RESEARCH ARTICLE

Nonlinear Dynamics of Green Innovation, Environmental Regulation, and Corporate Carbon Abatement

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Abstract

Corporate carbon abatement is commonly modeled as a direct and continuous response to environmental regulation or green innovation. Such representations cannot fully explain persistent differences among firms facing similar policy conditions or distinguish the pressure required to initiate a low-carbon transition from that needed to sustain it. This study develops a three-state continuous-time model linking corporate carbon abatement, accumulated green innovation, and effective regulatory pressure, with green market preference treated as an external demand-pull parameter. Green innovation lowers the marginal cost of abatement, abatement generates learning and commercial returns to innovation, and regulatory pressure responds to residual emissions while relaxing as abatement and credible green capability improve. The regulation–innovation channel is non-monotonic: moderate pressure strengthens innovation incentives, whereas the marginal stimulus declines at high pressure. The analysis combines equilibrium and stability analysis, saddle-node verification, attraction-basin classification, temporary-policy simulations, regime mapping, and global sensitivity analysis. Under the baseline configuration, two locally stable equilibria are separated by a saddle, and nondegenerate saddle-node thresholds generate hysteresis between low- and high-abatement states. Green market preference lowers the transition threshold, while temporary regulation succeeds only when both intensity and duration are sufficient. Regulatory adjustment may generate damped oscillatory modes, but no Hopf crossing is detected within the examined range. The findings show that regulatory pressure acts as an adaptive state-transition mechanism through which initial capability, policy timing, and market demand shape persistent corporate carbon-abatement outcomes. Broader stress tests confirm that bistability is conditional and can disappear under weak innovation capacity or combined adverse structural conditions. The numerical procedures developed here—multistart root finding,adaptive Runge–Kutta integration, analytical Jacobian evaluation,and Latin-hypercube sensitivity design—are transferable tocontinuous-time state-transition models in other applied domains.

Submitted: 29 July 2026
Revised: 17 August 2026
Accepted: 20 August 2026
Published: 15 October 2026

Vol. 3, **No.** 1, 2027.
10.62762/JNSPM.2026.289453

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Citation
Sun, F. (2026). Nonlinear Dynamics of Green Innovation, Environmental Regulation, and Corporate Carbon Abatement. *Journal of Numerical Simulations in Physics and Mathematics*, 3(1), 19–37.

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Keywords: Corporate carbon abatement, green innovation, environmental regulation, regulatory thresholds, green market preference.

1 Introduction

Corporate carbon abatement has become a central issue in operations, capital allocation, innovation, and strategic governance. Firms are expected to reduce direct emissions, improve energy efficiency, redesign production processes, and develop lower-carbon products while retaining competitiveness. Climate-mitigation pathways require technological and organizational changes at substantial scale, making corporate behavior a decisive part of the transition [1]. Yet corporate responses remain persistently heterogeneous. Some firms accumulate green technologies and embed abatement into routine decision-making, whereas others make only limited adjustments even when they operate under similar regulatory regimes.

Environmental regulation is commonly presented as the principal external driver of this transition. Emission standards, taxes, trading schemes, disclosure requirements, and penalties increase the cost of carbon-intensive activity. The Porter hypothesis further suggests that well-designed regulation may stimulate innovation that offsets part of the compliance burden [2]. Subsequent studies show that this effect is conditional rather than automatic: enforcement credibility, financing conditions, technological opportunity, policy design, and firm capability determine whether regulatory pressure becomes productive innovation or merely additional cost [3, 4]. Meta-analytic evidence likewise indicates that average competitiveness effects conceal considerable firm- and context-level heterogeneity [5].

Green innovation supplies the technological basis for persistent carbon abatement. Cleaner equipment, process redesign, material substitution, energy-management systems, and low-carbon products can reduce the cost of abatement and create new revenue opportunities. Eco-innovation research has therefore emphasized the joint influence of regulatory push, technology push, and market pull [6]. Firm-level evidence connects green research and development with lower emissions and improved performance, although the benefits depend on the capacity to convert research expenditure into operational technology [7]. Recent research also finds that environmental regulation can promote corporate green innovation, but the magnitude and timing vary markedly [8, 9].

Most empirical models nevertheless remain marginal and approximately linear. Regulation is treated as an explanatory variable whose coefficient indicates an average increase or decrease in innovation or emissions. Green innovation is often treated as a contemporaneous input or mediator rather than as an accumulated state. These approaches are valuable for estimating average effects, but they are less suited to explaining four observations. First, firms can remain in low-abatement states despite nonzero regulation. Second, firms with similar current incentives may follow different long-run paths because their inherited technology and organizational capability differ. Third, the pressure required to initiate transition can exceed the pressure needed to maintain it after complementary capabilities accumulate. Fourth, regulatory authorities do not remain passive: weak abatement can trigger stronger enforcement, while sustained corporate improvement can justify policy relaxation.

These observations imply a state-dependent and jointly adaptive process. Green innovation lowers the technical and organizational cost of deeper abatement. A larger innovation stock can therefore raise the feasible and profitable abatement level. Abatement, in turn, validates green investments, develops specialized routines, generates implementation learning, and increases the return to subsequent innovation. This reciprocal reinforcement may create increasing returns and multiple equilibria. Regulation introduces a balancing feedback: insufficient abatement increases regulatory activation, while improved abatement and innovation reduce the need for continued pressure. The policy response is not necessarily monotonic in its effect on innovation. Moderate regulation can induce research and technology adoption, whereas very high compliance burdens can reduce the marginal innovation stimulus by diverting managerial and financial resources away from long-horizon research [10, 11].

The possibility of multiple stable states has direct policy relevance. Under bistability, an incremental policy change may produce little permanent improvement while a firm remains inside the low-abatement attraction basin. Once regulatory pressure crosses an upper threshold, the low state disappears or becomes inaccessible, forcing a transition toward a high-abatement state. After green capabilities have accumulated, pressure can be

reduced without immediately reversing the transition. The resulting hysteresis separates transition initiation from transition maintenance and makes intervention strength, duration, and timing analytically important.

Market demand can alter these boundaries. Consumers and business customers that value lower-carbon products increase the private return to environmental innovation. Evidence links consumer environmental preference with corporate green technology investment [12], while willingness to pay for credible eco-labeled products indicates a market reward for verified environmental performance [13]. Green demand may therefore complement regulation by strengthening both innovation incentives and the revenue consequences of abatement. Within a state-transition framework, the relevant question is whether market preference shifts the policy threshold required to escape low-abatement lock-in.

This study addresses five questions. First, under what conditions can corporate carbon abatement, green innovation, and responsive regulation generate multiple equilibria? Second, how does green innovation affect abatement through both productivity and cost-reduction channels? Third, how does endogenous regulation alter the stability and accessibility of low- and high-abatement states? Fourth, how does green market preference shift the regulatory thresholds? Fifth, when policy pressure is temporary, what combinations of intensity and duration produce a persistent transition rather than a temporary response?

To answer these questions, the study develops a three-state continuous-time model. Corporate carbon abatement $A(t)$, the green innovation stock $K(t)$, and effective regulatory or enforcement pressure $R(t)$ are normalized state variables. Green market preference G is an external parameter. Hill functions represent threshold-like capability effects, a state-dependent cost term captures innovation-enabled reductions in marginal abatement cost, a non-monotonic regulatory term represents innovation stimulation followed by a declining marginal stimulus at high pressure, and a responsive enforcement equation links policy intensity to residual emissions and accumulated corporate capability. The analysis combines positive invariance, equilibrium identities, a three-dimensional analytical Jacobian, Routh–Hurwitz stability conditions, fold-point solving with nondegeneracy checks, equilibrium-branch tracing, basin classification, temporary-intervention experiments, feedback-regime mapping, and global sensitivity analysis.

The study makes three contributions. First, it moves beyond a symmetric two-variable reinforcement model by assigning distinct economic mechanisms to abatement, innovation, and regulation. Innovation changes the cost structure of abatement; abatement creates learning returns; and regulation responds to corporate performance rather than remaining fixed. Second, it identifies two regulatory pressure thresholds and shows that green market preference reduces the pressure needed to eliminate low-abatement lock-in. Third, it separates equilibrium transition from regulatory adjustment speed. The model permits damped regulatory oscillations, but the threshold result remains when no Hopf crossing occurs, demonstrating that bistability is generated by the state structure rather than by an oscillatory instability.

