.0843e+014	−9.4872e+013	−4.0659e+013	−9.1484e+013	−8.1319e+013	−2.4802e+014	−1.6264e+014	−9.4872e+013	−5.6923e+013
	27	2.0502e+015	1.7939e+015	7.6881e+014	1.7298e+015	1.5376e+015	4.6898e+015	3.0753e+015	1.7939e+015	1.0763e+015

S_3, C_3 and T_3 represent net returns, management costs, and taxes payable, respectively, from the nine destinations taking by vehicles in Benin Station, and N denotes Naira.

Since 120 buses were purchased and distributed equally to all the six stations at the beginning of the planning horizon and the stations allocated the buses in the first stage of the operation as follows: At time 0, we have

$$y_{k,0}^{\pi_0^i} = (0,0,0,0,0,0,0,0,0), i = 1,2,\ldots,6.$$

At time 1, Benin Statiion has the following allocation poilcy to all the destinations:

$$s_0^1 = y_{k,1}^{\pi_1^1} = (3,3,2,1,2,1,4,3,1).$$

At time 1, Lagos Statiion has the following allocation poilcy to all the destinations:

$$s_0^2 = y_{k,1}^{\pi_1^2} = (2,3,3,2,4,1,2,2,1).$$

At time 1, Warri Statiion has the following allocation poilcy to all the destinations:

$$s_0^3 = y_{k,1}^{\pi_1^3} = (5,1,1,3,2,4,2,1,1).$$

At time 1, Abuja Statiion has the following allocation poilcy to all the destinations:

$$s_0^4 = y_{k,1}^{\pi_1^4} = (1,3,2,4,3,2,2,2,1).$$

At time 1, PH Statiion has the following allocation poilcy to all the destinations:

$$s_0^5 = y_{k,1}^{\pi_1^5} = (2,1,4,3,4,3,1,1,1).$$

At time 1, Ibadan Statiion has the following allocation poilcy to all the destinations:

$$s_0^6 = y_{k,1}^{\pi_1^6} = (3,2,4,1,3,2,2,2,1).$$

And $x_1^{i,k} = 18$ for all i and k. The company further set aside $\xi = 60\%$ of the total returns from each stations as management costs at time period t. Again, $a_y = 5\%$ of the remainder is paid to government as tax. It was further estimated that out of the number of breakdown vehicles per week, 85% will join the functioning vehicles in the next period. The aim of this paper is to determine the expected optimal net return, optimal management costs and optimal tax payable over a 27-week period.

The decay rate matrix for the company that represent the effect of bad roads on the vehicles from the station i to destination k via route j is given as

$$\tilde{\delta}_{i,j,k} = \begin{pmatrix} \frac{1}{10} & \frac{1}{30} & \frac{4}{45} & \frac{2}{45} & \frac{1}{18} & \frac{1}{145} & \frac{7}{180} & \frac{1}{26} & \frac{1}{20} \\ \frac{1}{35} & \frac{1}{13} & \frac{1}{9} & \frac{2}{15} & \frac{2}{35} & \frac{2}{35} & \frac{3}{40} & \frac{1}{45} & \frac{1}{21} \\ \frac{4}{45} & \frac{2}{9} & \frac{1}{16} & \frac{1}{45} & \frac{1}{18} & \frac{2}{23} & \frac{2}{45} & \frac{1}{15} & \frac{1}{22} \\ \frac{2}{45} & \frac{2}{15} & \frac{1}{45} & \frac{1}{12} & \frac{1}{15} & \frac{1}{18} & \frac{4}{45} & \frac{1}{12} & \frac{1}{23} \\ \frac{1}{18} & \frac{2}{35} & \frac{1}{18} & \frac{1}{15} & \frac{1}{17} & \frac{2}{23} & \frac{1}{45} & \frac{3}{40} & \frac{1}{24} \\ \frac{1}{45} & \frac{2}{35} & \frac{2}{23} & \frac{1}{18} & \frac{1}{40} & \frac{1}{15} & \frac{1}{9} & \frac{1}{30} & \frac{1}{25} \\ \frac{7}{180} & \frac{3}{40} & \frac{2}{45} & \frac{1}{15} & \frac{1}{45} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{26} \\ \frac{1}{13} & \frac{1}{45} & \frac{1}{15} & \frac{1}{12} & \frac{3}{40} & \frac{1}{30} & \frac{1}{12} & \frac{1}{18} & \frac{1}{30} \end{pmatrix}. \tag{26}$$

The following transition probability matrix represents the company policies on vehicle allocation from station i to destination k via route j.

$$A^i(j,k;t) = \left\{ \begin{matrix} \frac{3}{20} & \frac{3}{20} & \frac{1}{10} & \frac{1}{20} & \frac{1}{10} & \frac{1}{20} & \frac{3}{20} & \frac{1}{5} & \frac{1}{20} \\ \frac{1}{10} & \frac{3}{20} & \frac{3}{20} & \frac{1}{10} & \frac{1}{5} & \frac{1}{20} & \frac{1}{10} & \frac{1}{10} & \frac{1}{20} \\ \frac{1}{4} & \frac{1}{20} & \frac{1}{20} & \frac{3}{20} & \frac{1}{10} & \frac{1}{5} & \frac{1}{10} & \frac{1}{20} & \frac{1}{20} \\ \frac{1}{20} & \frac{3}{20} & \frac{1}{10} & \frac{1}{5} & \frac{3}{20} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{20} \\ \frac{1}{10} & \frac{1}{20} & \frac{1}{5} & \frac{3}{20} & \frac{1}{5} & \frac{3}{20} & \frac{1}{20} & \frac{1}{20} & \frac{1}{20} \\ \frac{3}{20} & \frac{1}{10} & \frac{1}{5} & \frac{1}{20} & \frac{3}{20} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{20} \\ \frac{1}{20} & \frac{1}{20} & \frac{1}{10} & \frac{3}{20} & \frac{1}{5} & \frac{3}{20} & \frac{3}{20} & \frac{1}{10} & \frac{1}{20} \\ \frac{1}{20} & \frac{3}{20} & \frac{1}{4} & \frac{1}{10} & \frac{1}{20} & \frac{1}{20} & \frac{1}{10} & \frac{3}{20} & \frac{1}{10} \end{matrix} \right\}. \tag{27}$$

Remark 4.1. *Note that the optimal policies for the stations give the management the actual number of vehicle(s) that should be allocated to each routes. It can be searched by computing:* $\pi_{i,t}^* = \max_{t\in T} \sum_{k=1}^{M} e^{\delta_{i,j,k}t} A^i(i,j,k;t) = \max_{t\in T} \sum_{k=1}^{M} e^{\delta_{i,j,k}t}\pi_{i,k}, i = 1,2,\ldots,N, j = 1,2,\ldots,m; t = 1,2,\ldots,T.$ *The maximum policy will be multiplied by the number of vehicles allocated to the station to obtain the actual number of vehicle(s) for each routes at time period t.*

Table 7. Optimal net returns, optimal management costs and optimal tax of nine routes for Abuja Station in the presence of breakdown vehicles at time T (in week), given that $\nu_0 = 20\%$, $\xi = 60\%$, $a_y = 5\%$, $\beta = 15\%$ and $\eta = 30\%$.

