REVIEW ARTICLE

Quantum Computing Essentials: Bridging Theory and Practice for New Learners

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Abstract

This paper investigates the core principles of quantum computation, providing an in-depth understanding of quantum phenomena and illustrating how these principles form the scientific foundation of the field. The pivotal physical concepts, such as properties of subatomic particles, including electrons and photons, as well as their mathematical description through linear algebra are examined. It focuses on the qubit, the quantum analogue of a classical bit, featuring properties like superposition, entanglement, and wave function collapse, which redefine the traditional concept of information processing. The mathematical structures that underlie quantum system modelling—vector spaces, tensor products, and Dirac notation—are also systematically explained. Essential components of quantum computation involve logic gates such as Pauli matrices and Hadamard gates, which demonstrate the novel application of qubits in unique ways. This sets up an important theme of the interdependence and potential for transformation of quantum states through the illustration of advanced ideas of quantum entanglement and teleportation. It makes an outstanding example of how quantum circuits exploit reversibility and superposition to solve complex problems at unprecedented levels of efficiency. The paper also highlights the Deutsch-Jozsa algorithm as a compelling example of quantum superiority over traditional computational approaches. Together, these findings emphasize the transformative potential of quantum computing for science and industry.

Keywords: quantum computing, qubit and superposition, quantum entanglement, quantum algorithms.

1 Introduction

Understanding quantum computation requires familiarity with fundamental concepts from physics, mathematics, and quantum mechanics. Key physical phenomena, such as the properties of electrons and photons, play a crucial role in this foundation. Electrons, subatomic particles with a charge of -1 in elementary charge units [1], have a charge that is equal in magnitude but opposite in sign to that of protons.

In 1905, Albert Einstein reshaped the scientific view of light by proposing the idea of photons while explaining the photoelectric effect. Einstein suggested that light is made up of individual units of energy,

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later called photons or light quanta, rather than a continuous wave. Photons are minute energy packets of electromagnetic radiation, characterized by their momentum p. Equations (1) and (2) describe the momentum and energy of a photon, respectively, highlighting their dependence on frequency ν and Planck's constant h. The momentum is also inversely proportional to the speed c, which is the speed of light:

$$p = \frac{h\nu}{c}, \tag{1}$$

$$E = h\nu. \tag{2}$$

Notably, photons are stable particles with a rest mass of zero, making them fundamental to understanding quantum electromagnetic interactions.

Spin is often defined as an intrinsic angular momentum. Recall that in classical mechanics, force is defined as a change in momentum. Furthermore, the energy of a system is defined in terms of motion or the rate of change of motion, which presupposes mass. The internal spin motion of a subatomic particle is closely related to its internal energy level. Elementary particles have this property even if they are not spinning. It is called intrinsic because it does not depend on external factors such as movement or location. Like classical angular momentum, quantum spin changes under rotations, but spin is quantized, which means that it admits only a discrete set of values.

The maximum spin of an elementary particle is given by the product of n (any integer or half-integer $n/2$ values) and the reduced Planck constant $\hbar$ $\left(\frac{h}{2\pi}\right)$. Fermions are ordinary particles that have a half-integer spin $\left(\frac{1}{2}\right)$, while bosons, like photons, have an integer spin of 1. Both electrons and photons have two possible spin states: spin "up" or "down." In mathematical terms, electrons have spin values of $+\frac{1}{2}\hbar$ or $-\frac{1}{2}\hbar$, corresponding to spin in the positive or negative rotation. Photons have spin values of $+\hbar$ and $-\hbar$, since they take integer spin values.

Motivation The accelerating development of quantum technologies is transforming the landscape of computation and information processing. While quantum computing holds the potential to solve problems that are intractable for classical computers, such as factoring large numbers and simulating complex quantum systems, it remains a challenging field for newcomers due to its reliance on abstract mathematical formalisms and unfamiliar physical principles. This paper is motivated by the need to bridge this gap between theoretical foundations and practical understanding. By systematically presenting essential concepts such as qubit representation, quantum gates, entanglement, and quantum algorithms in an accessible manner, we aim to equip readers with a solid starting point for deeper exploration. This foundational knowledge is critical not only for academic study but also for fostering innovation in emerging areas like quantum cryptography, optimization, and machine learning.