The remainder of the paper is organized as follows. Section 2 develops the theoretical mechanisms. Section 3 formulates the model and derives boundedness, equilibrium, and stability conditions. Section 4 explains the numerical design. Section 5 reports equilibria, basins, bifurcations, policy thresholds, and sensitivity results. Sections 6 and 7 discuss contributions and implications. Section 8 concludes.

2 Theoretical Background and Mechanisms

2.1 Green innovation as an accumulated corporate capability

Green innovation includes products, processes, equipment, knowledge, employee skills, and organizational routines that reduce environmental burdens [6]. It is more accurately represented as an accumulated capability than as a one-period expenditure. Research projects generate codified and tacit knowledge; production trials reveal implementation constraints; and repeated use creates complementary routines. The effectiveness of a given innovation expenditure therefore depends on the stock inherited from earlier periods.

The stock perspective also introduces depreciation. Green technologies become obsolete, equipment loses efficiency, specialized employees leave, and unsupported routines decay. A firm that stops investing can therefore lose part of its earlier capability. This accumulation-depreciation structure is central to path dependence: two firms facing the same current policy can respond differently because their inherited

green technology stocks differ [14, 15].

Innovation has two effects on abatement in the present model. The first is a productivity effect: once a critical mass of complementary technology is available, further abatement becomes technically feasible. The second is a cost-reduction effect: green technology lowers the marginal resource cost of deeper abatement. This second channel distinguishes the present model from a simple positive-input equation. It allows innovation to change the shape of the corporate abatement response rather than merely adding a constant incentive.

2.2 Abatement as a source of innovation returns and learning

The relationship is reciprocal. Corporate abatement can increase the return to green innovation through implementation learning, validation of capital expenditure, data generation, supplier coordination, and market credibility. Directed technical-change theory shows that relative returns influence the direction of innovation [16]. Carbon prices and environmental policy can therefore redirect technological development toward cleaner alternatives [17, 18]. At the firm level, the implementation of abatement projects generates information about bottlenecks and commercially valuable applications that may not be visible during laboratory research.

This feedback does not imply that every abatement action produces valuable innovation. Symbolic compliance, inexpensive offsets, and isolated efficiency campaigns may generate little organizational learning. The model therefore uses a threshold-like response: abatement contributes weakly to innovation at low levels, rises rapidly near a capability threshold, and saturates as the available learning opportunities become exhausted. Such nonlinearity is consistent with evidence that the technology-emissions relationship is not uniformly linear [19, 20].

Additional empirical evidence links environmental innovation and green-technology capability to lower carbon emissions across national and urban contexts [21, 22].

2.3 Environmental regulation as adaptive pressure

Environmental regulation affects both abatement and innovation. It raises the expected cost of residual emissions, coordinates investment expectations, and can select against inefficient technologies [2–5]. Evidence from emissions-control policies and environmental laws suggests that regulatory design influences corporate innovation, operating risk, and low-carbon transformation [10, 23, 24]. The model therefore represents the effective regulatory or enforcement pressure experienced by the focal firm as a state variable rather than treating formal statutory stringency as a fixed firm-level coefficient. The coupled unit of analysis is a firm-regulator system: the firm does not choose $R(t)$, and $R(t)$ does not represent the entire legal regime.

Hierarchical supervision mechanisms can strengthen the effectiveness of firm-level enforcement signals: evidence from centralized inspection systems shows that upper-level oversight substantially increases local enforcement intensity, supporting the interpretation of $R(t)$ as a realized and adjustable enforcement pressure rather than a fixed statutory parameter [25].

Recent firm-level evidence further links environmental regulation to corporate carbon emissions and green innovation through dynamic channels [26]. The salience of regulatory pressure can be amplified by information-dissemination environments: new media coverage intensifies the observable compliance signals firms face, thereby strengthening the regulation-to-innovation link [27].

Sector-specific and game-theoretic studies likewise show that the regulation–carbon relationship depends on industry context and strategic response [28, 29].

Review and cross-country evidence indicate that policy design and policy stringency condition innovation, competitiveness, and productivity outcomes [30, 31].

The regulatory equation contains two mechanisms. First, insufficient abatement raises policy activation. Regulators intensify monitoring or enforcement when residual emissions remain high. Second, policy pressure relaxes as firms achieve higher abatement and accumulate credible green capability. This does not imply complete deregulation; it represents adaptive allocation of enforcement attention within a normalized system. The resulting balancing loop can create lagged adjustment and temporary overshooting.

The state $R(t)$ should therefore be interpreted as realized firm-level regulatory exposure rather than as any single statutory instrument. Carbon prices, emissions standards, disclosure rules, and formal enforcement mandates define the institutional environment that feeds baseline activation and residual-emissions responsiveness,

whereas realized $R(t)$ reflects how strongly those rules are experienced by the focal firm. In empirical work, $R(t)$ could be operationalized by combining observable enforcement exposures, such as inspection frequency, enforcement notices, penalty incidence or severity, and emissions-trading compliance exposure, with jurisdiction- or sector-level measures of regulatory stringency. Existing studies illustrate the use of emission-control mandates and composite policy-stringency measures [23, 31]. Regulatory uncertainty or political turnover is not represented as a separate state in the deterministic baseline; it could enter empirically or in future extensions through time-varying regulatory parameters or stochastic shocks.

The innovation effect of regulation is non-monotonic. Moderate pressure can stimulate research and adoption by increasing the return to cleaner technology. At high pressure, however, compliance expenditure, liquidity constraints, managerial distraction, and short-term risk avoidance can weaken the additional innovation stimulus [11]. Environmental regulation at high intensity also elevates firm-level operating risk, which further constrains the capacity for long-horizon research investment [10]. The model captures this channel with $Re^{-\zeta R}$, which rises for $R < 1/\zeta$ and falls for $R > 1/\zeta$. Because this term remains nonnegative, the specification represents a declining regulation-induced innovation contribution at high pressure, not a net negative effect relative to zero regulation. The direct compliance effect on abatement remains positive and saturating.

2.4 Green market preference as demand-pull reinforcement

Green market preference increases the commercial value of lower-carbon products and credible environmental performance. Demand-pull can raise the return to both abatement and innovation [12, 13]. It also affects the policy problem: when market rewards are strong, firms may enter the high-abatement basin at lower baseline regulatory activation. Market preference is retained as an external parameter because its purpose is to shift the corporate-regulatory state structure rather than to create an additional fourth state equation. The core specification keeps G exogenous for state-space transparency, while Section 5.8 reports a robustness check in which market preference is partially linked to realized regulatory pressure.

Market demand cannot fully replace governance. Preferences can be noisy, product claims can be difficult to verify, and greenwashing can weaken the link between environmental performance and revenue [32]. Regulation may therefore remain necessary to establish measurement credibility and prevent low-cost symbolic responses. The expected relationship is partial substitution rather than complete equivalence.

2.5 Feedback structure and state transitions

Figure 1 summarizes the model. The central reinforcing loop links innovation and abatement: a larger green innovation stock lowers abatement cost, and greater abatement produces learning and commercial returns that stimulate further innovation. Regulatory pressure directly promotes abatement, whereas improved abatement and accumulated green capability reduce the need for continued enforcement. Green market preference strengthens the private return to both abatement and innovation. The regulation-innovation link is non-monotonic: the innovation contribution rises under moderate pressure but declines marginally at high pressure. The distinction between reinforcing and balancing loops follows established system-dynamics practice [33].

When the reinforcing loop is weak, the system has a unique equilibrium and policy changes generate gradual adjustment. When the loop is sufficiently steep, multiple equilibria can emerge. Responsive regulation then changes both the positions of these equilibria and the routes by which firms move between them. The central proposition is that low-carbon transition depends not only on incentive levels but also on whether policy and market forces move the firm across a state boundary.