		R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	R_9
S	T	N	N	N	N	N	N	N	N	N
	1	1.1563e+006	3.0164e+006	2.0110e+006	6.1669e+006	4.6252e+006	3.2175e+006	1.5082e+006	1.5082e+006	1.0055e+006
	2	2.3027e+006	1.0501e+007	7.0008e+006	2.1469e+007	1.6102e+007	1.1201e+007	5.2506e+006	5.2506e+006	3.5004e+006
	3	1.0929e+007	2.8510e+007	1.9007e+007	5.8287e+007	4.3716e+007	3.0411e+007	1.4255e+007	1.4255e+007	9.5034e+006
	4	2.6999e+007	7.0433e+007	4.6955e+007	1.4400e+008	1.0800e+008	7.5128e+007	3.5216e+007	3.5216e+007	2.3478e+007
	5	6.2996e+007	1.6434e+008	1.0956e+008	3.3598e+008	2.5198e+008	1.7529e+008	8.2169e+007	8.2169e+007	5.4779e+007
S_4	6	1.3994e+008	3.6506e+008	2.4337e+008	7.4635e+008	5.5976e+008	3.8940e+008	1.8253e+008	1.8253e+008	1.2169e+008
	7	2.9504e+008	7.6967e+008	5.1311e+008	1.5735e+009	1.1802e+009	8.2098e+008	3.8483e+008	3.8483e+008	2.5656e+008
	8	5.8490e+008	1.5258e+009	1.0172e+009	3.1195e+009	2.3396e+009	1.6276e+009	7.6292e+008	7.6292e+008	5.0861e+008
	20	−9.1743e+008	−2.3933e+009	−1.5955e+009	−4.8930e+009	−3.6697e+009	−2.5528e+009	−1.1966e+009	−1.1966e+009	−7.9777e+008
	26	1.8022e+013	4.7015e+013	3.1343e+013	9.6119e+013	7.2089e+013	5.0149e+013	2.3507e+013	2.3507e+013	1.5672e+013
	27	−2.5899e+014	−6.7562e+014	−4.5041e+014	−1.3813e+015	−1.0359e+015	−7.2066e+014	−3.3781e+014	−3.3781e+014	−2.2521e+014
	1	1.8257e+006	4.7628e+006	3.1752e+006	9.7373e+006	7.3030e+006	5.0803e+006	2.3814e+006	2.3814e+006	1.5876e+006
	2	6.3560e+006	1.6581e+007	1.1054e+007	3.3899e+007	2.5424e+007	1.7686e+007	8.2904e+006	8.2904e+006	5.5270e+006
	3	1.7256e+007	4.5016e+007	3.0011e+007	9.2033e+007	6.9025e+007	4.8017e+007	2.2508e+007	2.2508e+007	1.5005e+007
	4	4.2630e+007	1.1121e+008	7.4140e+007	2.2736e+008	1.7052e+008	1.1862e+008	5.5605e+007	5.5605e+007	3.7070e+007
	5	9.9468e+007	2.5948e+008	1.7299e+008	5.3049e+008	3.9787e+008	2.7678e+008	1.2974e+008	1.2974e+008	8.6494e+007
C_4	6	2.2096e+008	5.7641e+008	3.8428e+008	1.1784e+009	8.8383e+008	6.1484e+008	2.8821e+008	2.8821e+008	1.9214e+008
	7	4.6585e+008	1.2153e+009	8.1018e+008	2.4845e+009	1.8634e+009	1.2963e+009	6.0763e+008	6.0763e+008	4.0509e+008
	8	9.2353e+008	2.4092e+009	1.6061e+009	4.9255e+009	3.6941e+009	2.5698e+009	1.2046e+009	1.2046e+009	8.0307e+008
	20	−1.4486e+009	−3.7789e+009	−2.5193e+009	−7.7257e+009	−5.7943e+009	−4.0308e+009	−1.8894e+009	−1.8894e+009	−1.2596e+009
	25	2.8456e+013	7.4234e+013	4.9489e+013	1.5177e+014	1.1383e+014	7.9183e+013	3.7117e+013	3.7117e+013	2.4745e+013
	27	−4.0893e+014	−1.0668e+015	−7.1118e+014	−2.1809e+015	−1.6357e+015	−1.1379e+015	−5.3338e+014	−5.3338e+014	−3.5559e+014
	1	6.0858e+004	1.5876e+005	1.0584e+005	3.2458e+005	2.4343e+005	1.6934e+005	7.9380e+004	7.9380e+004	5.2920e+004
	2	2.1187e+005	5.5270e+005	3.6846e+005	1.1300e+006	8.4747e+005	5.8954e+005	2.7635e+005	2.7635e+005	1.8423e+005
	3	5.7521e+005	1.5005e+006	1.0004e+006	3.0678e+006	2.3008e+006	1.6006e+006	7.5027e+005	7.5027e+005	5.0018e+005
	4	1.4210e+006	3.7070e+006	2.4713e+006	7.5787e+006	5.6840e+006	3.9541e+006	1.8535e+006	1.8535e+006	1.2357e+006
	5	3.3156e+006	8.6494e+006	5.7662e+006	1.7683e+007	1.3262e+007	9.2260e+006	4.3247e+006	4.3247e+006	2.8831e+006
T_4	6	7.3653e+006	1.9214e+007	1.2809e+007	3.9282e+007	2.9461e+007	2.0495e+007	9.6069e+006	9.6069e+006	6.4046e+006
	7	1.5528e+007	4.0509e+007	2.7006e+007	8.2818e+007	6.2114e+007	4.3209e+007	2.0254e+007	2.0254e+007	1.3503e+007
	8	3.0784e+007	8.0307e+007	5.3538e+007	1.6418e+008	1.2314e+008	8.5661e+007	4.0153e+007	4.0153e+007	2.6769e+007
	20	−4.8286e+007	−1.2596e+008	−8.3975e+007	−2.5752e+008	−1.9314e+008	−1.3436e+008	−6.2981e+007	−6.2981e+007	−4.1988e+007
	26	9.4854e+011	2.4745e+012	1.6496e+012	5.0589e+012	3.7942e+012	2.6394e+012	1.2372e+012	1.2372e+012	8.2482e+011
	27	−1.3631e+013	−3.5559e+013	−2.3706e+013	−7.2698e+013	−5.4523e+013	−3.7929e+013	−1.7779e+013	−1.7779e+013	−1.1853e+013

S_4, C_4 and T_4 represent net returns, management costs, and taxes payable, respectively, from the nine destinations taking by vehicles in Benin Station, and N denotes Naira.

Remark 4.2. *The fact is that the decisions or policies taken by the stations are mainly from the fleet managers. A better policy yields a higher returns, and vice versa. So, stations returns is a dependent of managerial skills and policies, that is, high return shows effective managerial policy. Hence, a destination with a highest return has the best policy, and the station with the highest returns has the best managerial policy.*

Table 3 shows the optimal net returns, optimal management costs and optimal tax of nine destinations from Benin Station in the absence of breakdown vehicles at time T (in week), given that $\nu_0 = 20\%$, $\xi = 60\%$, $a_y = 5\%$, and $\eta = 30\%$. The table is made up of three compartments. The first compartment represents the discounted expected optimal returns from the nine destinations from Benin Station over a 27-week. The second compartment represents the management costs for each of the destination over a period of 27 weeks. The third compartment represents the amount of tax payable to government from each of the destination over a period of 27 weeks. From the first one week of the first compartment of Table 3, Benin to Sapele yields a net return of the sum of 1,809,900 Naira, in the second week, it yields a net return of 6,877,500 Naira, in the third week it yields the sum of 21,067,000 Naira, in the eighth week, the return is 3,797,700,000 Naira, in the 20th week, it yields the sum of 1.5182e+017 Naira, and at the 27th week, the return is 4.2509e+017 Naira. Similar discussion go to destination 2- destination 9 in the first compartment. It is imperative to observe that after 27 weeks of the operations, Benin to Ibadan yields the highest return of 1.3225e+018 Naira, follow by Benin to Ilorri with the return of 9.4465e+017 Naira, follow by Benin to Auchi with the return of 5.0381e+017 Naira and so on, but Benin to Warri yields the least return of 1.2989e+017 Naira. From the first one week of the second compartment of Table 3, Benin to Sapele yields a management costs of 2.8577e+006 Naira, in the second week, it yields a management of 1.0859e+007 Naira, in the third week it yields a management costs of 3.3263e+007 Naira, in the eighth week, a management costs is 5.9964e+009 Naira, in the 20th week, it yields the sum of 1.3928e+015 Naira, and at the 27th week, the management costs is 6.7120e+017 Naira. Similar discussion go to destination 2- destination 9 in the second compartment. It is imperative to observe that after 27 weeks of the operations, Benin to Ibadan yields the highest management costs of 2.0882e+018 Naira,

Table 8. Optimal net returns, optimal management costs and optimal tax of nine routes for Port Harcourt Station in the presence of breakdown vehicles at time T (in week), given that $\nu_0 = 20\%$, $\xi = 60\%$, $a_y = 5\%$, $\beta = 15\%$ and $\eta = 30\%$.

