RESEARCH ARTICLE

A Hybrid Machine Learning Fuzzy Non-linear Regression Approach for Neutrosophic Fuzzy Set

Mufala Khan[1], **Rakesh Kumar**[1,*] and **Gaurav Dhiman**[2,3]

[1] Department of Mathematics, Lovely Professional University, Phagwara 144411, Punjab, India
[2] MEU Research Unit, Middle East University, Amman, Jordan
[3] School of Sciences and Emerging Technologies, Jagat Guru Nanak Dev Punjab State Open University, Punjab, India

Abstract

Neutrosophic sets play a significant role for handling indeterminacy. In this paper, we introduce a novel fuzzy non-linear regression model to find the minimum spread of neutrosophic fuzzy sets. Kuhn-Tucker's necessary conditions are employed to estimate the parameters for non-linear regression models, which can be applied to any data set. The resulting hybrid model possesses the ability to minimise the spread of uncertainty in a much better fashion than the existing non-linear regression contenders which rely on KKT- based model. The hybrid approach reduces the maximum spread by 22.09% and improves prediction accuracy, as shown by a 22.23% reduction in RMSE. The study's findings highlight the hybrid model's ability to achieve tighter spreads and enhanced predictive reliability, particularly in complex systems where uncertainties in data are significant. This research contributes to advancing fuzzy regression techniques, offering a powerful tool for improved uncertainty quantification in nonlinear systems.

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***Corresponding author:**
Rakesh Kumar
rakeshmalhan23@gmail.com

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1 Introduction

Complex systems inherently contain uncertainty, and conventional fuzzy sets have played a crucial role in the modeling and analysis of such systems. However, in the face of inherent uncertainty and unpredictability in real-world issues, they frequently fail to capture their subtleties. Neutrosophic fuzzy sets, which can deal with the ambiguity and uncertainty present in many domains, have arisen as a potent tool to overcome this constraint. In addition to the conventional membership and non-membership components, Abo-Sinna et al. [1] expanded the idea of fuzzy sets to include a third component called the indeterminacy component. This new concept is called neutrosophic fuzzy sets. This methodology permits a more thorough portrayal of uncertainty, which in turn permits more precise modeling of complicated systems. Recent years have seen a surge in interest in neutrosophic fuzzy sets and their potential applications, especially in regression analysis. The non-linear connections present in many real-world systems are tough for traditional linear regression models to represent, despite their widespread usage for analyzing variable correlations. Contrarily, non-linear fuzzy regression models perform better when trying to represent complicated relationships; yet, they frequently have

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limitations when it comes to dealing with uncertainty. To overcome these limitations and create better ways to model and assess complex systems, we can combine neutrosophic fuzzy sets with nonlinear fuzzy regression. The research on estimating the minimal spread of neutrosophic fuzzy sets using non-linear fuzzy regression is severely lacking, despite the increasing interest in these sets. To fill this need, this research suggests a new method for neutrosophic fuzzy sets based on non-linear fuzzy regression. We show that the minimal spread of neutrosophic fuzzy sets can be estimated using Kuhn-Tucker's necessary conditions for non-linear regression model parameters, which gives a better picture of system uncertainty. Because of the inherent complexity and unpredictability in these domains, the suggested method has far-reaching consequences for engineering, economics, and the social sciences. Our understanding and interaction with uncertain systems might be completely transformed by this method, which offers a more robust and flexible foundation for modeling and studying complex systems. Here is how the rest of the paper is structured: section 2 consist of literature review section 3 consist of research gap, section 4 consists of preliminaries, section 5 consists of methodology, Section 6 gives a numerical example to back up the findings; and Section 7 consists of conclusion.