3 Model Formulation

3.1 State variables and response function

Let $A(t) \in [0,1]$ denote normalized corporate carbon abatement, $K(t) \in [0,1]$ the normalized green innovation stock, and $R(t) \in [0,1]$ the effective regulatory or enforcement pressure experienced by the focal firm. $R(t)$ is an endogenous policy-response state in the coupled firm-regulator system rather than the formal statutory stringency of the entire jurisdiction. Green market preference is denoted by $G \in [0,1]$. The smooth threshold response is

$$H(x;\kappa,h) = \frac{x^h}{\kappa^h + x^h}, \qquad \kappa > 0,\ h > 1. \tag{1}$$

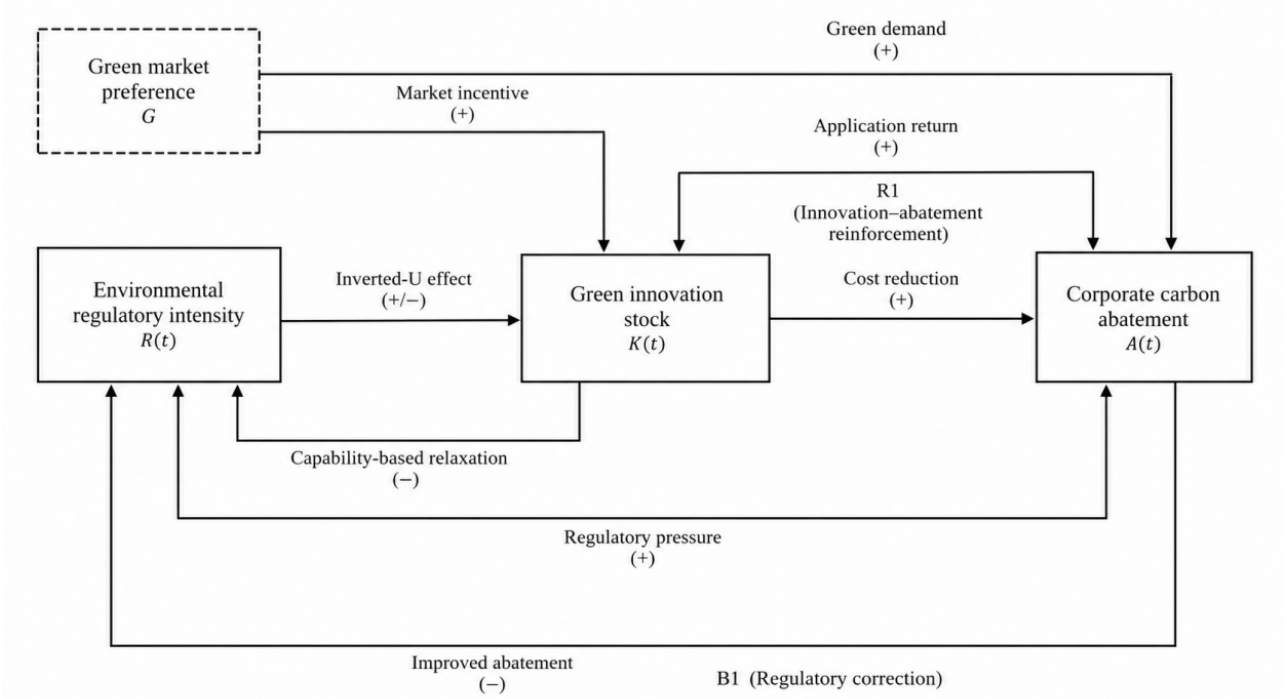


Figure 1. Core feedback structure linking corporate carbon abatement, green innovation stock, endogenous effective regulatory pressure, and green market preference.

Its derivative is

$$H_x(x;\kappa,h)=\frac{h\kappa^h x^{h-1}}{\left(\kappa^h+x^h\right)^2}. \tag{2}$$

The Hill form captures weak effects below a capability threshold, rapid gains near the threshold, and saturation at high levels.

3.2 Corporate carbon-abatement equation

The abatement equation is

$$\begin{aligned}\dot{A}=\lambda_A\Big\{&(1-A)\big[\alpha_0+\alpha_G G+\alpha_R H(R;\kappa_R,p)\\&+\alpha_K H(K;\kappa_K,m)\big]\\&-A\Big[\frac{c_0+c_1A}{1+\eta K}+\delta_A\Big]\Big\}.\end{aligned} \tag{3}$$

The first bracket is the expansion force. α_0 is baseline abatement effort, $\alpha_G G$ is demand pull, $\alpha_R H(R)$ is a saturating compliance response, and $\alpha_K H(K)$ is the productivity contribution of accumulated green capability. The factor $(1-A)$ represents the remaining feasible abatement space.

The second bracket is the effective abatement cost and decay. The numerator c_0+c_1A implies rising marginal difficulty as abatement deepens. Green innovation reduces this cost through the denominator $1+\eta K$. The derivative

$$\frac{\partial}{\partial K}\left(\frac{c_0+c_1A}{1+\eta K}\right)=-\frac{\eta\,(c_0+c_1A)}{(1+\eta K)^2}<0 \tag{4}$$

formalizes the cost-reducing role of innovation. δ_A captures deterioration of unsupported abatement routines and equipment.

3.3 Green-innovation equation

The green innovation stock evolves according to

$$\begin{aligned}\dot{K}=\lambda_K\Big\{&(1-K)\big[\beta_0+\beta_A H(A;\kappa_A,n)\\&+\beta_G G+\beta_R R e^{-\zeta R}\big]-\delta_K K-\xi_K K^2\Big\}.\end{aligned} \tag{5}$$

β_0 is baseline innovation, $\beta_A H(A)$ represents learning and commercial returns generated by abatement, and $\beta_G G$ captures market reward. The regulatory innovation term is

$$\Phi(R)=Re^{-\zeta R},\qquad \Phi'(R)=e^{-\zeta R}(1-\zeta R). \tag{6}$$

The regulation-induced innovation contribution $\Phi(R)$ rises when $R<1/\zeta$ and declines when $R>1/\zeta$. Because $\Phi(R)\geq 0$, the specification does not impose a net negative innovation effect relative to zero regulation. The maximum of this channel occurs at

$$R_{\text{innov}}^{\max}=\frac{1}{\zeta}. \tag{7}$$

This value maximizes only the modeled regulation-induced innovation term; it is not a

social-welfare optimum. $\delta_K K$ represents depreciation and $\xi_K K^2$ declining marginal knowledge productivity and coordination complexity. The stock-loss specification is consistent with organizational-learning perspectives that emphasize limits to accumulated learning and with system-dynamics representations of stock depreciation and capability erosion [14, 33].

3.4 Endogenous effective regulatory-pressure equation

Regulatory intensity adjusts according to

$$\dot{R} = \lambda_R \Big\{ \big[\rho_0 + \rho_E H(1-A;\kappa_E,q)\big](1-R) - \big(\mu_0 + \mu_A A + \mu_K K\big)R \Big\}. \tag{8}$$

ρ_0 is baseline regulatory activation and is the principal bifurcation parameter. It represents the institutional baseline that activates firm-level enforcement pressure; the realized state $R(t)$ adjusts endogenously and is not controlled by the firm. $\rho_E H(1-A)$ raises enforcement when residual emissions remain high. The factor $(1-R)$ limits further policy escalation. The relaxation term increases with current abatement and accumulated innovation, reflecting reduced enforcement need as credible corporate capability improves. λ_R controls regulatory adjustment speed.

For a given (A, K), the equilibrium regulatory intensity is explicit:

$$R^*(A,K) = \frac{\rho_0+\rho_E H(1-A;\kappa_E,q)}{\rho_0+\rho_E H(1-A;\kappa_E,q)+\mu_0+\mu_A A+\mu_K K}. \tag{9}$$

Equation (9) shows that regulation rises with baseline activation and the residual-emissions response, but falls as abatement and green capability increase.

3.5 Positive invariance and boundedness

Define $\Omega = [0,1]^3$. At $A = 0$, the right-hand side of (3) is nonnegative; at $A = 1$, it equals $-\lambda_A\left[(c_0+c_1)/(1+\eta K)+\delta_A\right] < 0$. At $K = 0$, the right-hand side of (5) is nonnegative; at $K = 1$, it equals $-\lambda_K(\delta_K+\xi_K) < 0$. At $R = 0$, the right-hand side of (8) is nonnegative; at $R = 1$, it equals $-\lambda_R(\mu_0+\mu_A A+\mu_K K) < 0$. Hence the vector field points inward on every face of Ω. Because the right-hand sides are continuously differentiable, a unique solution exists for every initial condition in Ω, and Ω is positively invariant.

3.6 Equilibrium identities

Let

$$U_A = \alpha_0 + \alpha_G G + \alpha_R H(R;\kappa_R,p) + \alpha_K H(K;\kappa_K,m), \tag{10}$$

$$C_A = \frac{c_0+c_1 A}{1+\eta K} + \delta_A, \tag{11}$$

$$U_K = \beta_0 + \beta_A H(A;\kappa_A,n) + \beta_G G + \beta_R R e^{-\zeta R}.$$

At equilibrium, (3) implies

$$A^* = \frac{U_A(A^*,K^*,R^*)}{U_A(A^*,K^*,R^*)+C_A(A^*,K^*)}. \tag{12}$$

Because C_A depends on A^*, (12) is an implicit fixed-point identity. From (5), K^* is the positive root of

$$\xi_K (K^*)^2 + (U_K+\delta_K)K^* - U_K = 0, \tag{13}$$

and therefore

$$K^* = \frac{-(U_K+\delta_K)+\sqrt{(U_K+\delta_K)^2+4\xi_K U_K}}{2\xi_K}. \tag{14}$$

Equations (9), (12), and (14) define the three-dimensional equilibrium fixed point.