		R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	R_9
S	T	N	N	N	N	N	N	N	N	N
	1	$1.3406e+006$	$9.0493e+005$	$5.7648e+006$	$4.6252e+006$	$5.6307e+006$	$2.7651e+006$	$1.5417e+006$	$1.3406e+006$	$1.8099e+006$
	2	$4.6269e+006$	$3.1231e+006$	$1.9895e+007$	$1.5963e+007$	$1.9433e+007$	$9.5429e+006$	$5.3209e+006$	$4.6269e+006$	$6.2463e+006$
	3	$1.2447e+007$	$8.4019e+006$	$5.3523e+007$	$4.2943e+007$	$5.2278e+007$	$2.5672e+007$	$1.4314e+007$	$1.2447e+007$	$1.6804e+007$
	4	$3.0495e+007$	$2.0584e+007$	$1.3113e+008$	$1.0521e+008$	$1.2808e+008$	$6.2896e+007$	$3.5069e+007$	$3.0495e+007$	$4.1168e+007$
	5	$7.0730e+007$	$4.7743e+007$	$3.0414e+008$	$2.4402e+008$	$2.9707e+008$	$1.4588e+008$	$8.1340e+007$	$7.0730e+007$	$9.5486e+007$
S_5	6	$1.5686e+008$	$1.0588e+008$	$6.7451e+008$	$5.4117e+008$	$6.5882e+008$	$3.2353e+008$	$1.8039e+008$	$1.5686e+008$	$2.1176e+008$
	7	$3.3245e+008$	$2.2441e+008$	$1.4296e+009$	$1.1470e+009$	$1.3963e+009$	$6.8569e+008$	$3.8232e+008$	$3.3245e+008$	$4.4881e+008$
	8	$6.6954e+008$	$4.5194e+008$	$2.8790e+009$	$2.3099e+009$	$2.8121e+009$	$1.3809e+009$	$7.6998e+008$	$6.6954e+008$	$9.0389e+008$
	20	$-5.2063e+007$	$-3.5143e+007$	$-2.2387e+008$	$-1.7962e+008$	$-2.1867e+008$	$-1.0738e+008$	$-5.9873e+007$	$-5.2063e+007$	$-7.0285e+007$
	26	$2.2554e+010$	$1.5224e+010$	$9.6982e+010$	$7.7811e+010$	$9.4726e+010$	$4.6517e+010$	$2.5937e+010$	$2.2554e+010$	$3.0448e+010$
	27	$-1.5155e+011$	$-1.0230e+011$	$-6.5166e+011$	$-5.2284e+011$	$-6.3650e+011$	$-3.1257e+011$	$-1.7428e+011$	$-1.5155e+011$	$-2.0459e+011$
	1	$2.1168e+006$	$1.4288e+006$	$9.1022e+006$	$7.3030e+006$	$8.8906e+006$	$4.3659e+006$	$2.4343e+006$	$2.1168e+006$	$2.8577e+006$
	2	$7.3056e+006$	$4.9313e+006$	$3.1414e+007$	$2.5204e+007$	$3.0683e+007$	$1.5068e+007$	$8.4014e+006$	$7.3056e+006$	$9.8625e+006$
	3	$1.9654e+007$	$1.3266e+007$	$8.4510e+007$	$6.7805e+007$	$8.2545e+007$	$4.0535e+007$	$2.2602e+007$	$1.9654e+007$	$2.6532e+007$
	4	$4.8150e+007$	$3.2501e+007$	$2.0705e+008$	$1.6612e+008$	$2.0223e+008$	$9.9309e+007$	$5.5373e+007$	$4.8150e+007$	$6.5003e+007$
	5	$1.1168e+008$	$7.5383e+007$	$4.8022e+008$	$3.8529e+008$	$4.6905e+008$	$2.3034e+008$	$1.2843e+008$	$1.1168e+008$	$1.5077e+008$
C_5	6	$2.4768e+008$	$1.6718e+008$	$1.0650e+009$	$8.5449e+008$	$1.0402e+009$	$5.1083e+008$	$2.8483e+008$	$2.4768e+008$	$3.3436e+008$
	7	$5.2493e+008$	$3.5433e+008$	$2.2572e+009$	$1.8110e+009$	$2.2047e+009$	$1.0827e+009$	$6.0367e+008$	$5.2493e+008$	$7.0865e+008$
	8	$1.0572e+009$	$7.1359e+008$	$4.5459e+009$	$3.6473e+009$	$4.4401e+009$	$2.1804e+009$	$1.2158e+009$	$1.0572e+009$	$1.4272e+009$
	20	$-8.2205e+007$	$-5.5488e+007$	$-3.5348e+008$	$-2.8361e+008$	$-3.4526e+008$	$-1.6955e+008$	$-9.4536e+007$	$-8.2205e+007$	$-1.1098e+008$
	26	$3.5611e+010$	$2.4038e+010$	$1.5313e+011$	$1.2286e+011$	$1.4957e+011$	$7.3448e+010$	$4.0953e+010$	$3.5611e+010$	$4.8075e+010$
	27	$-2.3929e+011$	$-1.6152e+011$	$-1.0289e+012$	$-8.2554e+011$	$-1.0050e+012$	$-4.9353e+011$	$-2.7518e+011$	$-2.3929e+011$	$-3.2304e+011$
	1	$7.0560e+004$	$4.7628e+004$	$3.0341e+005$	$2.4343e+005$	$2.9635e+005$	$1.4553e+005$	$8.1144e+004$	$7.0560e+004$	$9.5256e+004$
	2	$2.4352e+005$	$1.6438e+005$	$1.0471e+006$	$8.4014e+005$	$1.0228e+006$	$5.0226e+005$	$2.8005e+005$	$2.4352e+005$	$3.2875e+005$
	3	$6.5512e+005$	$4.4220e+005$	$2.8170e+006$	$2.2602e+006$	$2.7515e+006$	$1.3512e+006$	$7.5339e+005$	$6.5512e+005$	$8.8441e+005$
	4	$1.6050e+006$	$1.0834e+006$	$6.9015e+006$	$5.5373e+006$	$6.7410e+006$	$3.3103e+006$	$1.8458e+006$	$1.6050e+006$	$2.1668e+006$
	5	$3.7226e+006$	$2.5128e+006$	$1.6007e+007$	$1.2843e+007$	$1.5635e+007$	$7.6779e+006$	$4.2810e+006$	$3.7226e+006$	$5.0256e+006$
T_5	6	$8.2559e+006$	$5.5727e+006$	$3.5500e+007$	$2.8483e+007$	$3.4675e+007$	$1.7028e+007$	$9.4943e+006$	$8.2559e+006$	$1.1145e+007$
	7	$1.7498e+007$	$1.1811e+007$	$7.5239e+007$	$6.0367e+007$	$7.3490e+007$	$3.6089e+007$	$2.0122e+007$	$1.7498e+007$	$2.3622e+007$
	8	$3.5239e+007$	$2.3786e+007$	$1.5153e+008$	$1.2158e+008$	$1.4800e+008$	$7.2681e+007$	$4.0525e+007$	$3.5239e+007$	$4.7573e+007$
	20	$-2.7402e+006$	$-1.8496e+006$	$-1.1783e+007$	$-9.4536e+006$	$-1.1509e+007$	$-5.6516e+006$	$-3.1512e+006$	$-2.7402e+006$	$-3.6992e+006$
	26	$1.1870e+009$	$8.0126e+008$	$5.1043e+009$	$4.0953e+009$	$4.9856e+009$	$2.4483e+009$	$1.3651e+009$	$1.1870e+009$	$1.6025e+009$
	27	$-7.9762e+009$	$-5.3840e+009$	$-3.4298e+010$	$-2.7518e+010$	$-3.3500e+010$	$-1.6451e+010$	$-9.1727e+009$	$-7.9762e+009$	$-1.0768e+010$

S_5, C_5 and T_5 represent net returns, management costs, and taxes payable, respectively, from the nine destinations taking by vehicles in Benin Station, and N denotes Naira.

follow by Benin to Ilorri with a management costs of 1.4915e+018 Naira, follow by Benin to Auchi with a management costs of 7.9549e+017 Naira and so on, but Benin to Warri yields the least management costs of 2.0509e+017 Naira. From the first one week of the third compartment of Table 3, Benin to Sapele yields a tax payable of the sum of 9.5256e+004 Naira, in the second week, it yields a tax payable of the sum of 3.6197e+005 Naira, in the third week it yields a tax payable of the sum of 1.1088e+006 Naira, in the eighth week, a tax payable is 1.9988e+008 Naira, in the 20th week, a tax payable is 4.6128e+013 Naira, and at the 27th week, a tax payable is 2.2373e+016 Naira. Similar discussion go to destination 2- destination 9 in the third compartment. It is important to observe that after 27 weeks of the operations, Benin to Ibadan yields the highest tax payable of the sum of6.9606e+016 Naira, follow by Benin to Ilorri with a tax payable of the sum of 4.9718e+016 Naira, follow by Benin to Auchi with a tax payable of 2.6516e+016 Naira and so on, but Benin to Warri yields the least tax payable of 6.8363e+015 Naira. It is observed that Benin to Ibadan yields the higher expected discounted returns, the higher the management costs and tax over time, which is an expected results. This result will now give the company a possible net returns, management costs and tax to be paid when buses are allocated from Benin to: Sapele, Ekpoma, Auchi, Warri, PH, Lagos, Ibadan, Ilorri and Abuja over time.