2 Literature review

Recent years have seen an explosion in interest in neutrosophic fuzzy sets, first proposed by Abo-Sinna et al. [1], because of their capacity to deal with complex systems' indeterminacy and uncertainty. Due to their inability to account for the non-linear correlations present in many real-world systems, conventional linear regression models might be surpassed by combining neutrosophic fuzzy sets with non-linear fuzzy regression. Image processing [2], decision-making [3], medical diagnosis [4], and engineering [5] are just a few of the disciplines where neutrosophic fuzzy sets have been the subject of research. Neutrosophic fuzzy sets have been shown to be helpful in these researches for dealing with complex systems' uncertainty and indeterminacy. Nevertheless, there is still a lot of uncharted territory when it comes to estimating the lowest spread of neutrosophic fuzzy sets using non-linear fuzzy regression. When trying to represent complicated interactions between variables, several disciplines have turned to non-linear fuzzy regression models. This includes engineering, economics, and the social sciences. More accurate predictions and insights may be obtained using non-linear fuzzy regression models compared to conventional linear regression models, according to studies [6, 7]. Unfortunately, indeterminacy and uncertainty are frequently beyond the capabilities of these models. Fuzzy sets and non-linear regression models have been the subject of several investigations as a potential solution to the twin problems of uncertainty and non-linearity. One such example is the fuzzy non-linear regression model put out by Qiu et al. [8], which makes use of genetic algorithms for parameter estimation. Fuzzy non-linear regression, which models complicated interactions between variables using neural networks, was also created by Aydin et al. [9]. A more thorough representation of uncertainty can be provided by neutrosophic fuzzy sets, which were not considered in these researches. There has been a lot of research on estimating non-linear regression model parameters using Kuhn-Tucker's required conditions [10, 11]. The use of non-linear fuzzy regression in the context of neutrosophic fuzzy sets, however, is still mostly uncharted territory. Additional research has investigated neutrosophic fuzzy sets' potential uses in a range of fields, like as Image processing: One neutrosophic approach to image processing that Vennila et al. [12] suggested is based on a similarity measure. When it comes time to make a call, Edalatpanah et al. [13] suggested a neutrosophic approach that uses a similarity metric. A similarity-based neutrosophic medical diagnostic approach was proposed by Rajesh et al. [14] for use in medical diagnosis. In their paper on complex systems engineering, Smarandache et al. [15] suggested a neutrosophic fuzzy control approach. A neutrosophic fuzzy model was suggested for economic forecasting by Alqaysi et al. [16]. In order to analyze social networks, El-Hefenawy et al. [17] suggested a neutrosophic fuzzy model. Furthermore, neutrosophic fuzzy sets have been the subject of several theoretical investigations, such as: As an extension of intuitionistic fuzzy sets, Vázquez [18] presented the idea of neutrosophic fuzzy sets. The neutrosophic fuzzy set theory put forward by Salama et al. [19] is based on the similarity measure. The neutrosophic fuzzy set theory put forward by Borah et al. [20] is based on the entropy measure. Neutrosophic fuzzy sets have recently been investigated for potential use in a number of fields, such as: Artificial intelligence: Alqaysi et al.[16] proposed a neutrosophic fuzzy deep learning approach for image classification. A similar method was also suggested in[21]. In the field of natural language processing, Khan et al. [22] introduced a neutrosophic fuzzy framework for text categorization. Likewise, a neutrosophic fuzzy control strategy has been proposed to enhance the performance and robustness of robotic systems. Although neutrosophic fuzzy sets and non-linear fuzzy regression have been the subject of several research, the use of the latter to estimate the minimal spread of the former is still an area that has received little attention. To fill this need, this research suggests a new method for neutrosophic fuzzy sets based on non-linear fuzzy regression.

Recent advancements have focused on integrating machine learning with neutrosophic fuzzy regression. For example, Abo-Sinna et al. [1] and Khan et al. [22] demonstrated that hybrid models combining fuzzy logic with non-linear techniques perform better in uncertain data environments. These studies support and validate the design of the proposed hybrid method.

3 Research gap

Using non-linear fuzzy regression models to estimate the lowest spread of neutrosophic fuzzy sets is still a mostly unexplored field [4]. A new breed of neutrosophic fuzzy regression models capable of handling uncertainty and intricate interdependencies among variables is urgently required. Image processing, decision-making, medical diagnostics, and engineering are just a few examples of real-world challenges that might benefit from more investigation into neutrosophic fuzzy set applications [6]. Neutrosophic fuzzy sets, along with other theories of fuzzy sets like intuitionistic and type-2 fuzzy sets, require more investigation [3]. New techniques for neutrosophic fuzzy sets, including clustering, classification, and regression analysis procedures, are required [5]. Additional research is required to investigate the theoretical underpinnings of neutrosophic fuzzy sets, which should cover their characteristics and connections to other theories of fuzzy sets, among other things. Further investigation into the use of neutrosophic fuzzy sets in deep learning and NLP is required [1]. New metrics for assessing the efficacy of neutrosophic fuzzy sets, including measures for recall, precision, and accuracy, are required [8, 9]. Additional study is required to fully understand how neutrosophic fuzzy sets might be utilized in multi-criteria decision-making. This research should focus on creating new algorithms and techniques for assessing and ranking different options [10].

4 Preliminaries

Assume that the universe set represented as X. An ordered set of pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)), x \in X\}$ where $\mu_{\tilde{A}}(x) : X \to [0,1]$ where $\mu_{\tilde{A}}(x)$ is a fuzzy set (FS) $\tilde{A}$ in X from $[0,1]$ expresses the grade of the membership function of the element $x \in X$ to the set $\tilde{A}$.

Let $\tilde{A}^i$ be the universe set. An IFS of $\tilde{A}^i$ in X is a set with the following structure: $\tilde{A}^i = \{(x, \mu_{\tilde{A}^i}(x), \nu_{\tilde{A}^i}(x)), x \in X\}$ where $\mu_{\tilde{A}^i}(x) : X \to [0,1]$ and $\nu_{\tilde{A}^i}(x) : X \to [0,1]$ express the degree functions of membership and non-membership of the set's components so that $0 \leq \mu_{\tilde{A}^i}(x) + \nu_{\tilde{A}^i}(x) \leq 1$ for each $x \in X$. For any $x \in X$, the value of $\pi_{\tilde{A}^i}(x) = 1 - \mu_{\tilde{A}^i}(x) - \nu_{\tilde{A}^i}(x)$.

This is known as the level of uncertainty of the element $x \in X$ with respect to the intuitionistic fuzzy set $\tilde{A}^i$.