3.7 Jacobian and local stability

Let f, g, and h denote the right-hand sides of (3), (5), and (8). The Jacobian is

$$J(A,K,R) = \begin{pmatrix} f_A & f_K & f_R \\ g_A & g_K & g_R \\ h_A & h_K & h_R \end{pmatrix}. \tag{15}$$

The nonzero entries are

$$f_A = \lambda_A\left[-U_A - C_A - \frac{Ac_1}{1+\eta K}\right], \tag{16a}$$

$$f_K = \lambda_A\left[(1-A)\alpha_K H_K'(K) + \frac{A\eta(c_0+c_1 A)}{(1+\eta K)^2}\right], \tag{16b}$$

$$f_R = \lambda_A(1-A)\alpha_R H_R'(R),$$

$$g_A = \lambda_K(1-K)\beta_A H_A'(A), \tag{16c}$$

$$g_K = \lambda_K\left[-U_K - \delta_K - 2\xi_K K\right],$$

$$g_R = \lambda_K(1-K)\beta_R e^{-\zeta R}(1-\zeta R), \tag{16d}$$

$$h_A = \lambda_R\left[-\rho_E H_E'(1-A)(1-R) - \mu_A R\right], \tag{16e}$$

$$h_K = -\lambda_R \mu_K R,$$

$$h_R = -\lambda_R\left[\rho_0 + \rho_E H(1-A) + \mu_0 + \mu_A A + \mu_K K\right]. \tag{16f}$$

The characteristic polynomial, with s denoting the characteristic root, is

$$\det(sI - J) = s^3 + a_1 s^2 + a_2 s + a_3, \tag{17}$$

where

$$a_1 = -\operatorname{tr}(J), \tag{18a}$$
$$a_2 = J_{11}J_{22} + J_{11}J_{33} + J_{22}J_{33} - J_{12}J_{21} - J_{13}J_{31} - J_{23}J_{32}, \tag{18b}$$
$$a_3 = -\det(J). \tag{18c}$$

The Routh–Hurwitz conditions for local asymptotic stability are

$$a_1 > 0, \qquad a_2 > 0, \qquad a_3 > 0, \qquad a_1 a_2 > a_3. \tag{19}$$

A saddle-node candidate satisfies $\det(J) = 0$ with one simple zero eigenvalue and the standard nondegeneracy and transversality conditions. A Hopf candidate would satisfy $a_1, a_2, a_3 > 0$ and $a_1 a_2 = a_3$, together with eigenvalue transversality. These conditions are checked numerically rather than assumed.

3.8 Economic interpretation and model boundaries

The state equations encode directional economic mechanisms rather than the first-order conditions of a static profit-maximization problem. Market demand and innovation-abatement complementarity raise the expansion forces; accumulated innovation lowers effective abatement cost; and regulatory pressure raises the cost of residual emissions. The model represents a focal firm or integrated business unit coupled to a responsive enforcement process. It does not distinguish industries, pollutant types, individual policy instruments, or strategic interaction among regulators and multiple firms. Parameters are normalized for mechanism identification rather than estimated as universal coefficients. Accordingly, the reported threshold values identify mechanisms within a normalized corporate-regulatory system; they are not sector forecasts or universal empirical coefficients.

4 Numerical Methods and Baseline Design

The equilibrium system was solved by multistart nonlinear root finding from a $7 \times 7 \times 7$ grid over the unit cube. Solutions with residual norm below 10^{-8} were retained and deduplicated at Euclidean tolerance 10^{-5}. The analytical Jacobian was evaluated at every equilibrium, and local stability was classified by eigenvalues and Routh–Hurwitz coefficients. Trajectories were calculated with an adaptive Runge–Kutta solver using relative tolerance 10^{-9}, absolute tolerance 10^{-11}, and maximum step 0.1. Standard nonlinear-dynamics and bifurcation-analysis procedures follow established references [34, 35].

The attraction-basin calculation used a 110×110 grid of (A_0, K_0) values while fixing $R_0 = R_L$. Each initial condition was integrated for 99 dimensionless time units by an independent fixed-step fourth-order Runge–Kutta routine, and the terminal state was classified by its Euclidean distance to E_L and E_H. This calculation therefore characterizes a two-dimensional slice of the three-dimensional attraction geometry rather than a complete three-dimensional basin surface.

Equilibrium branches were traced on 241 values of baseline activation ρ_0. Fold candidates were obtained by jointly solving $f = g = h = 0$ and $\det(J) = 0$. At each candidate, the code verified a simple zero eigenvalue and evaluated the standard transversality coefficient $w^{\mathsf{T}} F_{\rho_0}$ and quadratic coefficient $\frac{1}{2} w^{\mathsf{T}} D_x^2 F(v, v)$, where v and w are right and left null vectors normalized by $w^{\mathsf{T}} v = 1$. Temporary policy experiments raised ρ_0 to ρ_p for duration T_p and then returned it to baseline; the terminal state was again assigned by nearest-equilibrium distance. Time is dimensionless, so reported durations indicate relative persistence requirements rather than months, quarters, or years.

Global sensitivity used a fixed-seed 500-point Latin-hypercube design over ten structural parameters, each varied independently within $\pm 15\%$ of baseline. Fold outcomes were reported separately as two positive folds, folds outside the positive policy interval, one recovered fold, baseline bistability with incomplete fold recovery, or a single state; incomplete recovery was not interpreted as proof that bistability was absent. Partial rank correlation coefficients (PRCCs) were computed only for draws with two positive recovered folds. All routines use random seed 20260729. To test whether the findings are confined to that local neighborhood, a second fixed-seed 500-point Latin-hypercube design expands the same ten structural parameters to $\pm 30\%$ of baseline (seed 20260815). Targeted structural stress tests separately raise c_0 and c_1 or reduce β_0 and β_A by 25% and 50%. The 269 original one-fold cases are diagnosed by recording which of the two prescribed fold-search seeds remains recoverable and comparing the associated parameter multipliers with the

two-positive-fold subset. Finally, the market-demand robustness check sets $G(R) = \min\{1, \gamma_0 + \gamma_1 R\}$, with $\gamma_0 \in \{0, 0.05, 0.10\}$ and $\gamma_1 \in \{0, 0.25, 0.50\}$, and recomputes the fold conditions using the derivative of the modified system.

Table 1 defines the variables and parameters. Table 2 reports the normalized baseline values. The values satisfy positive invariance, economically interpretable saturation, and a baseline range containing three equilibria. No empirical firm is claimed to possess these exact values.

The baseline $G = 0$ isolates internal and regulatory mechanisms before market demand is introduced comparatively. The baseline is not selected as a universal estimate. Figure 5 and the Latin-hypercube analysis explicitly examine whether the state structure persists beyond this calibration.

5 Numerical Results

5.1 Multiple equilibria and local stability

At the baseline setting $G = 0$ and $\rho_0 = 0.1000$, the model admits three equilibria. Table 3 summarizes their coordinates, characteristic roots, Routh–Hurwitz coefficients, and local classifications.

The equilibrium ordering is substantively meaningful. The low corporate state requires higher long-run regulation because residual emissions remain large. The high state combines stronger abatement and innovation with lower regulatory intensity because capability accumulation reduces the enforcement need. Thus the model does not equate strong environmental performance with permanently maximal regulation.

Numerical basin classification further confirms that the two stable equilibria possess distinct attraction regions. With regulatory pressure fixed at the low-equilibrium level, initial abatement–innovation combinations on opposite sides of the numerically identified basin boundary (separatrix) converge to different long-run states. This provides a dynamical explanation for persistent firm heterogeneity under otherwise identical structural parameters.

5.2 Initial conditions and endogenous regulatory paths

Figure 2 reports four trajectories under the same parameters. Low initial abatement and innovation converge to E_L, accompanied by a rise in regulatory intensity toward R_L. A moderate initial state can also return to the low equilibrium when the complementary capability combination remains below the separatrix. By contrast, an innovation-rich firm with low initial abatement converges to E_H: its inherited technology rapidly reduces abatement cost, after which learning supports continued innovation. A firm beginning at very high abatement and innovation also converges to E_H, but the variables decline toward their sustainable levels.