Table 4 shows the optimal net returns, optimal management costs and optimal tax of nine destinations from Benin Station in the presence of breakdown vehicles at time T (in week), given that $\nu_0 = 20\%$, $\xi = 60\%$, $a_y = 5\%$, $\beta = 15\%$, and $\eta = 30\%$. The table is made up of three compartments, as well. The first compartment represents the discounted expected optimal returns from the nine destinations from Benin Station over a 27-week, when vehicle breakdown is considered. In the first one week, Benin to Sapele yields a net return of 1,809,900 Naira, management costs of 2.8577e+006 and tax payable of 9.5256e+004. In the second week, the net return of 6.3061e+006 Naira, management costs of 9.956e+006 and tax payable of 3.3190e+005. In the third week, it yields a net return of 1.7206e+007 Naira, management costs of 2.7167e+007. In the eighth week, a net return of 1.0956e+009 Naira, management costs of 1.7300e+009. In the 20th week, it yields a sum of 2.5540e+008 Naira, management costs of 4.0326e+008 and tax payable of 1.3442e+007,

Table 9. Optimal net returns, optimal management costs and optimal tax of nine routes for Ibadan Station in the presence of breakdown vehicles at time T (in week), given that $\nu_0 = 20\%$, $\xi = 60\%$, $a_y = 5\%$, $\beta = 15\%$ and $\eta = 30\%$.

		R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	R_9
S	T	N	N	N	N	N	N	N	N	N
	1	2.3126e+006	2.0110e+006	3.6868e+006	7.7087e+005	3.2678e+006	1.6758e+006	1.4747e+006	3.0164e+006	8.3790e+005
	2	8.0682e+006	7.0159e+006	1.2862e+007	2.6894e+006	1.1401e+007	5.8465e+006	5.1450e+006	1.0524e+007	2.9233e+006
	3	2.2039e+007	1.9164e+007	3.5135e+007	7.3463e+006	3.1142e+007	1.5970e+007	1.4054e+007	2.8746e+007	7.9851e+006
	4	5.5085e+007	4.7900e+007	8.7816e+007	1.8362e+007	7.7837e+007	3.9917e+007	3.5127e+007	7.1850e+007	1.9958e+007
	5	1.3102e+008	1.1393e+008	2.0887e+008	4.3672e+007	1.8513e+008	9.4939e+007	8.3546e+007	1.7089e+008	4.7470e+007
S_6	6	2.9967e+008	2.6058e+008	4.7773e+008	9.9888e+007	4.2344e+008	2.1715e+008	1.9109e+008	3.9087e+008	1.0857e+008
	7	6.5933e+008	5.7333e+008	1.0511e+009	2.1978e+008	9.3166e+008	4.7778e+008	4.2044e+008	8.6000e+008	2.3889e+008
	8	1.3893e+009	1.2080e+009	2.2147e+009	4.6308e+008	1.9631e+009	1.0067e+009	8.8590e+008	1.8121e+009	1.0080e+009
	20	1.8552e+009	1.6132e+009	2.9576e+009	6.1841e+008	2.6215e+009	1.3444e+009	1.1830e+009	2.4198e+009	6.7218e+008
	25	5.6133e+010	4.8812e+010	8.9488e+010	1.8711e+010	7.9319e+010	4.0676e+010	3.5795e+010	7.3217e+010	2.0338e+010
	27	−3.7613e+011	−3.2707e+011	−5.9963e+011	−1.2538e+011	−5.3149e+011	−2.7256e+011	−2.3985e+011	−4.9061e+011	−1.3628e+011
	1	3.6515e+006	3.1752e+006	5.8212e+006	1.2172e+006	5.1597e+006	2.6460e+006	2.3285e+006	4.7628e+006	1.3230e+006
	2	1.2739e+007	1.1078e+007	2.0309e+007	4.2464e+006	1.8001e+007	9.2314e+006	8.1236e+006	1.6617e+007	4.6157e+006
	3	3.4798e+007	3.0259e+007	5.5476e+007	1.1599e+007	4.9172e+007	2.5216e+007	2.2190e+007	4.5389e+007	1.2608e+007
	4	8.6976e+007	7.5631e+007	1.3866e+008	2.8992e+007	1.2290e+008	6.3026e+007	5.5463e+007	1.1345e+008	3.1513e+007
	5	2.0687e+008	1.7988e+008	3.2979e+008	6.8956e+007	2.9231e+008	1.4990e+008	1.3192e+008	2.6983e+008	7.4952e+007
C_6	6	4.7316e+008	4.1144e+008	7.5431e+008	1.5772e+008	6.6859e+008	3.4287e+008	3.0172e+008	6.1716e+008	1.7143e+008
	7	1.0410e+009	9.0526e+008	1.6596e+009	3.4702e+008	1.4710e+009	7.5438e+008	6.6386e+008	1.3579e+009	3.7719e+008
	8	2.1936e+009	1.9074e+009	3.4970e+009	7.3118e+008	3.0996e+009	1.5895e+009	1.3988e+009	2.8612e+009	7.9477e+008
	20	−3.0191e+008	−2.6253e+008	−4.8131e+008	−1.0064e+008	−4.2661e+008	−2.1878e+008	−1.9252e+008	−3.9380e+008	−1.0939e+008
	26	8.8631e+010	7.7071e+010	1.4130e+011	2.9544e+010	1.2524e+011	6.4226e+010	5.6519e+010	1.1561e+011	3.2113e+010
	27	−5.9389e+011	−5.1643e+011	−9.4678e+011	−1.9796e+011	−8.3919e+011	−4.3036e+011	−3.7871e+011	−7.7464e+011	−2.1518e+011
	1	1.2172e+005	1.0584e+005	1.9404e+005	4.0572e+004	1.7199e+005	8.8200e+004	7.7616e+004	1.5876e+005	4.4100e+004
	2	4.2464e+005	3.6926e+005	6.7697e+005	1.4155e+005	6.0004e+005	3.0771e+005	2.7079e+005	5.5388e+005	1.5386e+005
	3	1.1599e+006	1.0086e+006	1.8492e+006	3.8665e+005	1.6391e+006	8.4054e+005	7.3968e+005	1.5130e+006	4.2027e+005
	4	2.8992e+006	2.5210e+006	4.6219e+006	9.6640e+005	4.0967e+006	2.1009e+006	1.8488e+006	3.7816e+006	1.0504e+006
	5	6.8956e+006	5.9961e+006	1.0993e+007	2.2985e+006	9.7437e+006	4.9968e+006	4.3972e+006	8.9942e+006	2.4984e+006
T_6	6	1.5772e+007	1.3715e+007	2.5144e+007	5.2573e+006	2.2286e+007	1.1429e+007	1.0057e+007	2.0572e+007	5.7144e+006
	7	3.4702e+007	3.0175e+007	5.5321e+007	1.1567e+007	4.9035e+007	2.5146e+007	2.2129e+007	4.5263e+007	1.2573e+007
	8	7.3118e+007	6.3581e+007	1.1657e+008	2.4373e+007	1.0332e+008	5.2984e+007	4.6626e+007	9.5372e+007	2.6492e+007
	20	−1.0064e+007	−8.7510e+006	−1.6044e+007	−3.3546e+006	−1.4220e+007	−7.2925e+006	−6.4174e+006	−1.3127e+007	−3.6463e+006
	25	2.9544e+009	2.5690e+009	4.7099e+009	9.8479e+008	4.1747e+009	2.1409e+009	1.8840e+009	3.8535e+009	1.0704e+009
	27	−1.9796e+010	−1.7214e+010	−3.1559e+010	−6.5988e+009	−2.7973e+010	−1.4345e+010	−1.2624e+010	−2.5821e+010	−7.1726e+009

S_6, C_6 and T_6 represent net returns, management costs, and taxes payable, respectively, from the nine destinations taking by vehicles in Benin Station, and N denotes Naira.

which shows a decline in net returns, management costs and tax. In the 26th week, the company will experience a negative returns due to breakdown of vehicles, and rise at the 27th week with a return of 1.2109e+011 Naira. Again, similar discussion go to destination 2- destination 9 in the first compartment of Table 4. It is observed that after 27 weeks of this operations, Benin to Ibadan yields the highest return of 3.7671e+011 Naira, follow by Benin to Ilorri with the return of 2.0908e+011 Naira, follow by Benin to Auchi with the return of 1.4351e+011 Naira and so on, but Benin to Warri again, yields the least return of 3.6998e+010 Naira. The second compartment in Table 4, represents the management costs for each of the destination over a period of 27 weeks. The third compartment represents the amount of tax payable to government from each of the destination over a period of 27 weeks, in the presence of vehicles breakdown. As in the case of absence of breakdown vehicles presented in Table 3, it is observed that the higher the expected discounted returns, the higher the management costs and tax over time, which is an expected results. Again, this result will now give the company a possible net returns, management costs and tax to be paid when buses are allocated from Benin to: Sapele, Ekpoma, Auchi, Warri, PH, Lagos, Ibadan, Ilorri and Abuja over time, in vehicle breakdown is considered.