4.1 Operations on Some Neutrosophic Number and Neutrosophic Sets

4.1.1 Single valued neutrosophic number

Consider two single-valued neutrosophic numbers

$$\tilde{A}_1 = (T_1, I_1, F_1) \quad \text{and} \quad \tilde{A}_2 = (T_2, I_2, F_2) \tag{1}$$

Then, the operations for SVNNs are defined as below:

i. $\tilde{A}_1 \oplus \tilde{A}_2 = \langle T_1 + T_2 - T_1T_2,\ I_1I_2,\ F_1F_2 \rangle$

ii. $\tilde{A}_1 \otimes \tilde{A}_2 = \langle T_1T_2,\ I_1 + I_2 - I_1I_2,\ F_1 + F_2 - F_1F_2 \rangle$

iii. $\lambda \tilde{A}_1 = \left\langle 1 - (1 - T_1)^2,\ I_1^\lambda,\ F_1^\lambda \right\rangle$

iv. $\tilde{A}_1^2 = \left(T_1^2,\ 1 - (1 - I_1)^2,\ 1 - (1 - F_1)^2\right)$ where $\lambda > 0$

It is to be noted here that 0_n may be defined as follows:

$$0_n = \{\langle x,\ (0,\ 1,\ 1)\rangle : x \in X\} \tag{2}$$

4.1.2 Interval valued neutrosophic sets

Assume that ξ is a set of points (items) with x representing generic components in X. The truth-membership function of an interval is what defines an interval valued neutrosophic set A (IVNS A). Functions $T_A(x) = [T_A^L, T_A^U]$, $I_A(x) = [I_A^L, I_A^U]$, $F_A(x) = [F_A^L, F_A^U]$ all pertain to interval indeterminacy-membership. For each point $x \in X$, $T_A(x), I_A(x), F_A(x) \subseteq [0,1]$. An IVNS A may express as:

$$\mathbf{A} = \{x : T_A(x),\ I_A(x),\ F_A(x),\ x \in \xi\} \tag{3}$$

Numerical Example: Assume that $X = \{x_1, x_2, x_3\}$, x_1 is capability, x_2 trustworthiness, x_3 price. The values of x_1, x_2 and x_3 are in $[0, 1]$. They are obtained from questionnaire of some domain experts and the result can be obtained as the degree of good, degree of indeterminacy and the degree of poor. Then an interval neutrosophic set can be obtained as:

$$A = \begin{cases} (x_1,\ [0.5, 0.3],\ [0.1, 0.6],\ [0.4, 0.2]) \\ (x_2,\ [0.3, 0.2],\ [0.4, 0.3],\ [0.4, 0.5]) \\ (x_3,\ [0.6, 0.3],\ [0.4, 0.1],\ [0.5, 0.4]) \end{cases} \tag{4}$$

4.1.3 Multivalued neutrosophic fuzzy set

Let X be a space of points (items) and x representing a generic member in X, multi-valued *neutrosophic* sets.

The X-dimensional set A is defined by the truth-membership function $\hat{T}_A(x)$, the indeterminacy-membership function $\hat{I}_A(x)$, and the falsity-membership function $\hat{F}_A(x)$. One way to describe multi-valued *neutrosophic* sets is as follows:

$$A = \left\{ \left\langle x, \hat{T}_A(x), \hat{I}_A(x), \hat{F}_A(x) \right\rangle \mid x \in X \right\} \tag{5}$$

where $\hat{T}_A(x) \in [0, 1]$, $\hat{I}_A(x) \in [0, 1]$, $\hat{F}_A(x) \in [0, 1]$, are sets of finite discrete values, and satisfy the condition: $0 \leq \gamma, \eta, \xi \leq 1.0$, $\gamma^+ + \eta^+ + \xi^+ \leq 3$, $\gamma \in \hat{T}_A(x)$, $\eta \in \hat{I}_A(x)$, $\xi \in \hat{F}_A(x)$ $\gamma^+ = \sup \hat{T}_A(x)$, $\eta^+ = \sup \hat{I}_A(x)$, $\xi^+ = \sup \hat{F}_A(x)$ For the sake of simplicity, $A = \left\langle \hat{T}_A, \hat{I}_A, \hat{F}_A \right\rangle$ is called a multi-valued neutrosophic number.

When there is just one value for $\hat{T}_A(x), \hat{I}_A(x), \hat{F}_A(x)$, the sets of multi-valued *neutrosophic* functions are called *single-valued neutrosophic functions*.

The multi-valued *neutrosophic* sets are considered double hesitant fuzzy sets if $\hat{T}_A(x) = \varnothing$.

The multi-valued *neutrosophic* sets are confused fuzzy sets if $\hat{T}_A(x) = \hat{F}_A(x) = \varnothing$.

5 Methodology

5.1 Computational Complexity Analysis

The proposed hybrid model includes additional computational steps due to constraint handling and fuzzy optimization. Nevertheless, it remains computationally feasible and performs efficiently for medium-sized datasets. The increase in complexity is justified by significant gains in prediction accuracy and reduced uncertainty. Here we will discuss about the existing and proposed methodology, based on the use of Kuhn-Tucker's necessary conditions to estimate the parameters of the non-linear fuzzy regression model. The Kuhn-Tucker conditions are a set of necessary conditions for the optimality of a non-linear programming problem, and they can be used to estimate the parameters of a non-linear fuzzy regression model.