2 Mathematical Foundations

A solid understanding of the mathematical principles underlying quantum computation is essential, particularly in areas that align with the phenomena being modeled. Linear algebra forms the cornerstone of these mathematical tools, and vector algebra is a foundational concept.

2.1 Vectors

A vector is a mathematical entity characterized by two primary components: magnitude and direction. The magnitude of a vector, denoted $|\mathbf{v}|$, can be calculated using the expression (3):

$$|\mathbf{v}| = \sqrt{x^2 + y^2}, \tag{3}$$

where $\mathbf{v}$ represents the vector and x and y are its components.

The expression in Equation (3) highlights how the magnitude is derived from the components of the vector, making vectors an essential tool for modeling physical phenomena in quantum mechanics and other domains.

2.2 Unit Vector

A unit vector is defined as a vector whose magnitude (or length) is exactly equal to 1. For instance, the vector $\langle 0,1\rangle$ qualifies as a unit vector because its magnitude, calculated using equation 3 equals 1

2.3 Orthonormal vector

Two vectors are orthogonal if and only if their inner (dot) product is equal to zero. When combinations of unit vectors are orthogonal to each other, these are called orthonormal bases.

2.4 Two-dimensional unit vector

Electron spin measurements result in two distinct outcomes, which can be effectively represented within

a two-dimensional vector space, $\mathbb{R}^2$. To capture these outcomes, unit vectors—vectors normalized to have a magnitude of 1—are used, limiting the possible results to two discrete values: 0 and 1. The operations performed on qubits in this framework can be visualized as rotations confined to a two-dimensional plane.

Although the full range of qubit states can be described by rotations on a three-dimensional sphere, the binary nature of measurement outcomes allows this system to be reduced to a two-dimensional representation. A more general framework would involve the use of complex numbers in $\mathbb{C}^2$, which incorporate both real and imaginary components, where $i = \sqrt{-1}$. However, to maintain simplicity, this representation focuses on real numbers in $\mathbb{R}^2$ and avoids the complexities associated with imaginary components.

2.5 Tensor product of two vectors

The tensor product is an operation used to combine two or more vectors or vector spaces into a larger composite space. Two vectors, say v and w, and their tensor product create a new vector that represents all possible combinations of the elements of v and w. To illustrate the concept, the following example is presented for consideration:

$$v \rightarrow \bigotimes w \rightarrow = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 4 & 5 \end{bmatrix} = \begin{bmatrix} 1.4 & 1.5 \\ 2.4 & 2.5 \\ 3.4 & 3.5 \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ 8 & 10 \\ 12 & 15 \end{bmatrix} \tag{4}$$

From equation 4, it can be seen that the two vectors v and w are multiplied such that each element of v is multiplied by each element of w to form the result.

In classical computing, the combined state of multiple bits is represented using the tensor product of individual bits, symbolized by $\otimes$.

3 Basic Notations for Quantum Computing

In classical computation, the bit is the fundamental unit of information, taking a value of either 0 or 1. Quantum computation, however, relies on a similar concept known as the quantum bit, or qubit. Unlike a classical bit, which can only exist in one of two distinct states [2], a qubit can exist in a superposition of the basis states $|0\rangle$ and $|1\rangle$, which correspond to the classical 0 and 1, respectively. The notation '$|\,\rangle$' is known as Dirac notation, which is commonly used in quantum mechanics to represent states. We will encounter it frequently, as it is the standard way to denote quantum states. The key difference between classical bits and qubits is that qubits can also acquire a state between $|0\rangle$ and $|1\rangle$, made possible by a linear combination of states often called superposition, as represented in Equation 5 below.