Table 5 shows the optimal net returns, optimal management costs and optimal tax of nine destinations from Lagos Station in the presence of breakdown vehicles at time T (in week), given that $\nu_0 = 20\%$, $\xi = 60\%$, $a_y = 5\%$, $\beta = 15\%$, and $\eta = 30\%$. The table is also made up of three compartments. The first compartment represents the discounted expected optimal returns from Abuja, Benin, Ibadan, Ilorri, Minna, Warri, PH, Kaduna and Kano, destinations from Lagos Station over a 27-week, when vehicle breakdown is considered. The second compartment in Table 5, represents the management costs for each of the destination over a period of 27 weeks. The third compartment represents the amount of tax payable to government from each of the destination over a period of 27 weeks, in the presence of vehicles breakdown. Here, in the first week, Lagos to Abuja yields a net return of 4,893,300 Naira, management costs of 7,726,300 Naira set aside and tax of 257,540 Naira paid. In the second week, the net return of 1.7049e+007 Naira, management costs of 2.6920e+007 Naira set aside and tax of 8.9734e+005 Naira paid. In the third

Table 10. Optimal net returns, management costs, tax for six stations from nine destination in the absence of breakdown vehicles for a period of twenty-seven weeks.

		S_1	S_2	S_3	S_4	S_5	S_6
	T	N	N	N	N	N	N
	1	$1.9490e+007$	2.7818e+007	2.4215e+007	2.4215e+007	2.5724e+007	1.9054e+007
	2	$7.4060e+007$	1.0571e+008	1.0897e+008	9.2018e+007	9.7749e+007	7.2405e+007
	3	$2.2686e+008$	3.2380e+008	4.0561e+008	2.8187e+008	2.9942e+008	2.2179e+008
	4	$6.5469e+008$	9.3447e+008	1.4438e+009	8.1344e+008	8.6410e+008	6.4006e+008
ER	5	$1.8526e+009$	2.6443e+009	5.0776e+009	2.3018e+009	2.4452e+009	1.8112e+009
	6	$5.2069e+009$	2.0837e+010	6.2310e+010	1.8139e+010	1.9268e+010	1.4272e+010
	7	$4.0896e+010$	5.8372e+010	2.1811e+011	5.0812e+010	5.3977e+010	3.9982e+010
	8	$1.1453e+011$	1.6347e+011	7.6341e+011	1.4230e+011	1.5116e+011	1.1197e+011
	20	$9.4993e+015$	1.3559e+016	7.3707e+017	1.1803e+016	1.2538e+016	9.2869e+015
	26	$4.5776e+018$	6.5338e+018	1.3549e+021	5.6876e+018	6.0418e+018	4.4753e+018
	27	$1.2817e+019$	1.8295e+019	4.7423e+021	1.5925e+019	1.6917e+019	1.2531e+019
	1	3.0773e+007	4.3924e+007	3.8235e+007	3.8235e+007	4.0616e+007	3.0085e+007
	2	1.1694e+008	1.6691e+008	1.7206e+008	1.4529e+008	1.5434e+008	1.1432e+008
	3	3.5820e+008	5.1127e+008	6.4043e+008	4.4505e+008	4.7277e+008	3.5019e+008
	4	1.0337e+009	1.4755e+009	2.2797e+009	1.2844e+009	1.3644e+009	1.0106e+009
	5	2.9252e+009	4.1753e+009	8.0173e+009	3.6345e+009	3.8609e+009	2.8598e+009
MC	6	8.2213e+009	1.1735e+010	2.8099e+010	1.0215e+010	1.0851e+010	8.0376e+009
	7	2.3051e+010	3.2901e+010	9.8384e+010	2.8640e+010	3.0424e+010	2.2535e+010
	8	6.4572e+010	9.2167e+010	3.4438e+011	8.0230e+010	8.5227e+010	6.3129e+010
	20	1.4999e+016	2.1408e+016	1.1638e+018	1.8636e+016	1.9796e+016	1.4664e+016
	26	7.2278e+018	1.0317e+019	2.1394e+021	8.9804e+018	9.5397e+018	7.0662e+018
	27	2.0238e+019	2.8886e+019	7.4878e+021	2.5145e+019	2.6711e+019	1.9785e+019
	1	1.0258e+006	1.4641e+006	1.2745e+006	1.2745e+006	1.3539e+006	1.0028e+006
	2	3.8979e+006	5.5637e+006	5.7352e+006	4.8431e+006	5.1447e+006	3.8108e+006
	3	1.1940e+007	1.7042e+007	2.1348e+007	1.4835e+007	1.5759e+007	1.1673e+007
	4	3.4458e+007	4.9183e+007	7.5991e+007	4.2813e+007	4.5479e+007	3.3687e+007
	5	9.7507e+007	1.3918e+008	2.6724e+008	1.2115e+008	1.2870e+008	9.5327e+007
Tax	6	2.7404e+008	3.9116e+008	9.3663e+008	3.4049e+008	3.6170e+008	2.6792e+008
	7	7.6835e+008	1.0967e+009	3.2795e+009	9.5466e+008	1.0141e+009	7.5117e+008
	8	2.1524e+009	3.0722e+009	1.1479e+010	2.6743e+009	2.8409e+009	2.1043e+009
	20	4.9996e+014	7.1362e+014	3.8793e+016	6.2119e+014	6.5988e+014	4.8878e+014
	26	2.4093e+017	3.4388e+017	7.1312e+019	2.9935e+017	3.1799e+017	2.3554e+017
	27	6.7459e+017	9.6288e+017	2.4959e+020	8.3817e+017	8.9037e+017	6.5951e+017

N denotes Naira, ER the expected net returns and MC management costs.

week, it yields a net return of 4.6397e+007 Naira, management costs of 7.3259e+007 Naira set aside and tax of 2.4420e+006 Naira paid. In the eighth week, a net return of 2.6485e+009 Naira, management costs of 4.1818e+009 Naira set aside and tax of 1.3939e+008 Naira paid. In the 20th week, it yields a sum of 1.1711e+009 Naira, management costs of 1.8491e+009 Naira set aside and tax of 6.1635e+007 Naira paid. At the 25th week, there was negative returns, negative management costs and negative tax payable, and at the 27th week, the return becomes 7.3550e+013 Naira, management costs of 1.1613e+014 Naira set aside and tax of 3.8711e+012 Naira paid. Similar discussion also go to destination 2- destination 9 in the first compartment of Table 5. It is observed that after 27 weeks of the company's operations, Lagos to Minna yields the highest return of 1.0075e+014 Naira with management costs of 1.5909e+014 Naira set aside and tax of 5.3028e+012 Naira paid, follow by Lagos to Benin with the return of 8.3122e+013 Naira with management costs of 1.3125e+014 Naira set aside and tax of 4.3748e+012 Naira paid, follow by Lagos to Abuja with the return of 7.3550e+013 Naira, management costs of 1.1613e+014 Naira set aside and tax of 3.8711e+012 Naira paid and so on, but Lagos to Ilorri, yields the least return of 1.4609e+013 Naira with management costs of 2.3067e+013 Naira set aside and tax of 7.6891e+011 Naira paid.

Similar discussion go to Tables 6, 7, 8 and 9.

Table 10 shows the optimal net returns of the six stations from nine destination in the absence of

Table 11. Optimal net returns, management costs and tax for the six stations from nine destination in the in the presence of breakdown vehicles for a period of twenty-seven weeks.