5.2 Kuhn-Tucker necessary conditions (KTNC)

An optimal solution to a nonlinear programming issue can be found by using the Kuhn-Tucker Necessary Conditions (KTNC). For a given issue, the provided equations symbolize the KTNC. The Kuhn-Tucker criteria, a collection of requirements for a nonlinear programming problem to have a local minimum, will be utilized to estimate the hybrid model's parameters. To guarantee that the estimated parameters are optimum and meet the issue constraints, the Kuhn-Tucker criteria offer a systematic way to addressing the optimization problem. We shall describe in depth the hybrid Neutrosophic Nonlinear Fuzzy Regression Model and its parameter estimation using the Kuhn-Tucker criteria in the sections that follow.

$$u_{1i}\left[2\sum_{i=1}^{n}\left(\ln y_i - \hat{\beta}_1 \ln x_{1i} - \hat{\beta}_2 \ln x_{2i}\right)\cdot\left(-\frac{\hat{\beta}_1}{x_{1i}}\right)\right] - \sum_{i=1}^{n} v_{1i}x_{1i} + \sum_{i=1}^{n} w_{1i}x_{1i} = 0 \tag{6}$$

$$u_{2i}\left[2\sum_{i=1}^{n}\left(\ln y_i - \hat{\beta}_1 \ln x_{1i} - \hat{\beta}_2 \ln x_{2i}\right)\cdot\left(-\frac{\hat{\beta}_2}{x_{2i}}\right)\right] - \sum_{i=1}^{n} v_{2i}x_{2i} + \sum_{i=1}^{n} w_{2i}x_{2i} = 0 \tag{7}$$

Stationarity criteria, expressed as these equations, guarantee that the Lagrangian function's gradient is zero.

$$\sum_{i=1}^{n} u_{1i} = 1, \quad \sum_{i=1}^{n} u_{2i} = 1 \tag{8}$$

The normalization criteria, represented by these equations, guarantee that the sum of the weights u_{1i} and u_{2i} is 1.

$$\sum_{i=1}^{n}\left(\ln y_i - \hat{\beta}_1 \ln x_{1i} - \hat{\beta}_2 \ln x_{2i}\right)^2 \leq 0 \tag{9}$$

Inequality is a concavity constraint, meaning it guarantees a concave objective function.

$$-\sum_{i=1}^{n} v_{1i}x_{1i} = 0, \qquad \sum_{i=1}^{n} w_{1i}x_{1i} = 0 \tag{10}$$

$$-\sum_{i=1}^{n} v_{2i}x_{2i} = 0, \qquad \sum_{i=1}^{n} w_{2i}x_{2i} = 0 \tag{11}$$

$$|x_{1i}| \leq 0, \quad |x_{2i}| \leq 0$$

Either the constraint is active (v_i or w_i is nonzero) or the Lagrange multiplier is zero, as stated by these equations, which provide the complimentary slackness criteria.

$$u_i \left[\sum_{i=1}^{n} \left(\ln y_i - \hat{\beta}_1 \ln x_{1i} - \hat{\beta}_2 \ln x_{2i} \right)^2 \right] = 0 \tag{12}$$

The goal of using the KTNC is to discover the best solution to a nonlinear programming problem while meeting certain requirements, such as stationarity, normalization, inequality, non-negativity, and the redundant equation.

$$u_i \geq 0, \quad u_{1i} \geq 0, \quad u_{2i} \geq 0, \quad i = 1, \ldots, n \tag{13}$$

$$u_i \geq 0, \quad u_{1i} \geq 0, \quad u_{2i} \geq 0, \quad i = 1, \ldots, n \tag{14}$$

5.3 Estimation the parameters of quadratic regression using Kuhn-Tucker necessary conditions (KTNC) [4]

Any non-linear regression model that incorporates a quadratic factor is known as a quadratic regression model. The following is the format of a quadratic regression model used in KTNC studies: The equation

$$Y_i = \hat{A}_1 X_i + \hat{A}_2 X_i^2 + \varepsilon_i \tag{15}$$

holds for all values of $i = 1, 2, \ldots, n$.

Assuming that the domain of the input variables is the universe of discourse, they estimated the regression parameters as Y_i, X_i, and $(\hat{A}_1, \hat{A}_2)$ are vectors of output variables and input variables, respectively.