$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle \tag{5}$$

where α and β are complex numbers. When a qubit is measured, the result will be either 0 or 1, with the likelihoods of these results being $|\alpha|^2$ and $|\beta|^2$, respectively. These probabilities must satisfy the normalization rule, which ensures that their sum equals one as given in 6

$$|\alpha|^2 + |\beta|^2 = 1 \tag{6}$$

The sum of the probabilities for all potential outcomes must be equal to one. Equation 6 ensures that this fundamental principle of probability is satisfied. Geometrically, this condition means that the qubit state vector is normalized to have a magnitude of 1. As a result, a qubit can be represented as a unit vector in a complex two-dimensional vector space [4].

One of the fascinating properties of qubits is their ability to exist in a superposition of multiple states simultaneously. However, once a measurement is made, the qubit state collapses to one of the basis states, either $|0\rangle$ or $|1\rangle$ [5]. Imagine spinning a coin. When the coin is spinning, one cannot see which state it is in, but once captured with a camera, only one side can be seen, that is, heads or tails [3]. This analogy illustrates how qubits work. This property is known as the wave function collapse.

Before introducing the concept of basis, it is essential to understand the standard notation used in linear algebra to appropriately interpret the relevant symbols.

In this context, row vectors are called bras and column vectors are called kets. These vectors are defined as follows in Equation 7:

$$\langle a| = \begin{bmatrix} 0 & 1 \end{bmatrix}, \quad |b\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{7}$$

The bras and kets can be multiplied when they have equal dimensions. The inner product of the bra $\langle a|$ and ket $|b\rangle$ is computed in Equation 8:

$$\begin{bmatrix} 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = (0 \cdot 1) + (1 \cdot 0) = 0 \tag{8}$$

There are three orthonormal bases for spin, as presented in Equations 9, 10, and 11. The first basis is given by:

$$\{|\uparrow\rangle, |\downarrow\rangle\} = \begin{bmatrix}1 & 0\end{bmatrix}, \begin{bmatrix}0 & 1\end{bmatrix} \tag{9}$$

The second basis is:

$$\{|\rightarrow\rangle, |\leftarrow\rangle\} = \left[\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right], \left[\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right] \tag{10}$$

Finally, the third basis is:

$$\{|\nwarrow\rangle, |\searrow\rangle\} = \left[\frac{1}{2}, -\frac{\sqrt{3}}{2}\right], \left[\frac{\sqrt{3}}{2}, \frac{1}{2}\right] \tag{11}$$

The first basis in Equation 9, indicated by the up and down arrows, is called the *standard basis* and corresponds to measuring spin along the vertical direction, or the z-axis. The second basis in Equation 10, represented by the right and left arrows, relates to measuring spin along the x-axis. The third basis in Equation 11 corresponds to spin measured along another axis.

When bras and kets of the same spin state are multiplied, their inner product equals 1, reflecting normalization, as shown in Figure 1. This demonstrates that quantum state vectors are unit vectors in Hilbert space, satisfying $\langle\psi|\psi\rangle = 1$, which ensures the probability interpretation of quantum mechanics remains consistent.

$$\langle\uparrow|\uparrow\rangle = \begin{bmatrix}1 & 0\end{bmatrix} \cdot \begin{bmatrix}1\\0\end{bmatrix} = 1$$

$$\langle\downarrow|\downarrow\rangle = \begin{bmatrix}0 & 1\end{bmatrix} \cdot \begin{bmatrix}0\\1\end{bmatrix} = 1$$

$$\langle\rightarrow|\rightarrow\rangle = \begin{bmatrix}\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{bmatrix} \cdot \begin{bmatrix}\frac{1}{\sqrt{2}}\\-\frac{1}{\sqrt{2}}\end{bmatrix} = 1$$

$$\langle\leftarrow|\leftarrow\rangle = \begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{bmatrix} \cdot \begin{bmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{bmatrix} = 1$$

$$\langle\searrow|\searrow\rangle = \begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\end{bmatrix} \cdot \begin{bmatrix}\frac{\sqrt{3}}{2}\\\frac{1}{2}\end{bmatrix} = 1$$

$$\langle\nwarrow|\nwarrow\rangle = \begin{bmatrix}\frac{1}{2} & -\frac{\sqrt{3}}{2}\end{bmatrix} \cdot \begin{bmatrix}\frac{1}{2}\\-\frac{\sqrt{3}}{2}\end{bmatrix} = 1$$

Figure 1. Inner products of identical spin states yield 1.