The regulatory path differs across basins. Regulation increases when a firm remains in the low state, but declines after a successful transition because the endogenous enforcement gap narrows. This distinction matters for policy evaluation. A decline in regulatory intensity after transition does not necessarily indicate policy failure; it can be the endogenous consequence of improved corporate capability.

5.3 Saddle-node bifurcation and hysteresis

Baseline regulatory activation ρ_0 changes the number and location of equilibria. Jointly solving the equilibrium equations and $\det(J) = 0$ identifies two saddle-node folds:

$$\rho_{\text{down}} = 0.0266, \qquad \rho_{\text{up}} = 0.2848. \tag{20}$$

At the lower fold, the high equilibrium and saddle are created at approximately $(A, K, R) = (0.5466, 0.4599, 0.1139)$. At the upper fold, the low equilibrium and saddle collide at approximately $(0.3316, 0.2289, 0.5853)$. Figure 3 traces the three state components along the equilibrium branches.

For $\rho_0 < \rho_{\text{down}}$, only the low state exists. Between the folds, low and high stable equilibria coexist. For $\rho_0 > \rho_{\text{up}}$, only the high state remains. The interval therefore exhibits hysteresis. A firm initially trapped in the low state must experience pressure near the upper threshold before the low equilibrium disappears. Once it has entered the high state, baseline regulatory activation can fall substantially before reversal occurs.

This separation distinguishes initiation from maintenance. Transition requires policy strong enough to overcome inherited technological and organizational disadvantages. Maintenance requires less pressure because accumulated innovation and abatement routines reinforce one another. The result is consistent with path dependence and increasing returns [15, 18], but the present mechanism additionally incorporates adaptive policy relaxation.

Table 1. Variables and parameters.

Symbol	Definition	Role
$A(t)$	Corporate carbon-abatement level	State variable, $[0, 1]$
$K(t)$	Green innovation stock	State variable, $[0, 1]$
$R(t)$	Effective regulatory or enforcement pressure experienced by the focal firm	State variable, $[0, 1]$
G	Green market preference	External parameter, $[0, 1]$
$\lambda_A, \lambda_K, \lambda_R$	Adjustment speeds	Time-scale parameters
α_0, β_0	Baseline abatement and innovation inputs	Preserve positive activity
α_G, β_G	Green market preference effects on abatement and innovation	Demand-pull channel
α_K, β_A	Innovation-abatement feedback strengths	Reinforcing loop
α_R	Direct regulatory abatement effect	Compliance pressure
β_R, ζ	Regulation-induced innovation effect and high-pressure decay	Non-monotonic channel
c_0, c_1, η	Abatement cost, marginal difficulty, and innovation cost reduction	Cost mechanism
$\delta_A, \delta_K, \xi_K$	Abatement decay, innovation depreciation, and complexity cost	Self-limitation
ρ_0, ρ_E	Baseline regulatory activation and residual-emissions response	Regulatory escalation
μ_0, μ_A, μ_K	Regulatory relaxation parameters	Adaptive enforcement
$\kappa_R, \kappa_K, \kappa_A, \kappa_E$	Half-saturation thresholds	Response thresholds
p, m, n, q	Hill exponents	Response steepness

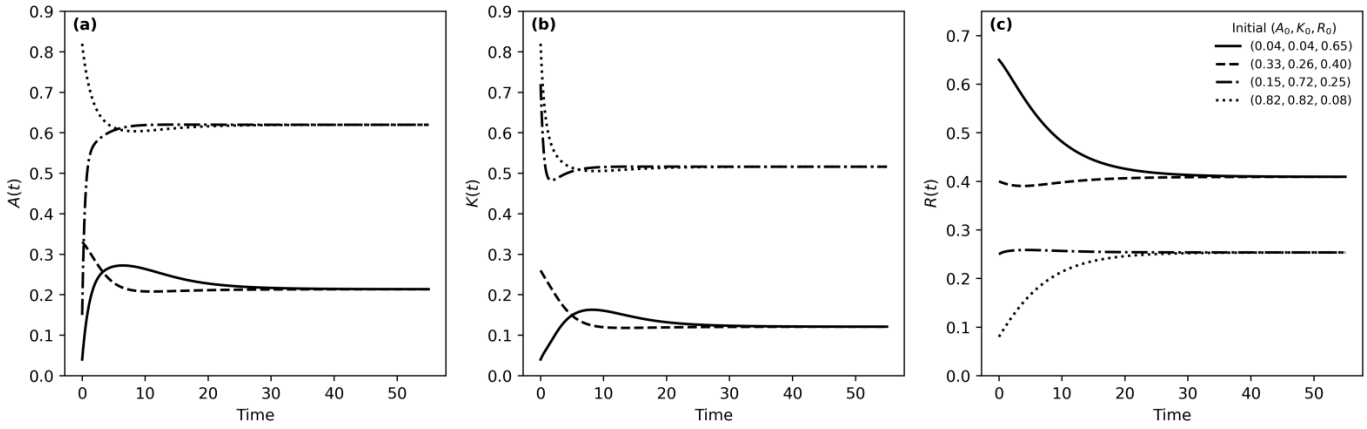


Figure 2. Corporate abatement, green innovation, and effective regulatory-pressure trajectories under heterogeneous initial conditions.

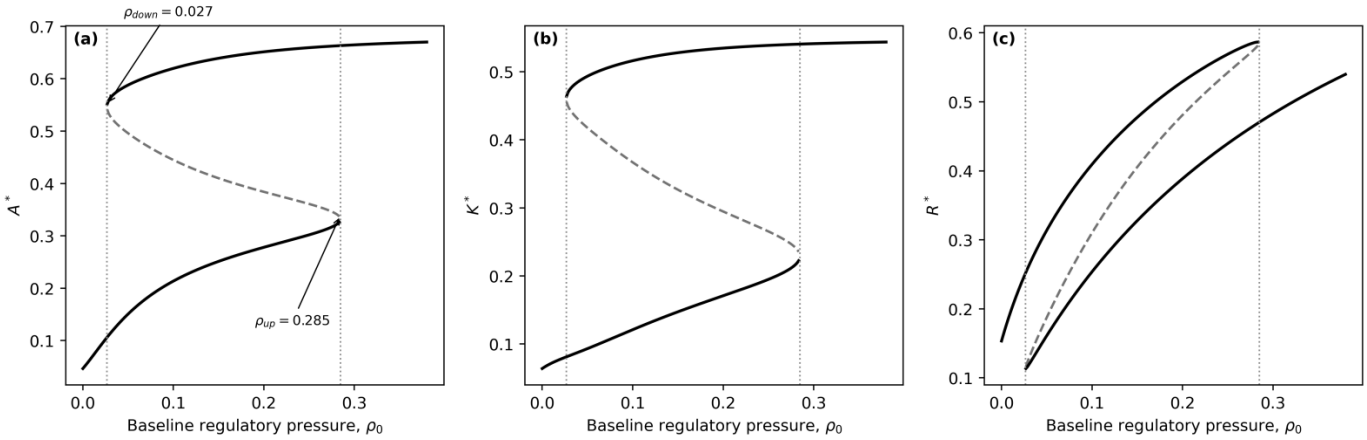


Figure 3. Saddle-node bifurcation of equilibrium abatement, innovation, and regulation with respect to baseline regulatory activation.

Table 2. Baseline normalized parameter values.

Parameter	Value	Parameter	Value
λ_A	1.000	λ_K	1.000
λ_R	0.350	α_0	0.012
α_G	0.120	α_R	0.250
α_K	1.380	κ_R	0.400
p	3.000	κ_K	0.500
m	4.000	c_0	0.490
c_1	0.210	η	0.800
δ_A	0.050	β_0	0.012
β_A	1.010	κ_A	0.550
n	4.000	β_G	0.100
β_R	0.240	ζ	1.800
δ_K	0.580	ξ_K	0.100
ρ_0	0.100	ρ_E	0.027
κ_E	0.500	q	4.000
μ_0	0.120	μ_A	0.180
μ_K	0.160	Baseline G	0.000

5.4 Green market preference and policy-region shifts

Green market preference shifts the folds downward. At $G = 0.05$, the upper threshold falls from 0.2848 to 0.2351; at $G = 0.10$, it falls to 0.2005; at $G = 0.20$, it falls to 0.1537; and at $G = 0.60$, it falls to 0.0608. The lower fold becomes negative near $G = 0.10$, meaning that a high state created under stronger earlier conditions can remain feasible at zero baseline regulatory activation within the normalized model.