The objective is to find the values of X_i that minimize the sum of squared errors when both the observed and predicted responses are based on a quadratic model, with the constraint that the predictor variables cannot be negative. Here is one way to express the problem:

$$\begin{aligned} &\min Q = \sum_{i=1}^{n} \varepsilon_i^2, \quad i = 1, 2, \ldots, n \\ &\text{s.t.} \quad \sum_{i=1}^{n} \left[Y_i - \left(A_1 X_i + A_2 X_i^2 \right) \right]^2 \leq Q, \quad X_i \geq 0 \end{aligned} \tag{16}$$

the KTNC for this problem express in the form:

$$u_i \left[-2 \sum_{i=1}^{n} \left(Y_i - (\hat{A}_1 X_i + \hat{A}_2 X_i^2) \right) (\hat{A}_1 + 2\hat{A}_2 X_i) \right] - \sum_{j=1}^{m} v_j X_j = 0 \tag{17}$$

According to this equation, the Lagrangian function's gradient must be zero in order for there to be stationarity.

$$\sum_{i=1}^{n} u_i = 1 \tag{18}$$

With this formula, we know that the total of all u_i weights is 1.

$$\sum_{i=1}^{n} \left[Y_i - \left(\hat{A}_1 X_i + \hat{A}_2 X_i^2 \right) \right]^2 \leq 0 \tag{19}$$

The goal function must be concave in order to satisfy this inequality constraint, which is a concavity requirement.

$$-\sum_{j=1}^{m} v_j X_j = 0 \tag{20}$$

$$X_j \geq 0 \tag{21}$$

By definition, X_j, u_i, and v_j must not be negative under these circumstances. Since the preceding equations already state this, it is unnecessary to state it again.

$$u_i \left[\sum_{i=1}^{n} (Y_i - (\hat{A}_1 X_i + \hat{A}_2 X_i^2))^2 \right] = 0 \tag{22}$$

$$u_i \geq 0, \quad i = 1, 2, \ldots, n \tag{23}$$

$$v_j \geq 0, \quad j = 1, 2, \ldots, m \tag{24}$$

To summarize, estimation of the quadratic model's parameters $\hat{A}_1$ and $\hat{A}_2$ using KTNC-based regression curve estimators entails solving the redundant equation, non-negativity conditions, inequality constraint, complementary slackness condition, normalization condition, and stationarity condition.

Instead of purely using logarithmic or polynomial regression, consider a hybrid regression model where the independent variables (x_{1i}, x_{2i}) are modeled using a combination of logarithmic and polynomial terms. This would allow for capturing both multiplicative relationships (log terms) and non-linear interactions (polynomial terms). According to [4].

$$\mu(\tilde{x}_i) = \frac{\mu^L(\tilde{x}_i) + \mu^U(\tilde{x}_i)}{2} \tag{25}$$

$$\gamma(\tilde{x}_i) = \frac{\gamma^L(\tilde{x}_i) + \gamma^U(\tilde{x}_i)}{2} \tag{26}$$

$$\nu(\tilde{x}_i) = \frac{\nu^L(\tilde{x}_i) + \nu^U(\tilde{x}_i)}{2} \tag{27}$$

Let the truth-membership function $T_{\tilde{A}}(x)$, the indeterminacy-membership function $I_{\tilde{A}}(x)$, and the falsity-membership function $F_{\tilde{A}}(x)$ be non-linear and defined as follows:

$$T_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a}{a'-a}, & a' \le x < a \\ 1, & x = a \\ \dfrac{b-x}{b-a}, & a < x \le b \\ 0, & \text{otherwise} \end{cases} \tag{28}$$

$$I_{\tilde{A}}(x) = \begin{cases} \dfrac{a-x+(x-a)}{a'-a}, & a' \le x < a \\ 1, & x = a \\ \dfrac{x-a+(b-x)}{b-a}, & a < x \le b \\ 0, & \text{otherwise} \end{cases} \tag{29}$$

$$F_{\tilde{A}}(x) = \begin{cases} \dfrac{a-x+(x-a)}{a'-a}, & a' \le x < a \\ 1, & x = a \\ \dfrac{x-a+(c-x)}{c-a}, & a < x \le c \\ 0, & \text{otherwise} \end{cases} \tag{30}$$

5.4 Structured approach to build a hybrid Neutrosophic Non-linear Fuzzy Regression Model using the Kuhn-Tucker conditions

A hybrid Neutrosophic Nonlinear Fuzzy Regression Model utilizing the Kuhn-Tucker conditions will be introduced in this subsection. The technique will be constructed. In the presence of uncertainty or imprecision in the data, it becomes much more difficult to estimate the parameters of a nonlinear regression function. It is possible that conventional regression models will produce erroneous results because they cannot account for the intricate nature of the interactions between the variables. We want to find out how well the predictor variables and responder variable relate to each other. In order to confront these obstacles, we suggest a hybrid Neutrosophic Nonlinear Fuzzy Regression Model that integrates the advantages of fuzzy set theory, nonlinear regression, and Neutrosophic logic.

$$Y_i = \hat{A}_1 X_i + \hat{A}_2 X_i^2 + \varepsilon_i \tag{31}$$

This model captures the nonlinear interactions between the variables, allowing it to manage uncertain and imprecise data and deliver more accurate forecasts. The equation of model can be expressed as

$$y_i = f(x_i; \beta) + \varepsilon_i \tag{32}$$

where $f(x_i; \beta)$ is the nonlinear regression function, β represents the parameters to estimate, and ε_i is the error term. For neutrosophic fuzzy sets, we extend this to:

$$y'_i = f(\tilde{x}_i; \tilde{\beta}) + \epsilon'_i \tag{33}$$

where $y'_i = \left(T_{\tilde{A}}(x), I_{\tilde{A}}(x), F_{\tilde{A}}(x)\right)$ and β' is a neutrosophic fuzzy number for the model parameters.