Conversely, when bras and kets corresponding to orthogonal (opposite) spin states are multiplied, their inner product equals zero, indicating orthogonality, as shown in Figure 2. This reflects the fundamental property that mutually exclusive quantum states are represented by orthogonal vectors in Hilbert space.

$$\langle\uparrow|\downarrow\rangle = \begin{bmatrix}1 & 0\end{bmatrix} \cdot \begin{bmatrix}0\\1\end{bmatrix} = 0$$

$$\langle\downarrow|\uparrow\rangle = \begin{bmatrix}0 & 1\end{bmatrix} \cdot \begin{bmatrix}1\\0\end{bmatrix} = 0$$

$$\langle\rightarrow|\leftarrow\rangle = \begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{bmatrix} \cdot \begin{bmatrix}\frac{1}{\sqrt{2}}\\-\frac{1}{\sqrt{2}}\end{bmatrix} = 0$$

$$\langle\leftarrow|\rightarrow\rangle = \begin{bmatrix}\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{bmatrix} \cdot \begin{bmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{bmatrix} = 0$$

$$\langle\searrow|\nwarrow\rangle = \begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\end{bmatrix} \cdot \begin{bmatrix}\frac{1}{2}\\-\frac{\sqrt{3}}{2}\end{bmatrix} = 0$$

$$\langle\nwarrow|\searrow\rangle = \begin{bmatrix}\frac{1}{2} & -\frac{\sqrt{3}}{2}\end{bmatrix} \cdot \begin{bmatrix}\frac{\sqrt{3}}{2}\\\frac{1}{2}\end{bmatrix} = 0$$

Figure 2. Inner products of orthogonal spin states yield zero.

4 Quantum Entanglement

Quantum entanglement is a phenomenon in which two particles become so deeply correlated that the state of one instantaneously influences the state of the other, regardless of the distance separating them—even if they are billions of light-years apart [6]. This striking property is often described as "spooky action at a distance," as illustrated in Figure 3. To illustrate this concept, imagine two individuals in different countries, each tossing a coin. Despite the separation, they observe that their outcomes are perfectly correlated. For example, they may always both get heads or both get tails, or their results may be oppositely correlated, with one obtaining heads whenever the other gets tails. This persistent correlation indicates that the two coins are entangled, exhibiting a form of quantum interdependence.

5 Quantum Logic Gates

While a classical computer relies on electrical circuits composed of wires and logic gates to process information, a quantum computer uses quantum gates to transmit and manipulate data. Two properties matter the most when it comes to quantum logic gates: (i) *Reversibility* and (ii) *Universality*. Reversibility refers to the principle that every operation, from the input stage to the output, should be capable of being reversed, allowing the system to return to the original input from the output. In other words, there must be a mechanism or function that can reverse the operation, ensuring that no information is discarded during the