Figure 4(a) divides the (G, ρ_0) plane into low-only, bistable, and high-only regions. The upper boundary has a negative slope, indicating partial substitution between green market preference and baseline regulatory activation. This substitution is not complete in institutional terms because credible market reward often depends on regulation, disclosure, and verification that limit greenwashing [32].

5.5 Temporary regulation and intensity-duration thresholds

Policy pressure is frequently temporary. Pilot standards, transition funds, enforcement campaigns, and accelerated compliance periods can raise pressure for a limited duration. The model sets the firm initially at E_L, raises baseline activation from $\rho_0 = 0.10$ to ρ_p for duration T_p, and then returns it to baseline.

Figure 4(b) reports the minimum dimensionless duration required to enter the high attraction basin. No pulse at or below approximately $\rho_p = 0.30$ succeeds within the 45-unit horizon. At $\rho_p = 0.31$, approximately 44.5 dimensionless time units are required. The threshold falls to about 28.5 at $\rho_p = 0.35$, 20.0 around $\rho_p = 0.40$, and approximately 14.0 at $\rho_p = 0.54$. Stronger intervention accelerates the rise in regulatory intensity and gives abatement and innovation more time to cross the separatrix before policy returns to baseline.

The result shows that nominal stringency alone is an incomplete measure of policy effectiveness. A strong policy withdrawn before corporate capability crosses the attraction boundary produces only a temporary response. A moderately stronger policy sustained for longer can generate a persistent transition. The relevant policy object is a nonlinear intensity-duration combination rather than a simple one-period treatment.

5.6 Regulatory adjustment speed and oscillatory modes

The three-state model permits complex eigenvalues and lagged adjustment. To determine whether the threshold results depend on oscillatory instability, λ_R was varied from 0.02 to 2.0 while the equilibrium states were held at their baseline locations. The dominant real parts remain negative for both stable equilibria across the examined range. Complex conjugate modes emerge over part of the range, producing damped oscillation, but their real parts do not cross zero.

Accordingly, no Hopf bifurcation is identified in the reported parameter interval. This negative result is important. The model is sufficiently rich to generate lagged and oscillatory adjustment, but the core bistability and hysteresis findings arise from saddle-node geometry rather than from a Hopf-type oscillatory instability. A Hopf transition could occur under other response coefficients or explicit policy delays, but it is not claimed here.

This speed sweep does not establish robustness to explicit institutional delay. If the enforcement response in Eq. (8) were driven by lagged abatement, for example by replacing the contemporaneous residual-emissions signal with $A(t - \tau)$, the model would become a delay differential system and the characteristic equation could admit oscillatory instability even when the no-delay system remains locally stable. Legislative lags, phased standards, and delayed enforcement can therefore change the oscillatory boundary; the reported no-Hopf result applies only to the contemporaneous-response

Table 3. Baseline equilibria and local stability.

Equilibrium	(A^*, K^*, R^*)	Characteristic roots (s_1, s_2, s_3)	$(a_1, a_2, a_3, a_1a_2 - a_3)$	Classification
E_L	$(0.2136, 0.1210, 0.4093)$	$(-0.9734, -0.3992, -0.1430)$	$(1.5156, 0.5849, 0.0556, 0.8309)$	Stable
E_M	$(0.4449, 0.3669, 0.3100)$	$(-2.3726, 0.3725, -0.1167)$	$(2.1168, -0.6504, -0.1032, -1.2735)$	Saddle
E_H	$(0.6197, 0.5162, 0.2537)$	$(-2.2993, -0.4172, -0.1656)$	$(2.8821, 1.4090, 0.1588, 3.9021)$	Stable

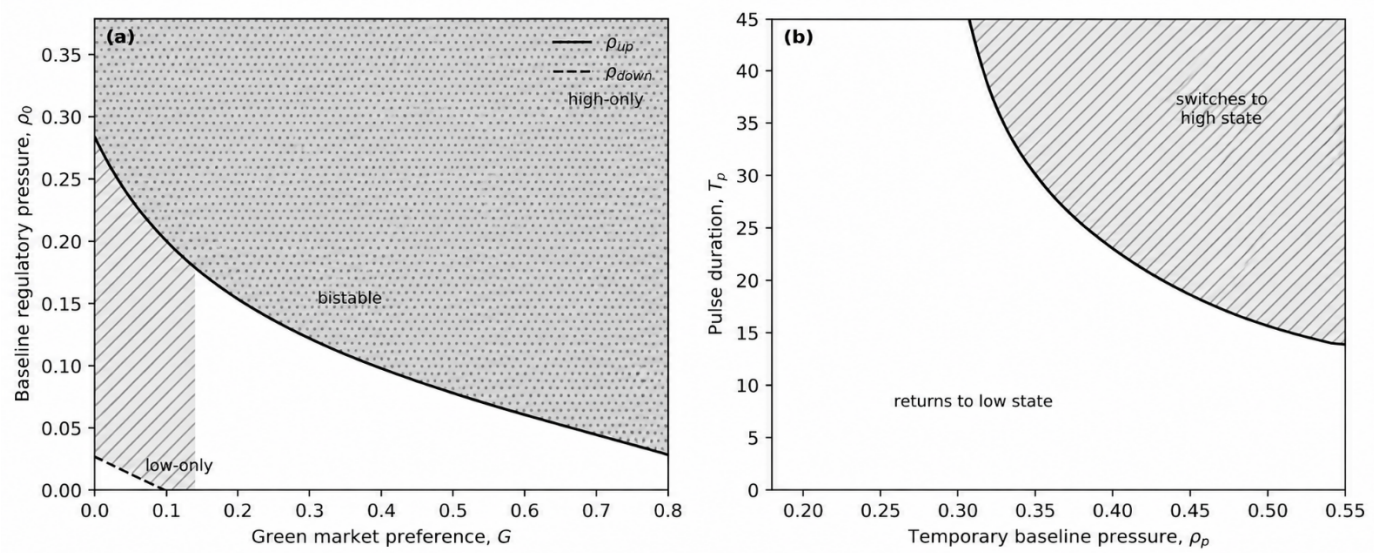


Figure 4. Policy thresholds: (a) regulatory-pressure and green-market-preference regions; (b) minimum duration of a temporary regulatory pulse required to switch from the low to the high state.

specification.

5.7 Feedback-strength regime map

Figure 5 varies the two reinforcing coefficients α_K and β_A while retaining the remaining baseline parameters. Weak feedback produces a unique low state. A broad intermediate region contains three equilibria and therefore supports path dependence. Only when both directions of the innovation-abatement feedback are sufficiently strong does the unique high state dominate at the baseline regulatory activation. The baseline point $(1.38, 1.01)$ lies inside, rather than on the edge of, the bistable region. This map shows that the three-equilibrium result is not generated by a single isolated coefficient pair, while also making clear that bistability is a conditional structural regime rather than a universal property.

The baseline values of α_K and β_A are dimensionless structural response strengths and cannot be compared one-for-one with reduced-form regression coefficients. Existing evidence nevertheless supports the directions represented by the two feedback channels: green innovation is associated with lower emissions, and environmental regulation can stimulate green technology innovation [7, 20]. Related firm- and policy-level studies also document dynamic links among regulation, innovation, and corporate carbon outcomes [23, 26]. Figure 5 should therefore be read as a structural regime map, not as an empirically estimated sectoral phase diagram.

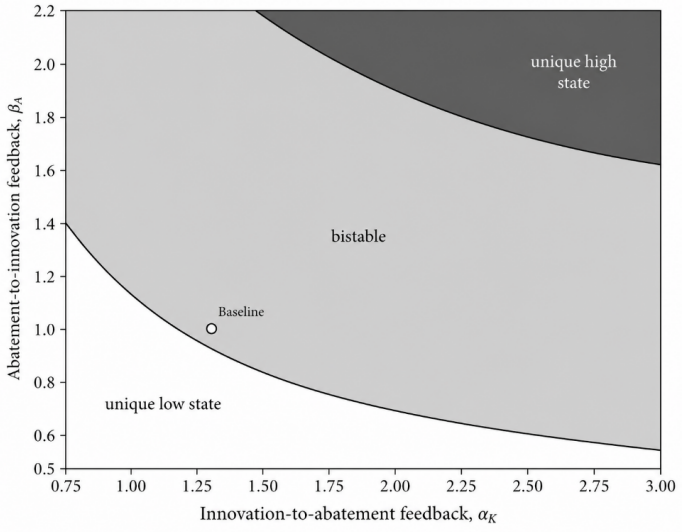


Figure 5. Equilibrium regimes across innovation-to-abatement and abatement-to-innovation feedback strengths at baseline regulatory activation. The open circle marks the baseline parameter pair.