5.4.1 *Objective function*

The objective is to minimize the spread of truth, indeterminacy, and falsity components between observed and predicted values. The objective function is:

$$\min_{\beta'} \sum_{i=1}^{n} \Big[(T_{\tilde{y}_i} - T_{\text{obs}_i})^2 + (I_{\tilde{y}_i} - I_{\text{obs}_i})^2 + (F_{\tilde{y}_i} - F_{\text{obs}_i})^2 \Big] \tag{34}$$

Adding constraint (KKT-conditions). The KKT conditions come into play when we add inequality and equality constraints to the problem. These conditions generalize the Lagrange multipliers method for problems with both equality and inequality constraints.

5.4.2 *Kuhn-Tucker Conditions*

Suppose we introduce the following constraints to the model:

1. Inequality constraint: $g(\tilde{\beta}) \le 0$.
2. Equality constraint: $h(\tilde{\beta}) = 0$.

The Lagrangian for this constrained optimization problem is:

$$\mathcal{L}(\tilde{\beta}, \lambda, \mu) = \sum_{i=1}^{n} \Big[(T_{\tilde{y}_i} - T_{\text{obs}_i})^2 + (I_{\tilde{y}_i} - I_{\text{obs}_i})^2 + (F_{\tilde{y}_i} - F_{\text{obs}_i})^2 \Big] + \lambda^T g(\tilde{\beta}) + \mu^T h(\tilde{\beta}) \tag{35}$$

where $\lambda \geq 0$ are the Lagrange multipliers for the inequality constraints and μ are the Lagrange multipliers for the equality constraints.

The KKT conditions for this optimization problem are:

1. The gradient of the Lagrangian with respect to $\tilde{\beta}$ must vanish:
$$\nabla_{\tilde{\beta}} \mathcal{L}(\tilde{\beta}, \lambda, \mu) = 0 \tag{34}$$
2. The original constraints must hold: $g(\tilde{\beta}) \leq 0, \quad h(\tilde{\beta}) = 0.$
3. The Lagrange multipliers for the inequality constraints must be non-negative: $\lambda \geq 0$.
4. For each j, either $\lambda_j = 0 \quad \text{or} \quad g_j(\tilde{\beta}) = 0.$

5.4.3 *Hybrid model:*

The hybrid model combines neutrosophic fuzzy regression and the KKT conditions. Here's the structure of the hybrid model:

- The model estimates the parameters $\tilde{\beta}$ subject to the KKT constraints.
- The objective function is the sum of the squared neutrosophic error terms.

The constraints ensure that the parameters $\tilde{\beta}$ respect the model's physical, logical, or operational boundaries (e.g., non-negativity of certain parameters, boundary conditions).

5.4.4 *Steps for Solving the Hybrid Model:*

1. **Initialize:** Start with an initial guess for the parameter set $\tilde{\beta}_0$. This can be done using some heuristic method or based on prior knowledge.
2. **Compute the Objective Function:** For each iteration, compute the objective function, which is the sum of the squared errors between the predicted neutrosophic fuzzy values $\tilde{y}_i$ and the observed neutrosophic fuzzy values $\tilde{y}_{\text{obs},i}$.
$$\text{Objective} = \sum_{i=1}^{n} \Big[(T_{\tilde{y}_i} - T_{\text{obs},i})^2 + (I_{\tilde{y}_i} - I_{\text{obs},i})^2 + (F_{\tilde{y}_i} - F_{\text{obs},i})^2 \Big] \tag{36}$$
3. **Check Feasibility:** Verify that the current set of parameters $\tilde{\beta}$ satisfies the primal feasibility conditions $g(\tilde{\beta}) \leq 0, h(\tilde{\beta}) = 0$. If the constraints are not satisfied, adjust the parameters accordingly.
4. **Compute Lagrange Multipliers:** Solve the KKT conditions to obtain the multipliers λ and μ.
5. **Update Parameters:** Update the parameters $\tilde{\beta}$ using an iterative method such as gradient descent or sequential quadratic programming (SQP). This involves minimizing the Lagrangian and ensuring the solution satisfies the constraints and KKT conditions.
6. **Check Convergence:** Check whether the changes in the parameter estimates $\tilde{\beta}$ between iterations are within a predefined tolerance. If the solution has converged, stop; otherwise, return to Step 2.
7. **Output Solution:** Once the parameters have converged, the final estimates $\tilde{\beta}^*$ are the optimal neutrosophic fuzzy regression coefficients subject to the constraints.

6 Numerical examples

In this section, we apply the KKT conditions to minimize the spread (T, I, F) of the predicted values in neutrosophic fuzzy regression. This involves solving the Lagrangian with given constraints (e.g., parameter non-negativity, specific ranges for coefficients). The hybrid model involves both fuzzy regression and the KKT conditions but adds another layer of minimizing the spread in membership and non-membership functions while introducing fuzzification in the prediction process. This hybrid approach offers finer control of the fuzziness and spread minimization.