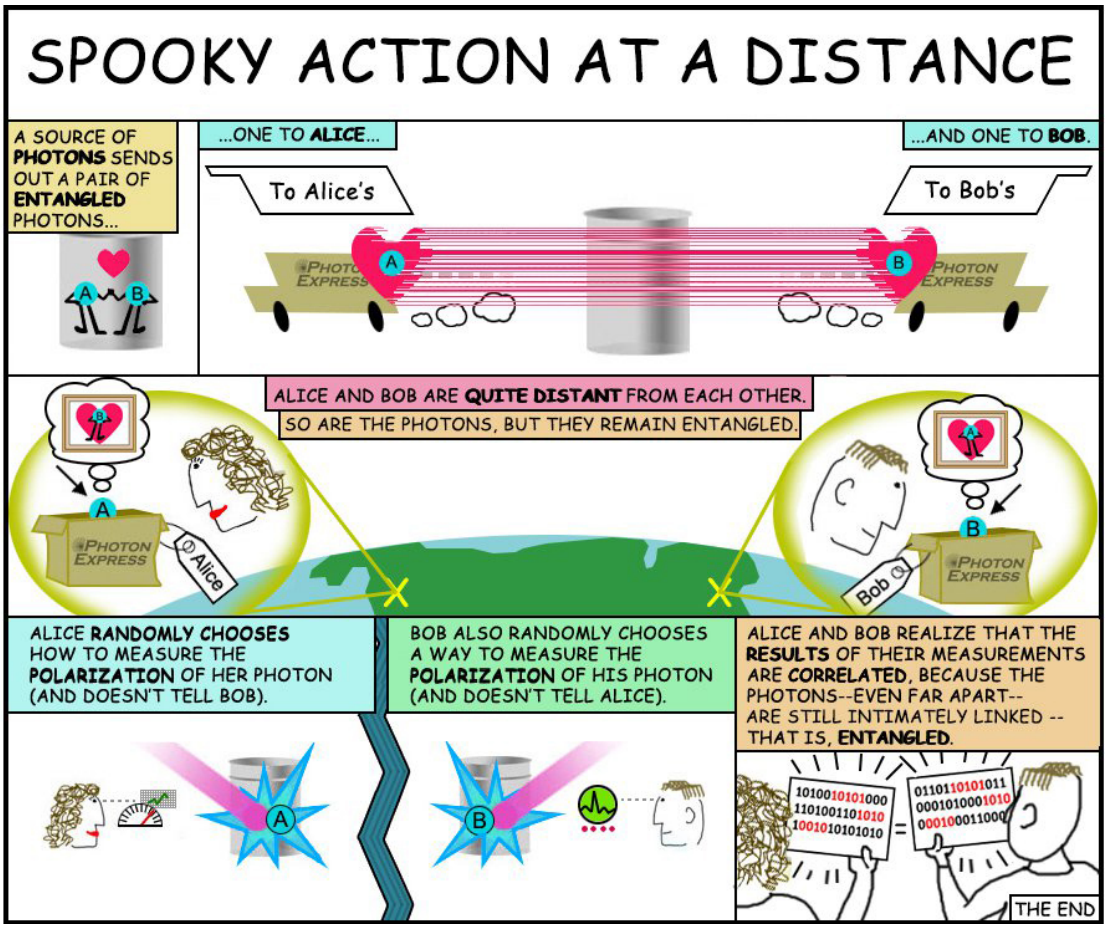

Figure 3. Illustration of quantum entanglement: A source emits a pair of entangled photons sent to two distant observers, Alice and Bob. Each independently measures their photon's polarization. Despite the separation and randomness in choosing measurement settings, the results are correlated—highlighting the non-locality of entanglement. Adapted from NASA/JPL-Caltech [7].

process. Universality refers to the ability of a logic gate to perform any possible operation on bits. A universal gate, such as the NAND gate, is capable of being combined in various ways to implement any computational function, making it fundamental to digital computing.

In classical computing, a NOT gate flips a bit from 0 to 1 or from 1 to 0. Similarly, in quantum computing, the quantum NOT gate operates linearly by swapping the amplitudes in a superposition state. It transforms a state like $\alpha|0\rangle + \beta|1\rangle$ into the state $\beta|0\rangle + \alpha|1\rangle$ by interchanging the coefficients α and β.

XOR and NAND gates are both irreversible, meaning they can only produce output from input. However, there are gates like the CNOT, Toffoli, and Hadamard gates that allow us to retrieve the input from the output alone. Among these, the Hadamard gate is particularly important. As shown in equation (12), the Hadamard gate takes an input vector, such as $(0, 1)$, and produces an output vector by multiplying the input by an orthogonal matrix, thereby putting the qubit into a superposition state.

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \cdot 0 + \frac{1}{\sqrt{2}} \cdot 1 \\ \frac{1}{\sqrt{2}} \cdot 0 + \left(-\frac{1}{\sqrt{2}}\right) \cdot 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} \tag{12}$$

Equation (12) illustrates how the Hadamard transformation operates on a single qubit basis vector. By applying this orthogonal matrix, the input state is transformed into an equal superposition of the computational basis states.