		S_1	S_2	S_3	S_4	S_5	S_6
	T	N	N	N	N	N	N
	1	1.9490e+007	2.7818e+007	2.4215e+007	2.4215e+007	2.5724e+007	1.9054e+007
	2	6.7907e+007	9.6925e+007	9.9830e+007	8.4302e+007	8.8778e+007	6.6475e+007
	3	1.8528e+008	2.6377e+008	3.2990e+008	2.2887e+008	2.3883e+008	1.8158e+008
	4	4.6281e+008	6.5457e+008	1.0110e+009	5.6542e+008	5.8512e+008	4.5385e+008
ER	5	1.1010e+009	1.5387e+009	2.9643e+009	1.3193e+009	1.3571e+009	1.0795e+009
	6	2.5219e+009	3.4575e+009	8.3566e+009	2.9306e+009	3.0098e+009	2.4690e+009
	7	5.5669e+009	7.4132e+009	2.2559e+010	6.1787e+009	6.3790e+009	5.4323e+009
	8	1.1798e+010	1.5056e+010	5.7824e+010	1.2249e+010	1.2847e+010	1.1446e+010
	20	2.7503e+009	6.6574e+009	3.9247e+011	-1.9213e+010	-9.9896e+008	-1.5754e+009
	26	1.3039e+012	4.1813e+014	3.5180e+017	-5.4237e+015	-2.9078e+012	-3.0990e+012
	27	-8.8623e+012	-5.8501e+015	-8.4990e+018	9.6036e+016	2.3697e+013	2.5630e+013
	1	3.0773e+007	4.3924e+007	3.8235e+007	3.8235e+007	4.0616e+007	3.0085e+007
	2	1.0722e+008	1.5304e+008	1.5763e+008	1.3311e+008	1.4018e+008	1.0496e+008
	3	2.9255e+008	4.1647e+008	5.2089e+008	3.6138e+008	3.7710e+008	2.8671e+008
	4	7.3075e+008	1.0335e+009	1.5963e+009	8.9276e+008	9.2388e+008	7.1661e+008
	5	1.7384e+009	2.4296e+009	4.6805e+009	2.0831e+009	2.1428e+009	1.7044e+009
MC	6	3.9819e+009	5.4592e+009	1.3195e+010	4.6273e+009	4.7523e+009	3.8984e+009
	7	8.7899e+009	1.1705e+010	3.5619e+010	9.7559e+009	1.0072e+010	8.5773e+009
	8	1.8629e+010	2.3773e+010	9.1301e+010	1.9341e+010	2.0285e+010	1.8073e+010
	20	4.3425e+009	1.0512e+010	6.1969e+011	-3.0336e+010	-1.5773e+009	-2.4875e+009
	26	2.0588e+012	6.6020e+014	5.5547e+017	-8.5637e+015	-4.5913e+012	-4.8931e+012
	27	-1.3993e+013	-9.2371e+015	-1.3420e+019	1.5164e+017	3.7416e+013	4.0468e+013
	1	1.0258e+006	1.4641e+006	1.2745e+006	1.2745e+006	1.3539e+006	1.0028e+006
	2	3.5740e+006	5.1013e+006	5.2542e+006	4.4369e+006	4.6725e+006	3.4987e+006
	3	9.7518e+006	1.3882e+007	1.7363e+007	1.2046e+007	1.2570e+007	9.5569e+006
	4	2.4358e+007	3.4451e+007	5.3210e+007	2.9759e+007	3.0796e+007	2.3887e+007
	5	5.7946e+007	8.0986e+007	1.5602e+008	6.9435e+007	7.1428e+007	5.6814e+007
Tax	6	1.3273e+008	1.8197e+008	4.3982e+008	1.5424e+008	1.5841e+008	1.2995e+008
	7	2.9300e+008	3.9017e+008	1.1873e+009	3.2520e+008	3.3573e+008	2.8591e+008
	8	6.2097e+008	7.9244e+008	3.0434e+009	6.4469e+008	6.7615e+008	6.0243e+008
	20	1.4475e+008	3.5039e+008	2.0656e+010	-1.0112e+009	-5.2577e+007	-8.2916e+007
	26	6.8627e+010	2.2007e+013	1.8516e+016	-2.8546e+014	-1.5304e+011	-1.6310e+011
	27	-4.6644e+011	-3.0790e+014	-4.4732e+017	5.0545e+015	1.2472e+012	1.3489e+012

N denotes Naira.

breakdown vehicles for a period of twenty-seven weeks. In the first one week of the operation, Benin Station has a total return of 19,490,000 Naira, Lagos Station has 27,818,000 Naira, Warri Station has 24,215,000 Naira, Abuja Station has 24,215,000 Naira, Port Harcourt Station has a return of 25,724,000 Naira and Ibadan Station has a return of 19,054,000 Naira. At 27th weeks of time, Benin Station has a total net return of 1.2817e+019 Naira, Lagos Station has 1.8295e+019 Naira, Warri Station has 4.7423e+021 Naira, Abuja Station has 1.5925e+019 Naira, Port Harcourt Station has a net return of 1.6917e+019 Naira and Ibadan Station has a return of 1.2531e+019 Naira. Again, Warri Station yields the highest net return, follow by Lagos Station, follow by Port Harcourt, follow by Abuja, and follow by Benin and Ibadan yields the lowest net return when there is no vehicle breakdown over time. Similar interpretations also go to the management costs and tax payable.

Table 11 shows the optimal net returns of the six stations from nine destination in the presence of breakdown vehicles for a period of twenty-seven weeks. In the first one week of the operation, Benin Station has a total return of 19,490,000 Naira, Lagos Station has 27,818,000 Naira, Warri Station has 24,215,000 Naira, Abuja Station has 24,215,000 Naira, Port Harcourt Station has a return of 25,724,000 Naira and Ibadan Station has a return of 19,054,000 Naira. At the second week, Benin Station has a total return of 6.7907e+007 Naira, Lagos Station has 9.6925e+007 Naira, Warri Station has 9.9830e+007

Table 12. Optimal returns, management costs and tax for six stations from nine destinations in the presence of breakdown vehicles at a 27- week, for the varying value of depreciation rate, ν.

	S_1	S_2	S_3	S_4	S_5	S_6		
T	N	N	N	N	N	N		
		1	1.9490e+007	2.7818e+007	2.4215e+007	2.4215e+007	2.5724e+007	1.9054e+007
		2	3.8810e+007	5.5392e+007	4.8216e+007	4.8217e+007	5.1221e+007	3.7941e+007
		3	5.4060e+007	7.7147e+007	6.7149e+007	6.7155e+007	7.1342e+007	5.2848e+007
ER	0.1	6	1.4533e+008	2.1418e+008	1.8017e+008	1.8030e+008	1.9163e+008	1.4200e+008
		15	2.5463e+009	3.6289e+009	2.9479e+009	3.0654e+009	3.3064e+009	2.4580e+009
		27	3.9570e+010	2.4773e+010	−9.0741e+007	2.4771e+010	4.2590e+010	2.8988e+010
		1	3.0773e+007	4.3924e+007	3.8235e+007	3.8235e+007	4.0616e+007	3.0085e+007
		2	6.1279e+007	8.7460e+007	7.6131e+007	7.6133e+007	8.0876e+007	5.9907e+007
		3	8.5358e+007	1.2181e+008	1.0603e+008	1.0603e+008	1.1265e+008	8.3444e+007
MC	0.1	6	2.2946e+008	3.3818e+008	2.8448e+008	2.8469e+008	3.0257e+008	2.2421e+008
		15	4.0205e+009	5.7299e+009	4.6546e+009	4.8401e+009	5.2206e+009	3.8810e+009
		27	6.2480e+010	3.9115e+010	−1.4328e+008	3.9112e+010	6.7247e+010	4.5770e+010
		1	1.0258e+006	1.4641e+006	1.2745e+006	1.2745e+006	1.3539e+006	1.0028e+006
		2	2.0426e+006	2.9153e+006	2.5377e+006	2.5378e+006	2.6959e+006	1.9969e+006
		3	2.8453e+006	4.0603e+006	3.5342e+006	3.5345e+006	3.7548e+006	2.7815e+006
Tax	0.1	6	7.6487e+006	1.1273e+007	9.4827e+006	9.4895e+006	1.0086e+007	7.4736e+006
		15	1.3402e+008	1.9100e+008	1.5515e+008	1.6134e+008	1.7402e+008	1.2937e+008
		27	2.0827e+009	1.3038e+009	−4.7759e+006	1.3037e+009	2.2416e+009	1.5257e+009
		1	1.9490e+007	2.7818e+007	2.4215e+007	2.4215e+007	2.5724e+007	1.9054e+007
		2	3.8772e+007	5.5337e+007	4.8169e+007	4.8170e+007	5.1171e+007	3.7904e+007
		3	5.3874e+007	7.6874e+007	6.6910e+007	6.6917e+007	7.1092e+007	5.2664e+007
ER	0.3	6	1.4097e+008	2.0737e+008	1.7429e+008	1.7458e+008	1.8567e+008	1.3765e+008
		15	3.9415e+008	3.0084e+008	8.5089e+007	2.7510e+008	3.9727e+008	3.0849e+008
		27	−7.4416e+015	−1.9576e+018	1.7329e+022	−1.1139e+018	−1.1646e+017	−1.4441e+017
		1	3.0773e+007	4.3924e+007	3.8235e+007	3.8235e+007	4.0616e+007	3.0085e+007
		2	6.1219e+007	8.7375e+007	7.6056e+007	7.6058e+007	8.0797e+007	5.9849e+007
		3	8.5064e+007	1.2138e+008	1.0565e+008	1.0566e+008	1.1225e+008	8.3153e+007
MC	0.3	6	2.2259e+008	3.2743e+008	2.7519e+008	2.7566e+008	2.9317e+008	2.1734e+008
		15	6.2234e+008	4.7501e+008	1.3435e+008	4.3437e+008	6.2727e+008	4.8709e+008
		27	−1.1750e+016	−3.0909e+018	2.7361e+022	−1.7588e+018	−1.8388e+017	−2.2802e+017
		1	1.0258e+006	1.4641e+006	1.2745e+006	1.2745e+006	1.3539e+006	1.0028e+006
		2	2.0406e+006	2.9125e+006	2.5352e+006	2.5353e+006	2.6932e+006	1.9950e+006
		3	2.8355e+006	4.0460e+006	3.5216e+006	3.5220e+006	3.7417e+006	2.7718e+006
Tax	0.3	6	7.4197e+006	1.0914e+007	9.1731e+006	9.1885e+006	9.7724e+006	7.2447e+006
		15	2.0745e+007	1.5834e+007	4.4784e+006	1.4479e+007	2.0909e+007	1.6236e+007
		27	−3.9166e+014	−1.0303e+017	9.1204e+020	−5.8627e+016	−6.1294e+015	−7.6006e+015
		1	1.9490e+007	2.7818e+007	2.4215e+007	2.4215e+007	2.5724e+007	1.9054e+007
		2	3.8772e+007	5.5337e+007	4.8169e+007	4.8170e+007	5.1171e+007	3.7904e+007
		3	5.3874e+007	7.6874e+007	6.6910e+007	6.6917e+007	7.1092e+007	5.2664e+007
ER	0.5	6	1.4097e+008	2.0737e+008	1.7429e+008	1.7458e+008	1.8567e+008	1.3765e+008
		15	3.9415e+008	3.0084e+008	8.5089e+007	2.7510e+008	3.9727e+008	3.0849e+008
		27	−7.4416e+015	−1.9576e+018	1.7329e+022	−1.1139e+018	−1.1646e+017	−1.4441e+017
		1	3.0773e+007	4.3924e+007	3.8235e+007	3.8235e+007	4.0616e+007	3.0085e+007
		2	6.1219e+007	8.7375e+007	7.6056e+007	7.6058e+007	8.0797e+007	5.9849e+007
		3	8.5064e+007	1.2138e+008	1.0565e+008	1.0566e+008	1.1225e+008	8.3153e+007
MC	0.5	6	2.2259e+008	3.2743e+008	2.7519e+008	2.7566e+008	2.9317e+008	2.1734e+008
		15	6.2234e+008	4.7501e+008	1.3435e+008	4.3437e+008	6.2727e+008	4.8709e+008
		27	−1.1750e+016	−3.0909e+018	2.7361e+022	−1.7588e+018	−1.8388e+017	−2.2802e+017
		1	1.0258e+006	1.4641e+006	1.2745e+006	1.2745e+006	1.3539e+006	1.0028e+006
		2	2.0406e+006	2.9125e+006	2.5352e+006	2.5353e+006	2.6932e+006	1.9950e+006
		3	2.8355e+006	4.0460e+006	3.5216e+006	3.5220e+006	3.7417e+006	2.7718e+006
Tax	0.5	6	7.4197e+006	1.0914e+007	9.1731e+006	9.1885e+006	9.7724e+006	7.2447e+006
		15	2.0745e+007	1.5834e+007	4.4784e+006	1.4479e+007	2.0909e+007	1.6236e+007
		27	−3.9166e+014	−1.0303e+017	9.1204e+020	−5.8627e+016	−6.1294e+015	−7.6006e+015