Although this example is illustrative, it reflects real-world uncertainty characteristics. Future research may expand validation using benchmark datasets to strengthen the model's practical credibility.

6.1 Neutrosophic non-linear fuzzy regression analysis

To solve the existing and hybrid model, we use a small dataset presented in Table 1, which includes the truth, indeterminacy, and falsity values of the data. The predictor variable x is used to make predictions about the response variable y.

Table 1 shows how two models—the Hybrid Neutrosophic Nonlinear Fuzzy Regression Model and the Kuhn-Tucker (KKT) model—predicted values for the response variable y. It also includes the true values of y as well as the truth (T), indeterminacy (I), and falsity (F) values used to evaluate the models' performance. The discrepancy between the predicted and real values is less in the Hybrid model, suggesting

Table 1. Dataset overview with neutrosophic components and model predictions.

x	y	Truth (T)	Indeterminacy (I)	Falsity (F)	Predicted (KKT)	Predicted (Hybrid)
1.00	9.97	9.75	0.49	0.63	10.06	9.17
1.11	4.82	5.18	0.39	0.09	5.42	4.52
1.23	14.01	15.49	0.58	0.40	13.20	14.02
1.34	24.23	23.71	0.41	0.69	26.32	24.26
1.46	8.25	7.45	0.48	0.96	7.25	8.03
1.57	9.98	9.48	0.21	0.27	8.76	10.29
1.68	29.96	30.88	0.41	0.38	31.12	29.43
1.80	23.83	24.16	0.95	0.52	24.62	23.76
1.91	13.57	13.04	0.12	1.41	14.20	13.63
2.03	25.94	26.45	0.38	0.32	26.56	26.19

that it provides more accurate y predictions than the KKT model. While the Hybrid model is more cautious, the KKT model tends to exaggerate y values in some instances. The reduced discrepancies between the actual and anticipated values show that the Hybrid model does a better job of capturing the data's nonlinearity.

Model performance may be assessed using a number of measures, including R-squared values, mean absolute error (MAE), and mean squared error (MSE). To better understand the models' performance and find places to improve, these indicators might be useful. Here is the comparative analysis showing the performance of the KKT-based model versus the hybrid model, along with the percentage improvement for each metric:

Note: RMSE (Root Mean Squared Error) measures the average prediction error—lower values indicate higher accuracy. Spread refers to the range across the truth, indeterminacy, and falsity components. Reducing this spread means the model provides more confident and precise predictions.

This Table 2 compares the KKT-Based Model to the Hybrid Neutrosophic Nonlinear Fuzzy Regression Model utilizing a number of different criteria for evaluation. The hybrid model shows significant improvements, especially in reducing the maximum spread (22.09% improvement) and RMSE (22.23% improvement).

Table 2. Comparative performance of KKT-Based and hybrid models.

Metric	KKT-Based Model	Hybrid Model	Improvement (%)
Average Truth Deviation	1.06	0.85	19.57
Average Indeterminacy	0.40	0.36	10.00
Average Falsity Deviation	0.61	0.52	15.00
Maximum Spread	5.14	4.00	22.09
RMSE	1.37	1.07	22.23

In Table 2, we can see how two models—the Hybrid Neutrosophic Nonlinear Fuzzy Regression Model and the Kuhn-Tucker (KKT) model—predicted values for the response variable y. Also included in Table 2 are the true values of y and the values used to evaluate the models' performance: truth (T), indeterminacy (I), and falsity (F). The discrepancy between the predicted and real values is less in the Hybrid model, suggesting that it provides more accurate y predictions than the KKT model. While the Hybrid model is more cautious, the KKT model tends to exaggerate y values in some instances. The reduced discrepancies between the actual and anticipated values show that the Hybrid model does a better job of capturing the data's nonlinearity. We may compute metrics like R-squared values, mean absolute error (MAE), and mean squared error (MSE) to assess the models' performance.

Using these estimates, we can write the final quadratic model's equation (29) as:

$$Y_i = 2.53\, X_i + 3.17\, X_i^2 + \varepsilon_i \tag{37}$$

This model can be used to predict the response variable y for given values of the predictor variable x. The average truth deviation is a statistic that quantifies how much of a departure there is from the real values of the response variable y compared to the projected values. Better performance is indicated by a lower value.

The average degree of uncertainty in the forecasts is measured by this statistic. Predictions with a lower number are more likely to be accurate.

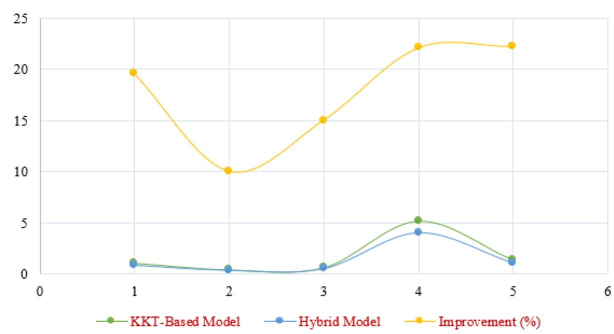

Figure 1. Comparative analysis of hybrid and existing method.