5.8 Global sensitivity and robustness

A 500-point Latin-hypercube experiment independently varied ten structural parameters by $\pm 15\%$. Two positive folds were robustly recovered in 199 draws. A further 26 draws produced two folds with at least one outside the positive policy interval, three retained three baseline equilibria although the fold search was incomplete, and three converged to a single low state. In 269 draws only one fold was recovered from the prescribed fold-search seeds; these cases are reported as incomplete fold recovery rather than interpreted as evidence of a single-equilibrium system. No draw was discarded as an unclassified numerical failure.

Among the 199 configurations with two positive recovered folds, the median lower threshold was 0.0599 with a 5th–95th percentile range of 0.0065–0.1623. The median upper threshold was 0.3025, with a range of 0.1913–0.6095. The share of recovered regimes is not a probability that real firms are bistable; it describes the numerical behavior of the normalized parameter neighborhood and separates structural changes from fold-search incompleteness. Table 4 reports the partial rank correlation coefficients (PRCCs) between the ten structural parameters and the two recovered regulatory thresholds.

The global results support the main directional interpretation. Stronger reciprocal innovation-abatement feedback lowers transition difficulty, whereas higher abatement cost and faster innovation depreciation raise it. The direct regulatory abatement channel is especially influential for the upper threshold. A larger ζ raises the upper threshold because the regulation-induced innovation contribution decays more rapidly at high pressure. These results are conditional on successful recovery of two positive folds and are not extrapolated to parameter regimes without that structure. The asymmetry for α_R is mechanistically consistent with the model: α_R enters the abatement expansion term directly, so near the upper fold a stronger compliance response most strongly erodes the low-abatement balance and facilitates elimination of the low branch. By contrast, creation of the high branch near the lower fold depends more strongly on the reciprocal α_K–β_A capability loop, which makes α_R less influential for ρ_{down}.

The expanded $\pm 30\%$ Latin-hypercube design provides a deliberately broader normalized stress test rather than an estimate of real-firm frequencies. Two positive folds were recovered in 120 of 500 draws, 291 draws yielded one recovered fold, 66 converged to a single low state, 12 retained baseline bistability despite incomplete fold recovery, and 11 produced folds outside the prescribed positive policy interval. The broader range therefore preserves a clear set of bistable configurations while also increasing the frequency of structural regimes in which a complete two-fold window is not recovered.

The 269 one-fold cases in the original $\pm 15\%$ design separate into two distinct boundary patterns rather than one homogeneous failure class. In 92 cases only the lower-fold seed recovered a fold; these cases had, on average, higher c_0 (1.074 times baseline) and δ_K (1.042), lower α_R (0.924), and lower β_A (0.950), consistent with a more difficult transition environment. In the remaining 177 cases only the upper-fold seed recovered a fold; these were instead associated with stronger α_K (1.041) and β_A (1.051), lower δ_K (0.950), and lower c_0 (0.964), consistent with the lower fold being shifted toward or beyond the edge of the searched policy region. The aggregate one-fold category therefore mixes two different structural boundaries and should not be interpreted as a single-equilibrium class.

Panel A of Table 5 reports numerical classifications, not probabilities for real firms. "Other" combines folds outside the prescribed positive policy interval and baseline bistability with incomplete fold recovery. Panel B uses a multi-start fold search plus equilibrium scans over $0 \le \rho_0 \le 0.8$; the stress multipliers are directional structural tests rather than calibrated steel-, cement-, or SME-specific estimates. The results narrow the generalizability claim: bistability is not confined to the immediate $\pm 15\%$ neighborhood, but it is also not universal. Higher abatement costs and weaker innovation capacity move the onset of bistability to substantially stronger policy activation, and sufficiently weak innovation capacity or combined disadvantage can leave only the low state over the scanned interval.

Partial endogeneity of green demand does not reverse the fold-shifting result. Under the robustness specification $G(R) = \min\{1, \gamma_0 + \gamma_1 R\}$, setting $\gamma_1 = 0.25$ yields upper thresholds of 0.1879, 0.1673, and 0.1501 for $\gamma_0 = 0$, 0.05, and 0.10, respectively; the corresponding exogenous-demand cases ($\gamma_1 = 0$) yield 0.2848, 0.2351, and 0.2005. With $\gamma_1 = 0.50$, the upper thresholds fall further to 0.1473, 0.1342, and 0.1225. Regulation-dependent demand

Table 4. Partial rank correlations with the recovered regulatory thresholds.

Parameter	PRCC with ρ_{down}	PRCC with ρ_{up}	Main interpretation
α_K	-0.872	-0.536	Stronger feedback lowers both thresholds
β_A	-0.909	-0.804	Stronger learning lowers both thresholds
c_0	0.816	0.914	Higher abatement cost raises thresholds
δ_K	0.911	0.890	Faster depreciation raises both thresholds
α_R	-0.168	-0.909	Direct regulatory effectiveness mainly lowers the upper threshold
ζ	0.148	0.633	Faster high-pressure decay raises the upper threshold

Table 5. Extended range and targeted structural stress tests.

Panel A. Latin-hypercube range expansion

Design	N	Two positive folds	One fold recovered	Single low / other
Original $\pm 15\%$	500	199	269	3 / 29
Extended $\pm 30\%$	500	120	291	66 / 23

Panel B. Targeted structural stress tests

Scenario	Parameter change	Recovered fold(s), ρ_0	Max equilibria, $0 \le \rho_0 \le 0.8$	Interpretation
Higher abatement cost	c_0, c_1 $+25\%$	0.1423	3	Bistability persists, but starts above baseline policy
Higher abatement cost	c_0, c_1 $+50\%$	0.7685	3	Bistability pushed to very high activation
Lower innovation capacity	β_0, β_A -25%	0.2674	3	Bistability persists only at stronger activation
Lower innovation capacity	β_0, β_A -50%	None	1	Single low state over scanned interval
Combined adverse structure	c_0, c_1 $+25\%$; β_0, β_A -25%	None	1	Single low state over scanned interval

credibility therefore strengthens rather than overturns the downward fold shift, while also showing why market preference and regulation should not be interpreted as institutionally interchangeable.

6 Discussion

6.1 Distinct mechanisms replace symmetric reinforcement

The model assigns different economic structures to the three states. Abatement balances expansion incentives against a marginal cost that rises with abatement and falls with innovation. Innovation accumulates through learning, market reward, and a non-monotonic regulatory channel. Regulation reacts to residual emissions and relaxes with credible corporate capability. The resulting complexity is not decorative. Each additional term corresponds to a distinct mechanism and produces testable comparative implications.

Green innovation is therefore not merely a mediator between regulation and emissions. It is a stock that changes the corporate cost structure. This helps explain why one-period research expenditure may have little effect when it remains below the capability threshold, while sustained accumulation can trigger a persistent transition. It also connects corporate environmental management to path-dependence theory and directed technical change [15, 16, 18].

6.2 Regulation changes both incentives and state geometry

The numerical results show that regulation does more than move a fixed equilibrium. By changing the number and stability of equilibria, baseline regulatory

activation alters the state geometry. Below the lower fold, high abatement is not self-sustaining. In the bistable region, both outcomes are possible and corporate history matters. Above the upper fold, the low state disappears.

This distinction clarifies why average marginal estimates can understate policy effects. Within an attraction basin, stronger regulation may produce only a gradual response. Near a fold, a small additional increase can eliminate a stable state and generate a large transition. Conversely, policy relaxation after transition may have little immediate effect because the firm remains on the high branch.

6.3 Endogenous regulation and apparent policy relaxation

The high-abatement equilibrium has lower regulatory intensity than the low-abatement equilibrium. This is not a contradiction. Regulation is responsive: poor performance raises enforcement, while accumulated capability reduces the need for continued pressure. A cross-sectional study could therefore observe a negative association between regulation and abatement even when regulation is causally important, because successful policy endogenously relaxes after improvement. Empirical work should distinguish initial enforcement, current enforcement, and cumulative policy exposure.