6 Bloch Sphere Notation

As per the normalization condition given in equation (6), the quantum state $\alpha|1\rangle + \beta|0\rangle$ can be expressed more generally as shown in equation (13):

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle \tag{13}$$

where θ, φ, and γ are real numbers. The parameters θ and φ define a point on the surface of a unit sphere in three-dimensional space, known as the Bloch sphere, as illustrated in Figure 4.

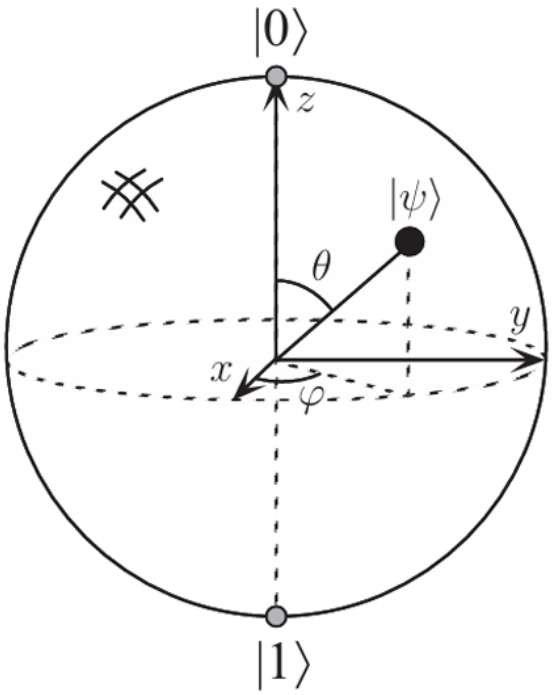

Figure 4. Bloch sphere representation of a qubit state $|\psi\rangle$, defined by angles θ and φ [4].

These matrices are fundamental in quantum computing and are widely used in the manipulation and measurement of qubits.

$$\sigma_0 \equiv I \equiv I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \sigma_1 \equiv \sigma_x \equiv X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_2 \equiv \sigma_y \equiv Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_3 \equiv \sigma_z \equiv Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Pauli-X Gate (Bit-flip Gate)

The Pauli-X gate is analogous to the classical NOT gate, as shown in Figure 5. It flips the state of a qubit.

- If the qubit is in state $|0\rangle$, it changes to $|1\rangle$
- If the qubit is in state $|1\rangle$, it changes to $|0\rangle$
- It corresponds to 180° rotation around x-axis

$$\text{Pauli-}X \quad -\boxed{X}- \quad \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Figure 5. Pauli X Gate.

Pauli-Y Gate (Bit and Phase-flip Gate)

The Pauli-Y gate flips the state of the qubit and also adds a phase factor of i, as shown in Figure 6.

- It changes $|0\rangle$ to i$|1\rangle$
- It changes $|1\rangle$ to -i$|0\rangle$
- It corresponds to a 180-degree rotation around the y-axis.

$$\text{Pauli-}Y \quad -\boxed{Y}- \quad \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

Figure 6. Pauli Y Gate.

Pauli-Z Gate (Phase-flip Gate)

The Pauli-Z gate flips the phase of the qubit, as shown in Figure 7.

- It leaves $|0\rangle$ unchanged.
- It changes $|1\rangle$ to $-|1\rangle$.
- It corresponds to a 180-degree rotation around the z-axis.

$$\text{Pauli-}Z \quad -\boxed{Z}- \quad \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Figure 7. Pauli Z Gate.

- **Controlled NOT Gate (CNOT GATE)**

  The CNOT gate works by entangling two input qubits by performing a bit-flip operation. It contains two types of inputs: control input and target input. When the control bit equals 1, the CNOT gate flips the target input [8]. When the control bit equals 0, the CNOT gate does nothing. In this manner, every output combination is traced back to one and only one input combination, as illustrated in Figure 8.