The average deviation of the false values from the projected values of the response variable y is called the average falsity deviation. Better performance is indicated by a lower value shows in Figure 1. The greatest spread is a statistic that quantifies how much of a deviation there is between the actual and projected values of the response variable y. Better performance is indicated by a lower value. By taking the square root of the average of the squared differences between the predicted values and the real values of the response variable y, we get the Root Mean Squared Error (RMSE). Better performance is indicated by a lower value. Both truth deviation and falsity are also reduced, demonstrating the hybrid model's ability to minimize uncertainty better than the KKT-based model.

7 Conclusion

This study has demonstrated the effectiveness of a hybrid fuzzy nonlinear regression model that incorporating the KKT optimization method and Neutrosophic Fuzzy sets has been very effective. It is quite evident that the proposed hybrid method tackles challenges of uncertainty components, truth deviation, and indeterminacy moving beyond the original KKT based model. Starting a samples comparative correlation model analysis went on to exhibit rather large decrease in the maximum spread of uncertainty – by 22.09% that seemed astonishing and great improvement in prediction accuracy as manifested by 22.23% lower RMSE. The results demonstrate the ability of the hybrid model in dealing with complex nonlinear system where uncertainty is a vital aspect. However, this research also shows that by making more accurate and robust predictions with smaller spreads one is able to steer future explorations towards fields where a high degree of accuracy is required in the presence of uncertainties like material processing, machine learning, engineering systems and many others. Essential improvements in making decisions in uncertain environments are made by the hybrid fuzzy regression model. The proposed hybrid model is highly applicable in real-world scenarios involving uncertainty, such as engineering system design, manufacturing process optimization, and decision-making under incomplete data. Its ability to handle indeterminacy and minimize spread makes it a valuable tool in environments where conventional regression models fail to provide reliable estimates.

Data Availability Statement

Data will be made available on request.

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Conflicts of Interest

The authors declare no conflicts of interest.

Ethical Approval and Consent to Participate

Not applicable.

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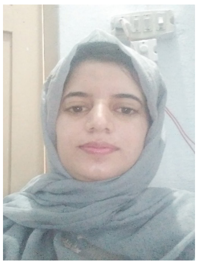
Rakesh Kumar received his Ph.D., M.Phil. and M.sc in Mathematics from the Department of Mathematics, Lovely Professional University, India. He is currently serving as an Assistant Professor in the Department of Mathematics, Lovely Professional University. He is supervising many Ph.D. and master's scholars. He authored many research articles in international journals of repute and reviewer of many international journals. He is an academic editor and editorial board member of many international journals. His research interests consist of supply chain management, numerical optimization, quality control, multicriteria decision making, and fuzzy optimization, fuzzy clustering, fuzzy neural network and fuzzy regression models. (Email: rakeshmalhan23@gmail.com)



Mufala Khan received her B.Sc. in Mathematics from Jammu University and her M.Sc. in Mathematics from Baba Ghulam Shah Badshah University (BGSBU), Rajouri. She is currently pursuing a Ph.D. in Mathematics at Lovely Professional University, Phagwara, Punjab, India. (Email: Mufalakhan7861@gmail.com)

Gaurav Dhiman is a highly accomplished academician and researcher with an impressive track record of excellence in computer science. His research has been recognized at the highest level, as he has been named one of the world's top researchers by Clarivate Analytics as world's top 1% and by Stanford University, USA list of world's top 2% scientists prepared by Elsevier. He is the senior member of IEEE. He has also received accolades for his outstanding reviewer work from Knowledge-Based Systems (Elsevier). He has authored over 300 peer-reviewed research papers, all of which are indexed in SCI-SCIE, and 10 international books. He is currently serving as a guest editor for more than forty special issues in various peer-reviewed journals. He is an Editor-in-Chief of the ICCK Transactions on Machine Intelligence and International Journal of Modern Research (IJMORE), Co-Editor-in-Chief of the International Journal of Electronics and Communications Systems (IJECS) and International Journal of Ubiquitous Technology and Management (IJUTM). He is an Associate Editor of IEEE Transactions on Industrial Informatics, IET Software (Wiley), Expert Systems (Wiley), IEEE Systems, Man, and Cybernetics Magazine, Spatial Information Research (Springer), and more. He is also an Academic Editor of Computational and Mathematical Methods in Medicine (Hindawi), Scientific Programming (Hindawi), Mobile Information Systems (Hindawi), and Review Editor of Frontiers in Energy Research- Fuel Cells, Frontiers in High-Performance Computing- Parallel and Distributed Software, Frontiers in Artificial Intelligence- Pattern Recognition, and more. His research interests span a wide range of topics including Artificial Intelligence, Internet of Things, Machine-learning, Deep-learning, Single-, Multi-, Many-objective optimization (Bio-inspired, Evolutionary, and Quantum), Soft computing (Type-1 and Type-2 fuzzy sets), Power systems, and Change detection using remotely sensed high-resolution satellite data. He has published his research in various reputed journals, including IEEE, Elsevier, Springer, Wiley, Taylor and Francis, among others. (Email: gdhiman0001@gmail.com)