The result also warns against treating high observed regulation as evidence of strong governance outcomes. High regulation can be a response to persistent poor performance. Conversely, moderate current pressure can coexist with strong environmental capability when earlier intervention has already shifted the firm to the high state.

6.4 High regulatory pressure and the declining innovation stimulus

The term $Re^{-\zeta R}$ adds a bounded non-monotonic innovation response. It remains nonnegative, so the model does not claim that high pressure has a net negative innovation effect relative to zero regulation. It shows instead that the additional regulation-induced innovation contribution declines beyond $R = 1/\zeta$, while the direct compliance channel remains positive. Policy packages focused exclusively on short-run compliance can therefore provide progressively less innovation stimulus and may fail to build the capability that makes abatement durable. Regulation should therefore combine credible constraints with mechanisms that preserve research capacity, such as predictable standards, transition finance, technical support, and sufficient implementation horizons [11, 36].

6.5 Market preference reduces but does not eliminate governance needs

Green market preference lowers the upper policy threshold and can sustain a high state after direct pressure falls. This supports the view that market pull complements regulatory push [6, 12]. However, demand-pull depends on credible information. If environmental claims are unverifiable, greenwashing can divert market rewards away from substantive innovation [32]. Regulation and market preference may therefore be institutionally complementary even when they appear partially substitutable in the mathematical threshold plane.

6.6 Relation to critical-transition research

The model draws on critical-transition logic but applies it to corporate capability and adaptive governance. The alternative stable states are not ecological regimes; they are organizational combinations of abatement, innovation, and policy pressure. The control parameter is baseline regulatory activation, while green market preference shifts the fold boundary. The absence of a Hopf crossing in the reported speed range further separates the mechanism from a purely cyclical policy story. The central transition is a fold-induced change in accessible corporate states [37–40].

The attraction geometry also clarifies how stochastic disturbances could matter. Energy-price spikes, supply-chain disruptions, technology failures, or abrupt financing shocks can temporarily displace $A(t)$, $K(t)$, or effective regulatory pressure without changing the underlying deterministic parameters. A firm close to the separatrix may therefore be pushed into the opposite basin by a finite shock, whereas a firm deep inside either basin would require a larger disturbance. This is the organizational analogue of a noise-induced transition and provides a concrete route for extending the deterministic model under uncertainty [40].

7 Managerial and Policy Implications

7.1 Implications for corporate managers

Managers should assess whether environmental initiatives are large enough to move the firm across a capability boundary. Small, disconnected projects can improve current performance without changing the long-run state. A firm with weak green technology

may require concentrated investment in equipment, data, employee capability, and supplier coordination rather than uniform incremental spending.

The attraction-basin result also suggests that the binding constraint differs among firms. A technology-rich firm with weak deployment may benefit most from implementation support and operating targets. A firm already undertaking abatement but lacking proprietary capability may require research partnerships or technology acquisition. The most efficient transition package raises the scarcer complementary state rather than treating abatement and innovation as interchangeable expenditures.

Managers should also recognize that regulatory pressure can decline after successful transition. The persistence of high abatement depends on maintaining the innovation stock, not on assuming that a one-time policy-induced change is permanent. Depreciation, employee turnover, and delayed capital renewal can gradually move the firm back toward the basin boundary.

7.2 Implications for regulators

Uniform policy intensity is unlikely to be equally effective across firms. Low-capability firms may need stronger and longer intervention to cross the transition boundary, while prepared firms can respond to lower pressure. Regulators can therefore combine minimum standards with capability-sensitive transition support.

The intensity-duration result indicates that short campaigns should not be judged only by immediate emission changes. A policy can produce a visible temporary response but fail to change the long-run state if it ends before innovation and abatement cross the separatrix. Conversely, a temporary but sufficiently sustained intervention can create a persistent effect after pressure returns to baseline.

Regulatory design should also protect innovation capacity. Excessively abrupt compliance demands can divert resources from long-term technology accumulation. Predictable trajectories, transition finance, technical assistance, and credible enforcement may be more effective than repeated short-lived escalations. The objective is not permanently maximal pressure but movement into a self-reinforcing high-capability state. As a model-based policy implication, green-bond and green-finance mechanisms may influence corporate innovation strategies by changing financing costs and policy incentives [41].

As a model-based prediction, capital access is likely to shift the same transition geometry. Higher financing costs or binding credit constraints would raise the effective resource burden of abatement and reduce the firm's capacity to accumulate green technology, effects analogous to higher c_0 and c_1 or weaker β_0 and β_A in the structural stress tests. The model therefore predicts that tighter finance should tend to move transition thresholds upward, while transition finance and green-finance instruments can lower the capability barrier by relaxing those effective costs [36, 41].

7.3 Implications for empirical research

The model generates several empirically testable propositions. First, the effect of regulation should depend on lagged green technology stocks. Second, policy initiation and maintenance effects should differ. Third, current regulatory intensity may be endogenous to past environmental performance. Fourth, firms near the transition boundary should display larger responses to small policy changes than firms far inside either basin. Fifth, green consumer demand should reduce the policy pressure required for transition, provided environmental claims are credible.

Testing these implications requires longitudinal data and methods that accommodate state dependence. Threshold panels, regime-switching models, dynamic treatment designs, and firm-level transition classification may be more informative than a single linear interaction term. Measures should distinguish green innovation input from accumulated usable capability. In empirical applications, $K(t)$ should be proxied as a stock rather than by contemporaneous R&D expenditure alone. Feasible measures include depreciated stocks of green invention patents, citation-weighted green patent stocks, cumulative adoption of identifiable low-carbon process technologies, environmental equipment or capital stocks, and perpetual-inventory measures constructed from historical green R&D; patent-based measures are particularly consistent with prior work linking green technology innovation to emissions outcomes [20]. By contrast, current-period green R&D is better interpreted as an innovation input feeding capability accumulation, consistent with evidence that R&D must be converted into operational eco-innovation before it affects firm performance [7]. The same distinction makes the model's first proposition empirically testable with lagged capability-stock measures rather than a single

contemporaneous expenditure variable.

8 Conclusion

This study develops a three-state framework for corporate carbon abatement, green innovation, and responsive environmental regulation. The model extends a simple mutual-reinforcement structure by introducing innovation-enabled cost reduction, rising marginal abatement difficulty, a non-monotonic regulation-induced innovation contribution, and endogenous policy adjustment. The state space is positively invariant, and local stability is evaluated through a three-dimensional Jacobian and Routh–Hurwitz conditions.

The baseline system contains two stable equilibria separated by a saddle. A low-abatement, low-innovation state coexists with a high-abatement, high-innovation state under the same baseline policy, while regulation is higher in the low state because residual emissions trigger enforcement. Fold-point solving and equilibrium-branch tracing identify lower and upper policy thresholds, producing hysteresis. Green market preference reduces the upper threshold, and temporary policy succeeds only when its intensity and duration jointly move the firm across the attraction boundary. Regulatory adjustment can create damped oscillatory modes, but no Hopf crossing is found in the reported range.

The main implication is that corporate low-carbon transition is not necessarily a smooth response to marginally stronger regulation. It can be a state transition governed by accumulated capability, endogenous policy response, market demand, and historical position. Policies should therefore be evaluated by whether they change the long-run state, not only by their short-run coefficient or immediate emissions response.

The model is deliberately stylized and abstracts from sectoral and firm-size heterogeneity, specific carbon-pricing and subsidy instruments, strategic interactions among firms, stochastic shocks, and explicit policy delays. Future research could calibrate the model using sector- or firm-level data, incorporate financial constraints, introduce heterogeneous firms and supply-chain spillovers, or examine rate-induced and noise-induced transitions. Such extensions should ensure that additional complexity remains grounded in identifiable economic and organizational mechanisms rather than complexity for its own sake. The extended $\pm 30\%$ and targeted structural stress tests reinforce this boundary: the bistable mechanism persists beyond the original local calibration in some regions, but severe innovation constraints or combined high-cost/low-capability conditions can eliminate the two-state structure over the examined policy interval. Likewise, the no-Hopf result should not be generalized to explicit legislative or enforcement delays. Stochastic energy, finance, technology, or supply-chain shocks may also induce basin switching when firms operate close to the separatrix.

Data Availability Statement

Data will be made available on request.

Funding

This work was supported without any funding.

Conflicts of Interest

The author declares no conflicts of interest.

AI Use Statement

The author declares that no generative AI was used in the preparation of this manuscript.

Ethical Approval and Consent to Participate

Not applicable.

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