- **Toffoli Gate** The Toffoli gate is a three-input, three-output logic gate. Among its inputs, two are designated as control bits, which remain unchanged during its operation, while the third is the target bit. The target bit is flipped only when both control bits are set to 1 [9]; otherwise, it remains unaffected. Importantly, the Toffoli gate is reversible, as applying it twice to the same inputs restores the system to its original state. Figure 9 represents the quantum CNOT gate output on standard bases.

CNOT

Input		Output	
x	y	x	x⊗y
|0>	|0>	|0>	|0>
|0>	|1>	|0>	|1>
|1>	|0>	|1>	|1>
|1>	|1>	|1>	|0>

Figure 8. Quantum CNOT gate on standard bases and tensor product output.

Inputs			Outputs		
a	b	c	a'	b'	c'
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	0	1
1	1	0	1	1	1
1	1	1	1	1	0

Figure 9. Quantum Toffoli gate on standard bases and tensor product output.

7 Quantum Circuits

In computer science, circuits are computational models where information travels through wires connected by gates. These gates perform operations that transform the information. Quantum circuits are an example of a computational model based on this general concept [10]. Each line in a quantum circuit diagram represents a wire that carries a qubit through the sequence of quantum gates.

Unlike classical circuits, the "wires" in quantum circuits do not simply represent the passage of time but can also correspond to transformations in a multi-dimensional quantum state space. This paper explores the concept of a swapping circuit, which exchanges the quantum states of two qubits.

Specifically, in this paper, a swapping circuit that swaps the states of two qubits that have computational basis states denoted as $|a, b\rangle$ is explained.

The circuit shown in Figure 10 performs a fundamental yet valuable operation: swapping the states of two qubits. To verify that this circuit achieves the swap operation, consider its effect on a computational basis state $|a, b\rangle$, where all additions are performed modulo 2. The sequence of gates systematically alters the qubits' states, ultimately interchanging their values. Thus, the circuit effectively swaps the states of the two qubits.

$$\begin{aligned} |a, b\rangle &\longrightarrow |a, a \oplus b\rangle \\ &\longrightarrow |a \oplus (a \oplus b), a \oplus b\rangle = |b, a \oplus b\rangle \\ &\longrightarrow |b, (a \oplus b) \oplus b\rangle = |b, a\rangle \,, \end{aligned}$$

Figure 10. swapping circuit.

8 Quantum Teleportation

A particle's quantum state can be transferred from one place to another without moving the particle itself. While science fiction portrays teleportation as the instant transport of objects, this is not the case in reality. Quantum teleportation relies on entanglement, where entangled particles have interdependent states. Regardless of the distance between them, a change in the state of one particle instantaneously affects the state of the other.

Imagine two individuals, commonly named Alice and Bob, who wish to exchange a quantum state. To do this, they begin by generating a pair of entangled particles. Entanglement means that the state of one particle is intrinsically connected to the state of the other, no matter how far apart they are. Alice keeps one of the entangled particles, and Bob takes the other. When Alice wants to transmit a quantum state to Bob, she performs a special type of measurement involving both her particle and the quantum state she intends to send. This operation alters her particle's state and entangles it with the target state. She then communicates the outcome of this measurement to Bob using a classical channel, such as a phone call or an email. Based on this information, Bob applies a corresponding operation to his particle, effectively recreating the original quantum state on his end. This process is known as *quantum teleportation*.

In 1997, a group of Austrian physicists led by Anton Zeilinger successfully demonstrated quantum state teleportation for the first time. By entangling two photons and performing a measurement on one, they were able to transfer its quantum state to the other. Later, in 2015, researchers at the National Institute of

Standards and Technology (NIST) extended the range of quantum teleportation to over 100 kilometres using optical fibre, achieving a transmission distance four times greater than previously possible.

Quantum teleportation plays a vital role in the development of a quantum internet. Such a network could revolutionize communication by enabling the highly secure transfer of quantum information across global distances.

9 Deutsch-Jozsa Algorithm

The Deutsch-Jozsa algorithm provides an efficient method for determining whether a given Boolean function is constant (produces the same output for all inputs) or balanced (produces 0 for half the inputs and 1 for the other half). Consider a Boolean function $f : \{0,1\}^n \to \{0,1\}$ that takes binary inputs and yields binary outputs.