Note that MC here stands for management costs, ER stands for expected returns and N denotes Naira.

Naira, Abuja Station has 8.4302e+007 Naira, Port Harcourt Station has a return of 8.8778e+007 Naira and Ibadan Station has a return of 6.6475e+007 Naira. At the 20th week, Benin Station has a total return of 2.7503e+009 Naira, Lagos Station has 6.6574e+009 Naira, Warri Station has 3.9247e+011 Naira, Abuja Station has -1.9213e+010 Naira, Port Harcourt Station has a return of -9.9896e+008 Naira and Ibadan Station has a return of -1.5754e+009 Naira. At 27th week, Benin Station has a total net return of -8.8623e+012 Naira, Lagos Station has -5.8501e+015 Naira, Warri Station has -8.4990e+018 Naira, Abuja Station has 9.6036e+016 Naira, Port Harcourt Station has a net return of 2.3697e+013 Naira and Ibadan Station has a return of 2.5630e+013 Naira. The negative returns implies that net returns went down due to breakdown of vehicles over time. This simply tells us that the station's policies and bad nature of the routes imparted significantly to the poor returns from the destinations. In the presence of vehicle breakdown, Abuja Station yields the highest net return, follow by Ibadan Station, follow by Port Harcourt Station, follow by Benin

Table 13. Optimal returns, management costs and tax for six stations from nine destinations in the presence of breakdown vehicles at a 27- week, for the varying value of discount rate η.