A function is classified as *constant* if all its outputs are identical, either all 0s or all 1s. Conversely, it is deemed *balanced* if the outputs are evenly distributed, with half of them being 0s and the other half 1s.

We consider four Boolean functions, denoted as f_0 to f_3. The function f_0 consistently outputs 0, regardless of whether the input is 0 or 1. Similarly, the function f_3 always outputs 1, irrespective of the input value. These two functions are classified as *constant*, as their outputs remain unchanged irrespective of the input. On the other hand, the function f_1 outputs 0 when the input is 0 and 1 when the input is 1, whereas f_2 produces the opposite behavior, returning 1 for an input of 0 and 0 for an input of 1.

In classical computation, determining whether a randomly chosen function among these is constant or balanced requires a minimum of two queries. To illustrate, we can input either 0 or 1 and observe the output. If we input 0, the output could be either 0 or 1. Similarly, if we input 1, the output could also be either 0 or 1. However, in both cases, the observed results do not allow us to conclusively identify whether the function is balanced or constant.

In contrast, a quantum algorithm can achieve this determination using only a single query. Deutsch demonstrated that by leveraging a Hadamard gate and utilizing both a control qubit and an input qubit, the distinction between balanced and constant functions can be made efficiently. The input information is first passed through the Hadamard gate, which places the qubits in a superposition state. For an input pair of $|0\rangle$ and $|1\rangle$, the resulting states after the Hadamard gate are $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$, respectively. The target qubit is then processed through the randomly chosen function f_x, enabling the algorithm to distinguish between balanced and constant functions with a single query.

Since the Hadamard gate is reversible, the operation f_x transforms the qubit into a superposition corresponding to the function's behavior. As shown in Equation (14), this results in one of four possible quantum states: a symmetric superposition, an antisymmetric superposition, or their negative counterparts.

$$\begin{aligned} &\frac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big), \quad \frac{1}{\sqrt{2}}\big(|0\rangle - |1\rangle\big), \\ &-\frac{1}{\sqrt{2}}\big(|0\rangle - |1\rangle\big), \quad -\frac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big). \end{aligned} \tag{14}$$

After applying f_x, the control qubit is again passed through the Hadamard gate to reverse the superposition. This process disentangles the information encoded in the phase, yielding one of the basis states shown in Equation (15).

$$\begin{aligned} f_0 &\to |0\rangle, \\ f_1 &\to |1\rangle, \\ f_2 &\to -|1\rangle, \\ f_3 &\to -|0\rangle. \end{aligned} \tag{15}$$

Then the qubit is measured. If the output is $|0\rangle$, the function must be constant, and if it is $|1\rangle$, the function must be balanced [11].

Even though the Deutsch oracle has no practical applications, it provides a powerful example of the advantages of quantum computing over classical computing.

The generalization of this method to n-bit inputs is known as the Deutsch-Jozsa algorithm. In classical computation, determining whether a function is balanced or constant in the worst-case scenario requires $2^{n-1} + 1$ queries. However, the Deutsch-Jozsa algorithm exploits quantum superposition and interference to identify the nature of the function with a single query, demonstrating a substantial computational advantage over classical approaches [1].

10 Conclusion

Quantum computing represents a transformative frontier in technological advancement, standing as one of the most significant breakthroughs beyond the capabilities of classical computing. By harnessing the principles of superposition, entanglement, and quantum logic gates, it offers unparalleled efficiency in addressing complex computational problems, as demonstrated by the Deutsch-Jozsa algorithm. The potential impact of quantum mechanics in computation is far-reaching, with anticipated applications in cryptography, optimization, and artificial intelligence. Despite ongoing challenges in error correction and practical implementation, the trajectory of quantum computing signals a paradigm shift that promises to redefine the future of science and industry, opening new avenues for computational innovation [9].

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Conflicts of Interest

The authors declare no conflicts of interest.

Ethical Approval and Consent to Participate

Not applicable.

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