	S_1	S_2	S_3	S_4	S_5	S_6		
T	N	N	N	N	N	N		
		1	$2.5058e+007$	$3.5766e+007$	$3.1134e+007$	$3.1134e+007$	$3.3073e+007$	$2.4498e+007$
		2	$4.9904e+007$	$7.1226e+007$	$6.2000e+007$	$6.2001e+007$	$6.5864e+007$	$4.8787e+007$
ER		3	$8.9399e+007$	$1.2758e+008$	$1.1105e+008$	$1.1105e+008$	$1.1798e+008$	$8.7394e+007$
	0.1	6	$5.1187e+008$	$7.3609e+008$	$6.3475e+008$	$6.3516e+008$	$6.7503e+008$	$5.0018e+008$
		15	$8.9829e+010$	$1.2607e+011$	$1.0603e+011$	$1.0905e+011$	$1.1714e+011$	$8.7007e+010$
		27	$5.2248e+013$	$4.9332e+013$	$7.6722e+012$	$4.6131e+013$	$6.1950e+013$	$4.4165e+013$
		1	$3.9565e+007$	$5.6473e+007$	$4.9159e+007$	$4.9159e+007$	$5.2221e+007$	$3.8681e+007$
		2	$7.8796e+007$	$1.1246e+008$	$9.7894e+007$	$9.7896e+007$	$1.0400e+008$	$7.7033e+007$
MC		3	$1.4116e+008$	$2.0144e+008$	$1.7534e+008$	$1.7535e+008$	$1.8628e+008$	$1.3799e+008$
	0.1	6	$8.0822e+008$	$1.1622e+009$	$1.0022e+009$	$1.0029e+009$	$1.0658e+009$	$7.8976e+008$
		15	$1.4184e+011$	$1.9906e+011$	$1.6742e+011$	$1.7219e+011$	$1.8496e+011$	$1.3738e+011$
		27	$8.2497e+013$	$7.7892e+013$	$1.2114e+013$	$7.2838e+013$	$9.7816e+013$	$6.9734e+013$
		1	$1.3188e+006$	$1.8824e+006$	$1.6386e+006$	$1.6386e+006$	$1.7407e+006$	$1.2894e+006$
		2	$2.6265e+006$	$3.7487e+006$	$3.2631e+006$	$3.2632e+006$	$3.4665e+006$	$2.5678e+006$
Tax		3	$4.7052e+006$	$6.7146e+006$	$5.8445e+006$	$5.8450e+006$	$6.2094e+006$	$4.5997e+006$
	0.1	6	$2.6941e+007$	$3.8741e+007$	$3.3408e+007$	$3.3429e+007$	$3.5528e+007$	$2.6325e+007$
		15	$4.7279e+009$	$6.6352e+009$	$5.5807e+009$	$5.7395e+009$	$6.1652e+009$	$4.5793e+009$
		27	$2.7499e+012$	$2.5964e+012$	$4.0380e+011$	$2.4279e+012$	$3.2605e+012$	$2.3245e+012$
		1	$2.2274e+007$	$3.1792e+007$	$2.7675e+007$	$2.7675e+007$	$2.9398e+007$	$2.1776e+007$
		2	$4.4359e+007$	$6.3312e+007$	$5.5111e+007$	$5.5112e+007$	$5.8546e+007$	$4.3367e+007$
ER		3	$7.0636e+007$	$1.0080e+008$	$8.7740e+007$	$8.7747e+007$	$9.3218e+007$	$6.9052e+007$
	0.2	6	$2.8405e+008$	$4.1295e+008$	$3.5224e+008$	$3.5247e+008$	$3.7459e+008$	$2.7757e+008$
		15	$1.7270e+010$	$2.4502e+010$	$2.0385e+010$	$2.0965e+010$	$2.2520e+010$	$1.6727e+010$
		27	$2.4440e+012$	$2.3329e+012$	$3.5889e+011$	$2.1579e+012$	$2.8979e+012$	$2.0659e+012$
		1	$3.5169e+007$	$5.0198e+007$	$4.3697e+007$	$4.3697e+007$	$4.6418e+007$	$3.4383e+007$
		2	$7.0041e+007$	$9.9966e+007$	$8.7017e+007$	$8.7019e+007$	$9.2440e+007$	$6.8474e+007$
		3	$1.1153e+008$	$1.5916e+008$	$1.3854e+008$	$1.3855e+008$	$1.4719e+008$	$1.0903e+008$
MC	0.2	6	$4.4851e+008$	$6.5202e+008$	$5.5617e+008$	$5.5653e+008$	$5.9147e+008$	$4.3826e+008$
		15	$2.7268e+010$	$3.8688e+010$	$3.2187e+010$	$3.3103e+010$	$3.5558e+010$	$2.6411e+010$
		27	$3.8590e+012$	$3.6835e+012$	$5.6666e+011$	$3.4072e+012$	$4.5756e+012$	$3.2620e+012$
		1	$1.1723e+006$	$1.6733e+006$	$1.4566e+006$	$1.4566e+006$	$1.5473e+006$	$1.1461e+006$
		2	$2.3347e+006$	$3.3322e+006$	$2.9006e+006$	$2.9006e+006$	$3.0813e+006$	$2.2825e+006$
		3	$3.7177e+006$	$5.3054e+006$	$4.6179e+006$	$4.6182e+006$	$4.9062e+006$	$3.6343e+006$
Tax	0.2	6	$1.4950e+007$	$2.1734e+007$	$1.8539e+007$	$1.8551e+007$	$1.9716e+007$	$1.4609e+007$
		15	$9.0893e+008$	$1.2896e+009$	$1.0729e+009$	$1.1034e+009$	$1.1853e+009$	$8.8037e+008$
		27	$1.2863e+011$	$1.2278e+011$	$1.8889e+010$	$1.1357e+011$	$1.5252e+011$	$1.0873e+011$
		1	$1.3921e+007$	$1.9870e+007$	$1.7297e+007$	$1.7297e+007$	$1.8374e+007$	$1.3610e+007$
		2	$2.7725e+007$	$3.9570e+007$	$3.4444e+007$	$3.4445e+007$	$3.6591e+007$	$2.7104e+007$
		3	$2.7592e+007$	$3.9376e+007$	$3.4273e+007$	$3.4276e+007$	$3.6413e+007$	$2.6973e+007$
ER	0.5	6	$2.7089e+007$	$4.1685e+007$	$3.3592e+007$	$3.3614e+007$	$3.5724e+007$	$2.6471e+007$
		15	$2.3966e+007$	$3.5992e+007$	$2.8290e+007$	$2.9095e+007$	$3.1253e+007$	$2.3213e+007$
		27	$1.2050e+007$	$1.2175e+007$	$1.7694e+006$	$1.0639e+007$	$1.4288e+007$	$1.0186e+007$
		1	$2.1981e+007$	$3.1374e+007$	$2.7311e+007$	$2.7311e+007$	$2.9012e+007$	$2.1489e+007$
		2	$4.3776e+007$	$6.2479e+007$	$5.4386e+007$	$5.4387e+007$	$5.7775e+007$	$4.2796e+007$
		3	$4.3567e+007$	$6.2172e+007$	$5.4116e+007$	$5.4120e+007$	$5.7494e+007$	$4.2590e+007$
MC	0.5	6	$4.2773e+007$	$6.5818e+007$	$5.3040e+007$	$5.3075e+007$	$5.6407e+007$	$4.1796e+007$
		15	$3.7842e+007$	$5.6829e+007$	$4.4668e+007$	$4.5939e+007$	$4.9346e+007$	$3.6653e+007$
		27	$1.9026e+007$	$1.9223e+007$	$2.7939e+006$	$1.6799e+007$	$2.2559e+007$	$1.6083e+007$
		1	$7.3269e+005$	$1.0458e+006$	$9.1035e+005$	$9.1035e+005$	$9.6705e+005$	$7.1631e+005$
		2	$1.4592e+006$	$2.0826e+006$	$1.8129e+006$	$1.8129e+006$	$1.9258e+006$	$1.4265e+006$
	0.5	3	$1.4522e+006$	$2.0724e+006$	$1.8039e+006$	$1.8040e+006$	$1.9165e+006$	$1.4197e+006$
Tax		6	$1.4258e+006$	$2.1939e+006$	$1.7680e+006$	$1.7692e+006$	$1.8802e+006$	$1.3932e+006$
		15	$1.2614e+006$	$1.8943e+006$	$1.4889e+006$	$1.5313e+006$	$1.6449e+006$	$1.2218e+006$
		27	$6.3420e+005$	$6.4076e+005$	$9.3129e+004$	$5.5995e+005$	$7.5198e+005$	$5.3609e+005$

Note that MC here stands for management costs,d ER stands for expected returns and N denotes Naira.

Station, and follow by Lagos Station, while Warri Station yields the lowest net return. It is observed that as time evolves, the effect of the presence of vehicle breakdown affects the returns from all the six stations. When compare with the values in Table 10, one will see that breakdown of vehicles reduces the returns from the investment over time.

Table 11 (second compartment) reports the optimal management costs for six stations under vehicle breakdown conditions over twenty-seven weeks. The results indicate that management costs increase over time as returns increase for all stations.

Also Table 11 shows the optimal tax payable by the six stations from nine destinations in the presence of breakdown vehicles for a period of twenty-seven weeks. It is also observed that as returns increases, the tax payable to government increases over time for all the stations.

Table 12 shows the optimal returns, management costs and tax for six stations from nine destinations in the presence of breakdown vehicles at a 27- week, for the varying value of depreciation rate, ν. We observed that as the depreciation rate increases, the expected net returns, management costs and tax for all the six stations, decreases. This again, simply tells us that as depreciation rate increase, will reduce the optimal return from the investment, thereby reducing management costs and taxes.

Table 13 shows the optimal returns, management costs and tax for six stations from nine destinations in the presence of breakdown vehicles at a 27- week, for the varying value of discount rate, η. We observed that as the discount rate increases, the expected net returns, management costs and tax for all the six stations, decreases. This again, simply tells us that as discount rate increase will reduce drastically, the optimal return from the investment, thereby reducing management costs and taxes.

5 Conclusion

This paper dealt with allocation of vehicles from stations to destinations via routes in order to maximize returns over finite time horizons. The expected returns were modelled as a function of the random failure of vehicles and depreciation rate. The rate of breakdown vehicle was assumed to be a strictly monotone increasing function of time. A model on allocation of the vehicles from different stations to different destinations through making choices of transiting using the best possible routes, in the presence of random breakdown of the vehicles was developed, and solved using dynamic programming approach. As a result, optimal expected discounted net returns, management cost and tax payable to government were obtained, in this paper.

It was observed that the higher the expected discounted returns, the higher the management costs and tax over time, which is an expected results. The result give the company a possible net returns, management costs and tax to be paid when buses are allocated from Benin Station to: Sapele, Ekpoma, Auchi, Warri, PH, Lagos, Ibadan, Ilorri and Abuja over time; Lagos Station to: Abuja, Benin, Ibadan, Ilorri, Minna, Warri, Port Harcourt, Kaduna, and Kano over time; Warri Station to: Benin, Lagos, Ibadan, Abuja, Kano, Kaduna, Onitcha, Enugu, and Abia over time, Abuja Station to: Lagos, Benin, Kaduna, Kano, Onitcha, Port Harcourt, Ibadan, Minna, Ilorri over time; Port Harcourt Station to Benin, Abuja, Lagos, Warri, Onitcha, Ibadan, Sapele, Auchi, and Agbor over time; and Ibadan Station to: Benin, Port Harcourt, Warri, Abuja, Lagos, Minna, Ilorri, Kano and Kaduna over time.

It was observed that in the absence of breakdown of vehicles that Warri Station and Abuja Station will yield the same net return, however, in the presence of vehicle breakdown, all stations saw reduced returns, with Warri Station suffering the most severe losses. This highlights that a station's specific allocation policies and the condition of its assigned routes are critical determinants of its financial resilience when vehicles are subject to stochastic failure. In the presence of vehicle breakdown, Port Harcourt Station yields the highest net return, follow by Benin Station, follow by Ibadan Station, follow by Abuja Station, and follow by Lagos Station, while Warri Station yields the lowest net return. It was again, found that as time evolves, the effect of the presence of vehicle breakdown affects the returns from all the six stations. It was also found that breakdown of vehicles reduces the returns from the investment over time.

Again, we also found that as returns increases, the tax payable to government increases over time for all the stations. It is also found that as returns increases, the tax payable to government increases over time for all the stations. It was found that as the depreciation rate increases, the expected net returns, management costs and tax for all the six stations, decreases. We found that as the discount rate increases, the expected net returns, management costs and tax for all the six stations, decreases.

Data Availability Statement

Data will be made available on request.

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Conflicts of Interest

The authors declare no conflicts of interest.

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Ethical Approval and Consent to Participate

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